Advanced Composites

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Lecture - 50
Principle of Virtual Work

Hello, welcome to Advanced Composites. Today is the second day of the ongoing week, that is the 9th week of this course and today, we will introduce a new topic, which is the Principle of Virtual Work. We will discuss this principle in context of are attempt to solve different types of problems related to finite plates. Now, earlier we had shown that we can solve some types of finite plate problems when we are able to select, functions for u v and w in such a way that these functions identically and exactly satisfy all the boundary conditions as well as the governing differential equation. But guessing such functions, when plates are of a complicated shape or when the lamination sequence is not very simple. In such situations, guessing such functions is most of the times not possible and in some cases, it is not necessarily obvious.

So, what do we do in such cases? Well we use this idea of principle of virtual work and it enables to us to solve a much larger, category of problems.

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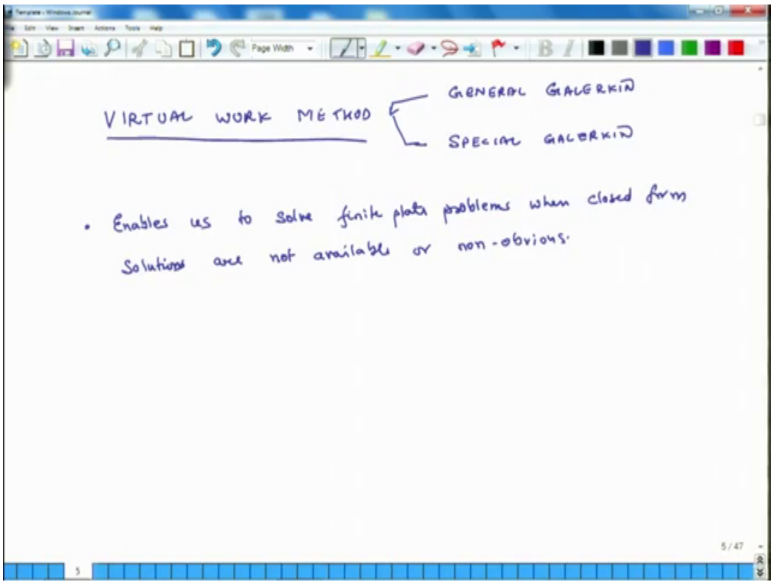

So, what we will discuss today is we will actually introduce, virtual work and the method and here, we will discuss two methods. So one is known as General Galerkin method and the other one is a Special Galerkin method. So, once again what is the advantage of these, this method? It enables us to solve finite plate problems, when closed form solutions are not available or non obvious.

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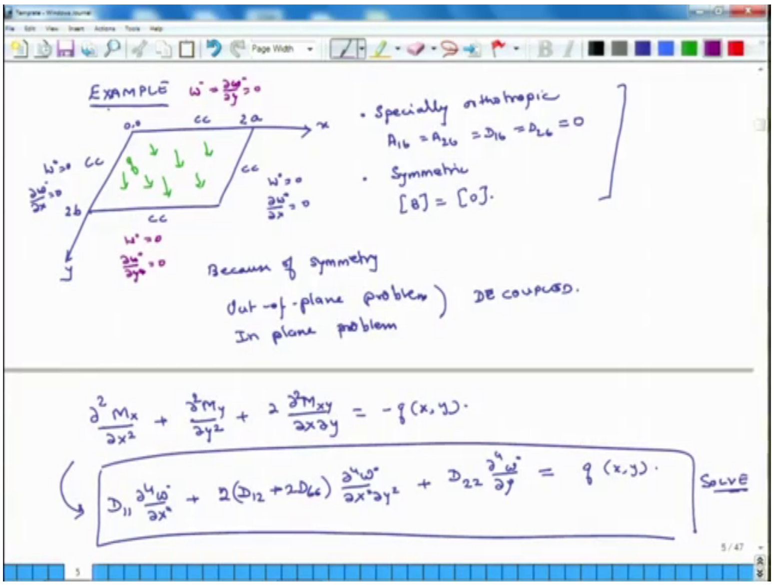

So, we will start with an example. So, this is a plate ok. So, this is 0, 0. So, this is my x axis, this is the y axis, the plate is k meters long and D meters wide and let us say the plate is clamped on all the four edges and we say that this is such a plate that it is a specially orthotropic. It is a specially orthotropic and so, if it is a specially orthotropic when, what is that mean? That A 16 is equal to A 26 is equal to D 16 is equal to D 26 is equal to 0 and we also say that the lamination sequence is symmetric and, because the lamination sequence is symmetric the B matrix is 0.

So, I will make a small correction here, we will say that the plate is 2 a long and 2 b y not a and b, it is dimensions are 2 a and 2 b ok. So, because of these conditions, because of a special orthography and symmetry actually it is primarily, because the symmetry; so, it is not, I will correct that. So, because of symmetry out of plane and in plane problem these are de coupled.

So, I do not have to solve the in plane problem and out of plane problem in a combined way, because they are disconnected. So, I can solve the third equation, equilibrium equations separately and it is solution, does not influence the solution for first two differential equations which are as well as the in plane part of the problem and we also say that this plate is, have is, having some normal load and that is q.

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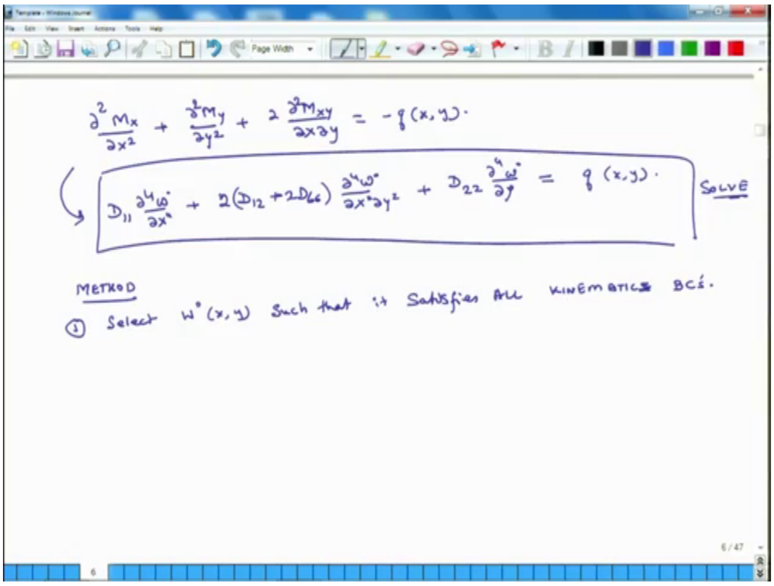

So, the out of plane problems governing differential equation is del 2 M x over del x square plus del 2 M y over del y square plus 2 del 2 M x y over del x del y equals minus q x y or if I express this M x M y and M x y in terms of ws. We have shown this earlier, this gives us D 1 1 del 4 over del x 4 plus 2 D 1 2 plus 2 D 6 6 del 4 over del x del y square. So, yeah plus D 2 2 del 4 w over del x del y 4 is equal to q x y.

So, this is the differential equation, we want to solve and we want to generate an expression for w which is based on this thing. So, here the method of virtual work is first step we select. So, what is the unknown in this equation, it is w naught? So, if w naught is unknown, we select w naught which is the function of sub x and y such that it satisfies all kinematic boundary conditions.

So, this is the significant difference. In the earlier approach, when we are solving for finite plate problem, we were selecting w which satisfied all the boundary conditions. Here, we are not satisfying all the boundary conditions, but we are satisfying those boundary conditions, which are related to kinematics of the problem. So, kinematics what does it mean that which tell, which are related to displacements and their derivatives ok.

So, in the clamped condition what are the boundary conditions, the boundary condition here is w naught is equal to 0 and del w is equal to 0 on this edge and also on this edge w naught is equal to 0 and del w over del x is equal to 0. Basically, the slope is 0 in x direction and w 0 in x direction on this edge and then on the other two edges w naught is equal to del 0 and del w over del y equals 0 and on that edge also w naught is equal to del w over del y is equal to 0 ok.

So, these are kinematic boundary conditions, if there were boundary conditions involving Ms or N s or M x y N x y, then those would be non kinematic boundary conditions. These boundary conditions, because they are directly related to u v and w we call them kinematic boundary conditions. So, we have to select a function, which satisfies all the kinematic boundary conditions. In earlier method which we had discussed, the function had to satisfy all the boundary conditions not only kinematic boundary conditions and also the governing differential equation.

Here, we are not saying that the formula or the relation for w need not satisfy the governing differential equation, it need not satisfy non kinematic boundary conditions, it only has to satisfy the kinematic boundary conditions. So, this is the first step. So, let us guess this. So, we will say that w x y is equal to, let us say w 1 1 times 1 minus cosine pi x over a times 1 minus cosine pi y over b.

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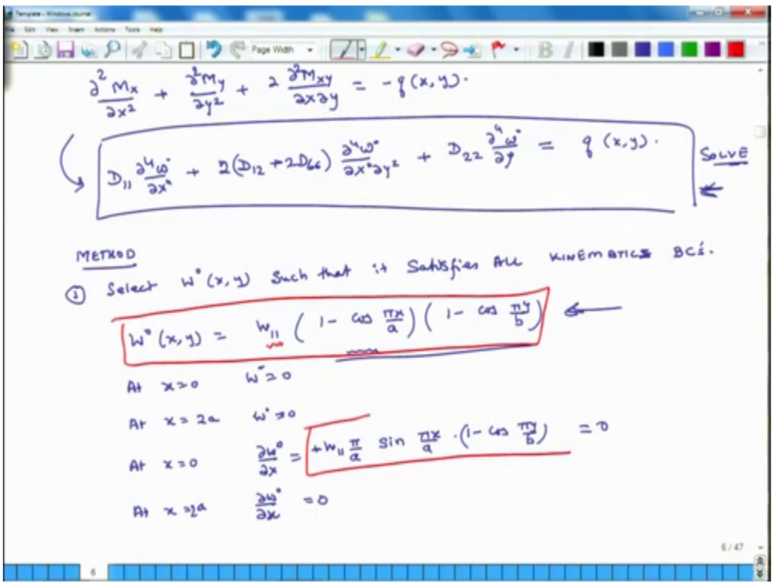

Let us see whether it satisfies the kinematic boundary conditions at x is equal to 0, at x is equal to 0 the boundary condition, that w should be 0 is satisfied, because then (Refer Time: 11:42) x is equal to 0, this term becomes 0. So, that is satisfied and at x is equal to 2 a I once again put x is equal to 2 a. So, cosine of 2 pi is again 1. So, this is also so, this is satisfied then what about slopes at x is equal to 0, the slope d w d x should be 0.

So, we see that del w over del x is equal to w 1 1 pi over a minus; so, minus this becomes plus sine pi x over a times 1 minus cosine pi x over a pi by over b excuse me. So, this is the slope in x direction. So, this is equal to 0 at x is equal to 0 and at x is equal to a x is equal to 2 a again when we put x is equal to 2 a sin of 2 pi 0. So, again this is equal to 0 ok. So, all the kinematic boundary conditions, these boundary conditions are satisfied.

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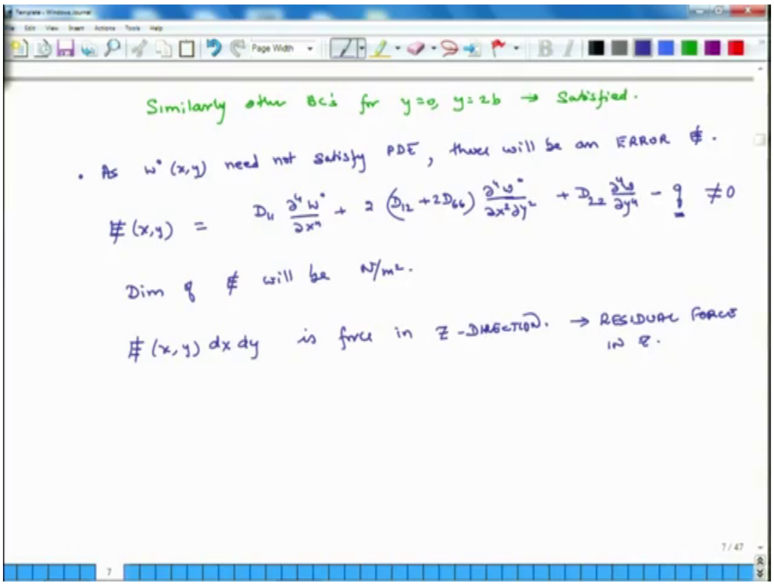

Now, we can and similarly other BC's for y is equal to 0 and y is equal to 2 b, they are also satisfied ok. So, first condition is satisfied. So, this is the first step. So, we are chosen a function, which satisfies all the kinematic boundary conditions. We do not care whether the function satisfies the governing differential equation or not and we also do not care that if there were some boundary conditions not related to u v and w and derivates of w.

You know if that, if these boundary conditions, where involving N s ms or N x y m x y then the relation for w need not satisfy those also. So, all the derivative boundary conditions are satisfied. So, now, we got to the next step as w x y need not satisfy the partial differential equation. So, if it was satisfying the partial differential equation, the left side and the right side of the equation would be identical and the difference between left and right side would be 0 right.

So, as w x y need not satisfy the partial differential equation, there will be an error, an error. So, we call it, we designate it as E and we put a cross mark on this. So what is the error? So, this error is the function of x y and that is equal to D 1 1 del 4. So, essentially what do, how do we calculate the error? We calculate the error by plugging in the governing differential equation, this is our governing differential equation, in this governing differential equation; we plug in the assumed relation for w.

So, in this governing differential equation, we have plugged in the assumed relation for w and, because the expression for w need not satisfy the partial differential equation it, may lead to an error and this error we designate it as E and then of course, crossed across it. So, this is the error and this if the expression for w was identically satisfying the differential equation, this error would be 0, but in this case, because it does not need not satisfy this need not be 0 ok. So, this there and it is a function of x and y, because w is a function of x and y. So, this is the second thing.

Now, let us look at what does this error mean? This error has a term q and q is the force in the z direction, distributed force in the z direction essentially, what this tells us is. So, what is the dimension of, this error the dimension of the error will be same as the dimension of q. So, dimension of error will be Newton's per meter square ok. So, force per unit area, the force per unit area.

So, if I multiply this force per unit area by a small element of area then I will get force in the z direction, because the dimension of E is Newton's per meter square direction of E is in the vertical direction, because q is in vertical direction. So, if I multiply this by small amount of area or some area, the overall error will represent an error in force in the z direction ok. So, error x y times d x d y is force in z direction. Now, what kind of force is this? If the expression for w was just the right one that it satisfied the differential equation.

This error would be 0, but it is in this case not necessarily 0. So, this is a residual force. So, it is a residual force in z direction, it is a residual force in z direction. Now, what we say is that if there is this residual force in the z direction and if I multiply it by some virtual displacement and I will explain what is a virtual displacement. So, if I, this is the residual force in z direction and if there is some virtual displacement in z direction then what is the product of virtual displacement and this error force, it will be work right.

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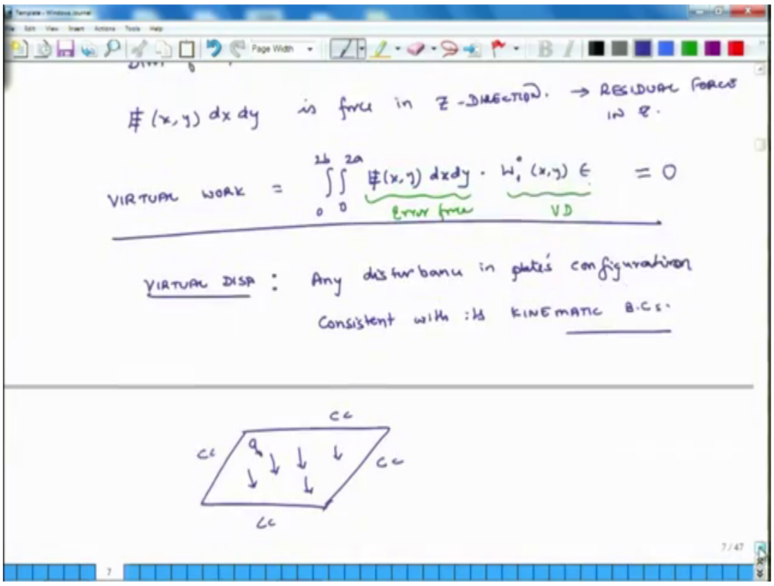

So; so, virtual work, virtual work in this case will be if I integrate this error times d x d y. So, this is the force in z direction and I multiplied this by a virtual displacement and I call it w 1. So, w was the actual displacement, this is w 1 and this is also a function of x and y. So, this is virtual work. So, I am integrating it from 0 to 2 a and 0 to 2 b and this force, this virtual displacement is extremely small. So, I multiply it by a parameter epsilon. So, this epsilon you can make it as small as possible 10 to the power of minus 20 10 to the power minus 30, it just a number ok, is just a number.

So, I multiply it by a parameter which is extremely small, because these virtual displacements, they have to be extremely small, extremely small. So, this is virtual work ok. Now, so, what is virtual work? Virtual work is the product of error force times virtual displacement and so, this is virtual displacement and this is error force. So, this is the product of, this thing ok. Now, we will explain what is virtual displacement. So, what is virtual displacement ?.

Virtual displacement is any disturbance in plates configuration, any disturbance in plates configuration consistent with it is kinematic BC's consistent, with it is kinematic BC's. So, what does it mean? So, suppose; so, this is the plate, it is clamped on all four sides and suppose, the plate is clamped and I, it is being acted upon virtual load q, you know not virtual load vertical load q, it is acted upon by this load q.

Then what is the; so, when it is so, this is the plate and it is at, it is experiencing some external load and as it experiences load, it gets deformed into some position and in this some position, if I disturb this plate very slightly and that slightly is assured by this parameter epsilon. What kind of disturbance? I can disturb, how can I disturb this? I can disturb it in any way, I can disturb it like this whatever shape does not matter any shape as long as I do not violate the kinematic boundary conditions; so, any function which does not violate the kinematic boundary condition.

So, in this case what are the kinematic boundary conditions that w 0 on all four sides and also del w del x is 0 on x is equal to 0 and x is equal to a and del w over del y is equal to 0 and x is equal to y is equal to 0 and y is equal b. So, any this disturbance in the configuration of the plate, which does not violate the kinematic boundary conditions, is the virtual displacement and it is a very small displacement, which I ensure through epsilon.

So, so, the principle of virtual works says that the body is going to be in equilibrium, if this virtual work is going to be equal to 0, this is the principle of virtual work ok. So, if the body is going to be in equilibrium then the integral of this virtual work over the domain of the thing has to be 0. So, now very quickly; so, we can so, we integrate it from 0 to 2 a and 0 to 2 b and what do we integrate error times virtual displacement.

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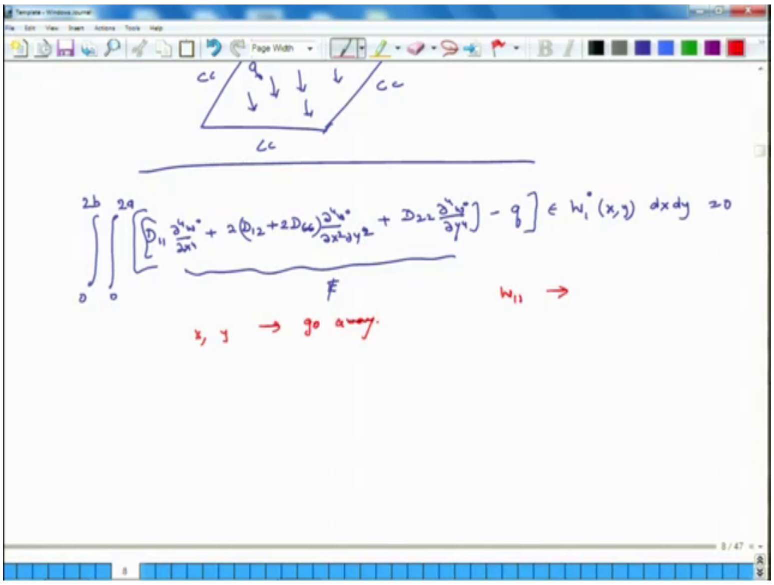

So, what is error? Error was D 1 1 del 4 w naught over del x 4 plus 2 D 1 2 plus 2 D 6 6 del 4 w naught over del x square del y square plus D 2 2 del 4 del y 4 minus q. So, this is the error now, and then I multiply it by virtual displacement, which is epsilon and any function. So; so, this is the virtual displacement and I can choose any function as long as it needs the kinematic boundary conditions.

I can choose even the same function, the similar function which I chose earlier, which is this thing. Does not matter right, I can choose the same function, I can chose the different function anyway. So, I have to choose some function w 1 and then I integrate it over the domain and I set this virtual work to be 0 and when I do this and when I integrate what will happen as I integrate over x and y, what will happen? Now, this w, what is this w naught? We had assumed w to be w was this thing ok.

So, here w 1 1 is unknown, if we know w 1 1, then we will be fine. So, that is what we are trying to find. So, when we do this integral, what will happen? w is a function of w 1 1 times 1 minus cosine and 1 minus cosine y over all that. So, here as I do the integral x y, they will go away, because when I integrate x and y. So, the only unknown in this equation will be w 1 1 and the only.

So, then I can solve for w 1 1 and get the solution and that will be an approximate solution. So, this is the overall principle of virtual work what we will do is, we will actually continue this discussion and solve this problem for, couple of examples. So, that you become much more clear about this whole thing and with that we close our discussion for today and tomorrow. We will continue with this discussion on virtual work.

Thank you.