Advanced Composites
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Lecture - 52
Virtual Work Method: Apply to Simply Supported Plate

Hello. Welcome to advanced composites. Today is the 4th day of the ongoing week which is the 9th week of this course. And we will continue our discussion on virtual work today as well as tomorrow and maybe even day after tomorrow. Because this is a very powerful technique to solve a lot of problems, we are closed form methods are not available, and we can get fairly good results, if we use the principle of virtual work to solve the problems which are of interest to us.

So, today we will discuss another problem in context of composite plates. And we will use the principle of virtual work to solve that problem.

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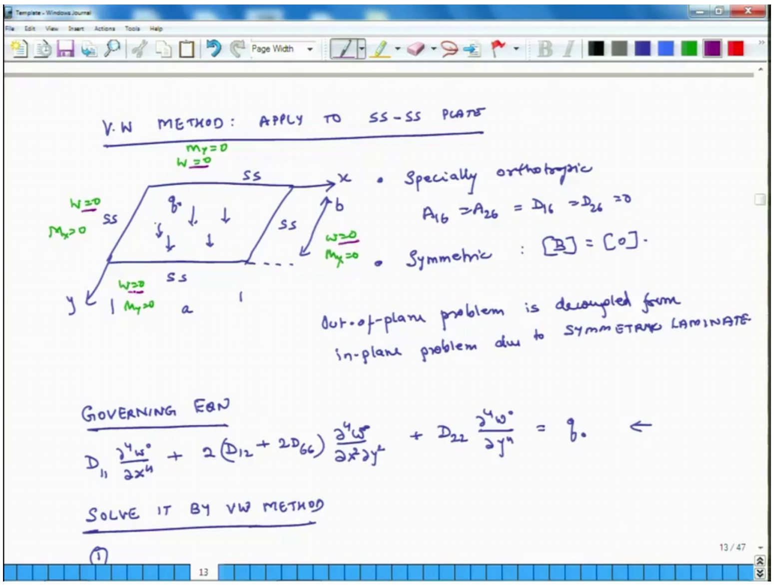

So, virtual work method, and here we are applying it to simply supported simply supported plate; which slowly making things more complex. So, consider a plate which is simply supported on all 4 sides. And it is seeing some normal distributed load which is uniform. So, it is q naught, and the plate is especially orthotropic. So, if it is specially orthotropic, it means A 1 6 is equal to A 2 6 is equal to D 1 6 is equal to D 2 6, they are all 0, and it is also symmetric. So, that implies the B matrix is a null matrix.

The dimensions of the plate are a so, this is my x axis, this is my y axis. So, the dimensions of the plate are a and this dimension is b. Let us write down the boundary conditions. So, boundary condition on this x axis is w is equal to 0, on all 4 edges and on edges x is equal to 0 and x is equal to a M x is equal to 0. Here M y equal 0 M x is equal to 0 and my equal 0. And we will not talk about in plane boundary conditions, because we are only interested in solving the out of plane problem, and because the plate is especially orthotropic and symmetric out of plane problem is decoupled because of symmetric from in plane due to symmetric, due to symmetric laminate. So, because the laminate is symmetric so, we only so, we will only discuss the out of plane problem, because we want to know what is the value of w at different points.

So now, next look at the governing equation. So, the governing equation for this we have shown in terms of w is this expression; with and this is q naught, because it is uniformly distributed. So, it is q naught. Now earlier we had solved this problem in an exact way by doing the following. First, we had decomposed q naught as in terms of it is Fourier series components. So, we had decomposed q naught into qmn sine m pi x over a sine m pi over b and summation of this using furrier series expansion. And then we also assume that w is equal to summation of w mn sine m pi x over a times sin m pi over b. So, using this method, and then we solve this and then we were able to solve this problem.

Now, what we will do is, we will consider the principle of virtual work to solve this problem. So, we will solve it by virtual work method. So, what is the first step? The first step is that we have to select w. W is what we want we have to select the function for w; which meets all the kinematic boundary conditions. So, which are the kinematic boundary conditions? W is equal to 0, w is equal to 0, w is equal to 0, w is equal to 0. So, any function which is 0 on all the edges of the plate that is a valid function that function need not ensure M x to be 0 and M y to be 0 on the edges of the plate. And the function need not satisfy the governing differential equation as well. So, all we are interested in satisfying the differential, the kinematic boundary condition.

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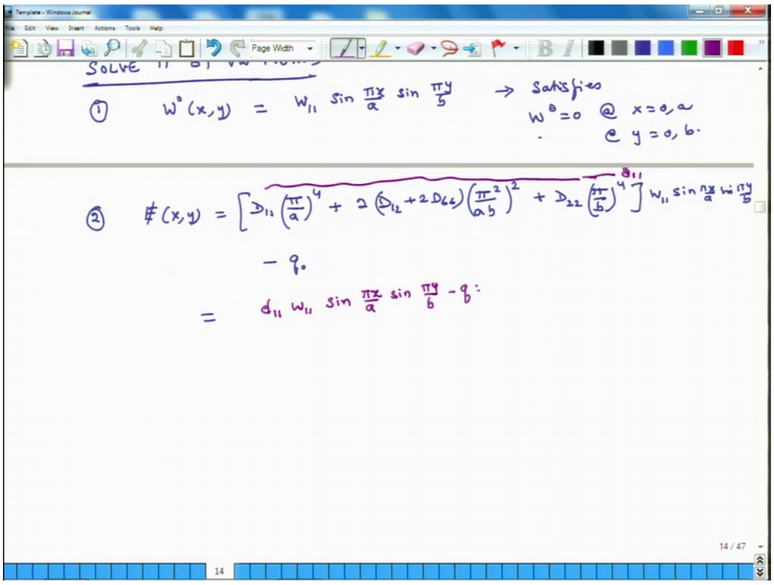

So, we say that w naught is equal to; let say we will just take a one term solution w 1 1 sin pi x over a sine pi y over b. Now this function, it satisfies w naught is equal to 0 at x is equal to 0 and a and at y is equal to 0 and b; it satisfies. And we do not worry whether it is going to satisfy the differential equation or not. Actually if you plug it in, it actually does not satisfy the differential equation.

Second, we compute the resident or the error force. So, how do we find the error force? We plug this in the differential equation and we and then we compute the error. So, the error is equal to D 1 1 pi over a to the power of 4 plus 2 D 1 2 plus 2 D 6 6, pi square over ab square plus D 2 2 pi over b 4. This entire thing multiplied by w 1 1 sin pi x over a sin pi y over b, minus q naught is error. So, just make things look simpler, we call this entire thing in the bracket as d 1 1. So, this is equal to d 1 1, it is not same as capital D 1 1. So, this is small d 1 1, w 1 1 sin pi x over a sin pi y over b minus q this is the error.

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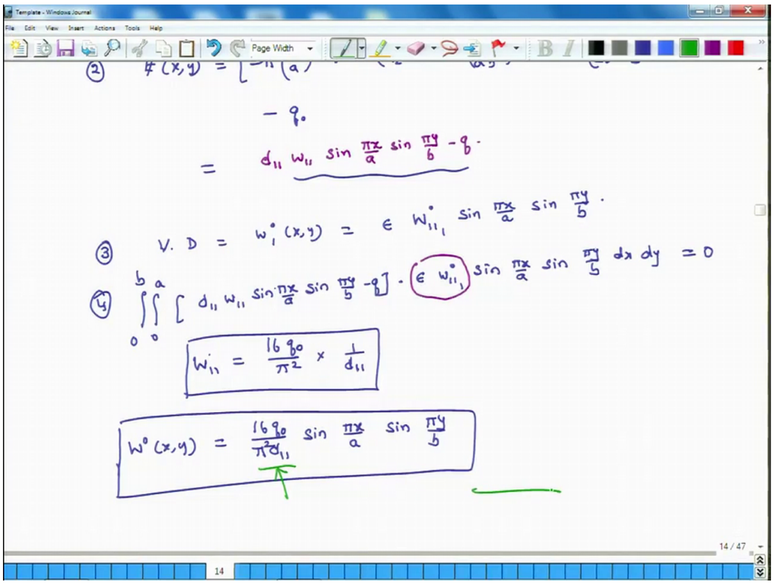

Third; what is the next step? We have to compute the virtual work, and for computing the virtual work, first thing is we have to select a virtual displacement field. And what kind of virtual displacement field is valid as long as it satisfies all the kinematic boundary conditions? Virtual displacement is we call it w 1 x comma y. And this function is it has a small parameter epsilon. And amplitude w 1 1 1; the last one implies that it is a virtual thing, and we will use a special Galerkin method. So, sin pi x over a sine pi y over b.

So now using this I compute virtual work and I equate it to 0. So, 0 to a 0 to b, this is my error force. So, this is d 1 1 w 1 1 sine pi x over a sine pi y over b minus q; times virtual displacement w 1 1 1 sin pi x over a sine pi y over b dx and dy. So, this is my entire virtual work and this should be equal to 0.

Now, if I do this integral, again this parameter epsilon 1 1 and w 1 1 1 is common. So, I can ignore this, and then when I do the integration all x and y or everything goes away, and the only thing I am left is a relation for w 1 1. So, when I do this I get w 1 1 equals 16 q naught over pi square into 1 over d 1 1.

And if you compare; so, what is my solution? So, my solution is, w naught xy is equal to 16 q naught over pi square D 1 1 sin pi x over a sin pi y over b. This is a one term solution, we could have used more terms in the system, but this is the one term solution. And if you compare the coefficient of the first term with the exact solution which we had developed a couple of weeks earlier, you will see that this coefficient 16 q naught over pi square d 1 1 is exactly the same thing what we had calculated earlier.

So, using this principal of virtual work, we arrive at this coefficient of the first term very easily right away. This is the very powerful technique. It is not that it will give me exact solution all the time. It just happens in this case that it is giving me the exact solution. The first term value is exactly the same. But in some cases it gives me very accurate results even in the very beginning. So, this is another example I wanted to give today. And we conclude our discussion for today. Tomorrow we will continue this discussion on the principle of virtual work. Till so far we have solved a problem of a beam and also of a plate by using just one term solutions. Now we will see how we can get multiple doing the same principle that is the principle of virtual work. So, that is all for today and I look forward to seeing you tomorrow.

Thank you.