Advanced Composites
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Lecture - 51
Virtual Work Method: Apply to Beam Problem

Hello. Welcome to Advanced Composites. Today is the third day of the ongoing week which is the 9th week of this course. Yesterday we started the discussion on the principle of virtual work in context of finding solutions for finite plate problems; where close form solutions or analytical solutions for not all that obvious.

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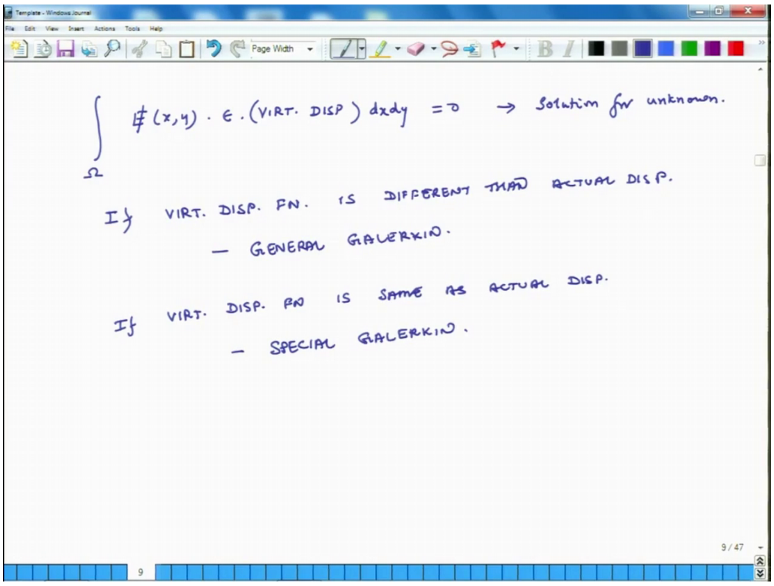

So, in that context we had said that get the solution we have to integrate error times virtual displacement. And, if I integrate it over the domain and equate it to 0, then I gets a solution for unknown. And we will actually do it so that things become clear. Now, I had said that, there are we will discuss 2 types of or 2 versions of this virtual work method.

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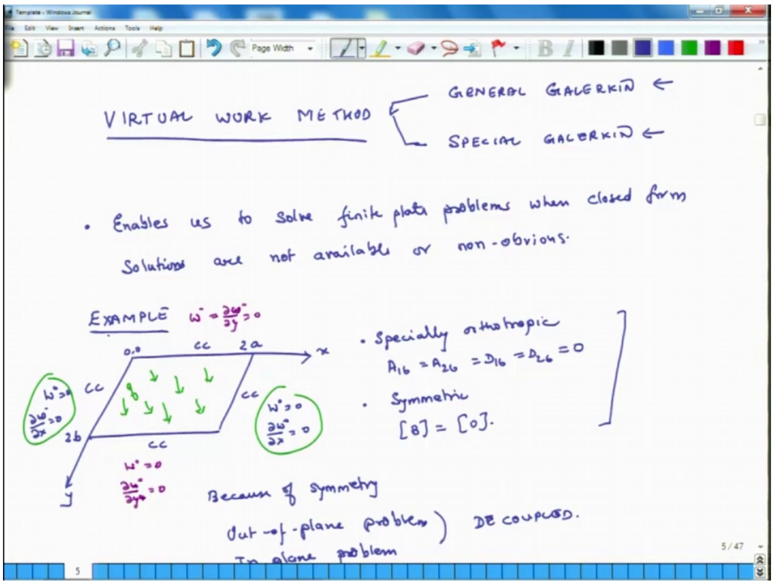

One is the general Galerkin method and the other one is special Galerkin method. So, just to distinguish between these 2, if virtual displacement function is different, then actual displacement if it is different, than that method is known as general Galerkin and if virtual displacement function is same as actual displacement, then we call it a special Galerkin, ok. So, these are the 2 approaches. So, let us first do an example.

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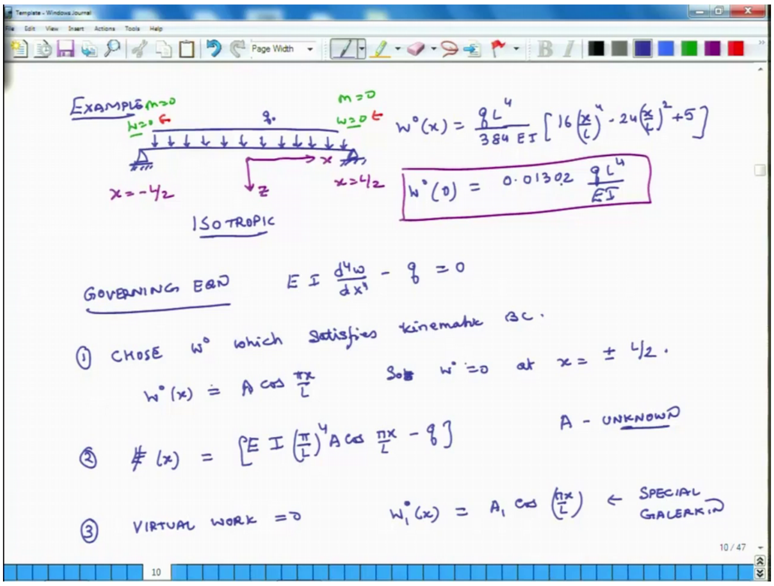

And we will start from a simple example. So, that things become clear so that simply. So, we will consider a the problem of a beam just a regular beam made up of isotropic material; which is uniformly loaded. So, the load intensity is q ok, q the original is in the centre, ok. And the vertical displacement is z. The overall length of the beam is L. So, this is x is equal to minus L over 2 and x is equal to L over 2

So, for this the exact close form solution from beam theory is that w naught; which is the function of x is equal to q L 4 divided by 384 times, excuse me there is a small error EI times 16 x over L 4 minus 24 x over L square plus 5. This is the general solution from beam theory; which you have done in your undergrad engineering classes, and w naught at 0, at 0 is basically I put x is equal to 0 in this case. So, it is 5 times 16 over 384. So, it is equal to 0.01302 q L 4 by EI. So, this is the exact solution.

Now, what we will do is, we will use principle of virtual work to solve this problem and see how accurate that solution is; because it is the principle of virtual work in general is gives us, approximate solution. So, this is an isotropic beam, we are not doing composite right now. So, what is the governing differential equation? The governing differential equation for a beam is EI d 4 w over d x 4 minus q is equal to 0. So, we have to solve this equation using principle of virtual work.

What are the boundary conditions? The boundary condition is what are the boundary condition? Boundary condition is w is equal to 0, and w is equal to 0 at z is equal to plus minus L over 2; so, w 0 at this end and w 0 at this end. What is the other boundary condition? Both the ends are pinned so, the moment is 0 at both the ends. So, m is equal to 0 and m is equal to 0 ok. So, the both are dependence. Now, what are the kinematic boundary conditions? The kinematic boundary condition is w is equal to 0 because it relates to displacement. The moment related boundary condition is not a kinematic boundary condition. So, what is the first step in developing the principle using principle of virtual work to solve this problem, is that we choose w which satisfies this boundary condition.

We choose w which satisfies this boundary condition. So, we choose it so, we guess we say that w is nothing but it is a function of x is equal to A cosine pi x over L, ok. So, A unknown, at x is equal to L over 2, it is cosine pi over 2. So, it is 0 and x is equal to minus L over 2, again it is cosine of minus pi over 2 it is 0. So, w naught is equal to 0 at x is equal to plus minus L over 2. So, all the kinematic boundary conditions are satisfied. And we do not care about the moment related boundary condition in this case we are not worried about that, we do not care about it. Second, we find the error using this function. So, error is we substitute this w in the governing differential equation. So, error in this case is a function of x, because there is no y, and this is equal to EI and I differentiate w 4 times. So, I get pi over L to the power 4 cosine pi x over L minus q, ok. So, that is the error, and of course this A here A. So, A is unknown we want to find A, if we know the A then we are good. So, this is my error.

The third thing is I have to now use this and create an expression for virtual work and equate that to 0. So, virtual work should be 0 and to find virtual work, I have to also assume a virtual displacement function. So, I say that virtual displacement function is w 1, and that is also a function of x. I can choose any function as long as it satisfies by kinematic boundary conditions. And I will use a similar function as A cosine pi x over L, because I will use the special Galerkin method, where the nature of virtual displacement and actual displacement is same, ok.

So, I say the w 1 x is equal to, let us say A 1, its amplitude cosine pi x over L. Now this virtual displacement also satisfies all the boundary conditions, kinematic boundary conditions. So, this is fine, ok. So, this is because I am using the special Galerkin method. So, if that is the case then virtual work is what integral from minus L over 2 to L over 2, error times virtual displacement. So, error times pi over L to the power of 4 times A cosine pi x over L minus q. This is the error times virtual displacement multiplied by the small parameters. So, that small parameter is epsilon, and the virtual displacement function is A 1 cosine pi x over L dx, ok.

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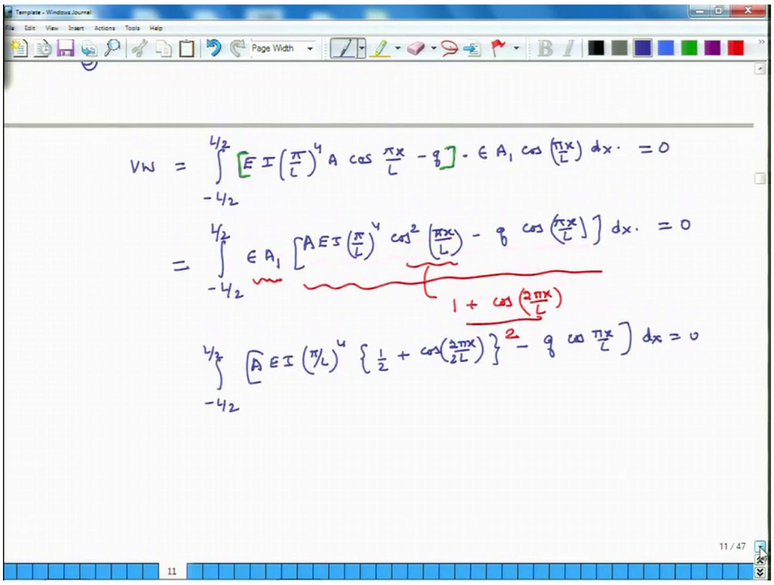

And this virtual work should be equal to 0. So, I process this further minus L over 2 to L over 2, and I get this bracket should be here, I remove this. So, this is epsilon A 1 EI pi over L to the power of 4 cosine square pi x over L minus q cosine pi x over L d x. And this should be equal to 0, ok.

Now, these E times epsilon is common to the whole thing so, I can ignore it, but later we will see in some cases we if this thing serves actually useful purposes. So, epsilon times A 1 in this case can be ignored, and if I integrate this entire function. So, what is cosine square pi x over L? This is nothing but 1 plus cosine 2 pi x over L divided by 2. Because cosine square theta is equal to 1 plus cosine 2 theta divided by 2, ok. So, I am going to ignore epsilon A 1 because it is common. So, I get minus L over 2 to L over 2.

Student: S p p p.

Yes, and there should be an A because that is what we are looking for. So, it is A EI pi over L to the power of 4 1 by 2 plus cosine 2 pi x over L over 2. So, it is 2 L minus q cosine pi x over L dx is equal to 0. So, when you do this whole integration and all that good stuff.

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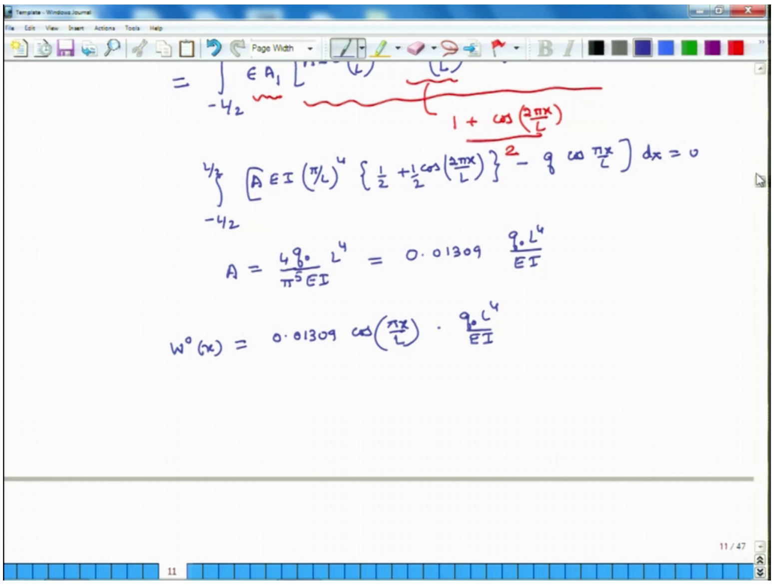

Basically you get from here A is equal to q naught divided by 4, q naught divided by pi to the power 5 EI.

Student: Why this is in (Refer Time: 15:50) by 2 L 2 pi x by 2 L, it should be.

Because, this is.

Student: 1 by 2.

This is half plus cosine 2 pi x.

Student: Pi L by 2.

I am sorry, this is L and there should be half here, ok. So, this is 4 q naught L 4 divided by pi to the power of pi EI and this is equal to 0.01309, q naught L 4 over EI, ok; so, w naught is 0.01309 cosine pi x over L. And what is its amplitude? Its amplitude is 0.01309 so, this has to be multiplied by q naught L 4 over EI.

So, what we have c is; that its amplitude is 0.01309 times q L 4 divided by EI. The exact solution is 0.10302, ok. So, the numbers are very close. So, in this case the principle of virtual work gives us a fairly good estimate of the displacement at the midpoint of the beam. And if we take more terms in the solution, we get more and more accurate results. So, this is how this approach works and before we close this for today couple of points.

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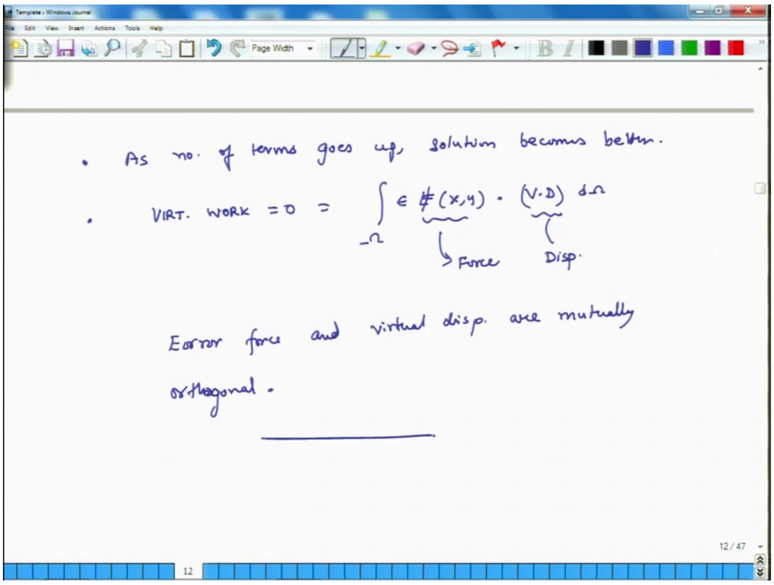

So, one thing we are seeing is that as number of terms goes up solution becomes better, ok. Second is that then we say that virtual work is equal to 0, what are we saying? We are saying that the integral over the domain over the overall domain of the problem, integral of what; error which is a function of x and y times virtual displacement d omega and multiplied by small parameter epsilon, this is to be 0. So, this is the virtual displacement and this is the error. What is this error? This is in this case it was force. And what is this? This is displacement.

Now, if the integral of the multiple of force and virtual displacement is 0, what does that mean? That the force and the displacement are mutually orthogonal, because when force and displacements are mutually orthogonal, their inner product or dot product is 0, ok. So, what this means is principal of virtual work implies is, that error force and virtual displacement are mutually orthogonal. So, this is what I wanted to explain.

Now, here I had chosen special Galerkin methods; where the virtual displacement was chosen as A 1 times cosine of pi x divided by L. But I could have chosen a different function, I could have chosen a different function also for w.

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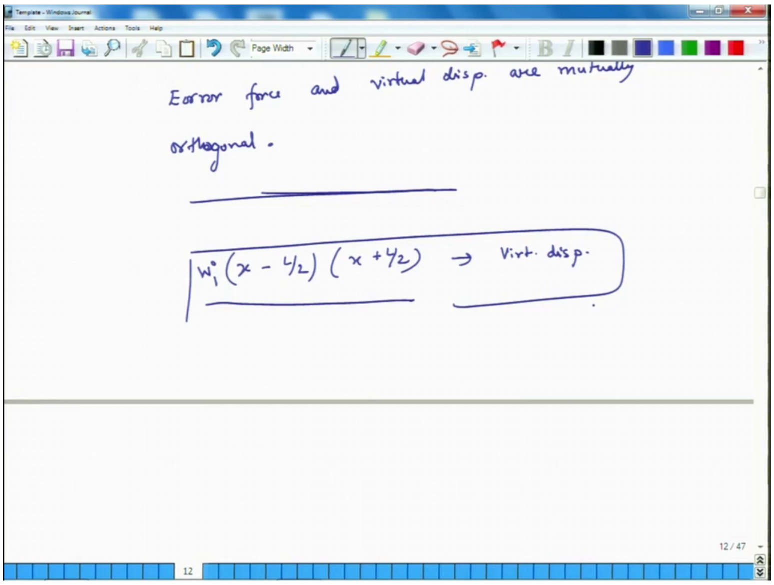

So, in this case I could have chosen x minus L by 2 and x plus L by 2 w 1. This is another possible virtual displacement because; this also meets our all kinematic boundary conditions. But, if I use this virtual displacement in this relation, then I would not use the special Galerkin method, I would use the general Galerkin method, ok. Because in general Galerkin method the virtual displacement function and the actual displacement function are not the same. In the special Galerkin function they are the same.

So, it makes us it requires less effort and less thinking. But I could have used other virtual displacement functions also. And I could there are series of all sorts of functions I could have used, ok. So, in this case this would have been the case of general Galerkin method. But anyway this concludes our discussion for today, and we will continue this discussion on virtual work theory tomorrow as well.

Thank you and have a good day.