Advanced Composites

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Lecture - 54
3rd Interpretation of Special Galerkin Method

Hello welcome to Advanced Composites. Today is the last day of the ongoing week which is the 9th week of this course, over this week we have discussed this principle of virtual work for several scenarios and we will continue this discussion today and may be also a part of the next week. And here what we plan to do today is bring out another facet of this principle of virtual work.

So, what we have shown till so, far is that the principle of virtual work says that the integral of the error force and the virtual displacement when you integrate it over the overall domain of the problem, then if you make it 0; then this helps you get the solution for the problem. So, this is one way of looking at things. Another way of looking at things was which is the secondary was that we said that because the virtual work is equal to 0. It implies that the error force and the virtual displacement they are mutually orthogonal because only, then your overall work is going to be 0. If they were not orthogonal, then the virtual work will not be 0 that was the second thing.

The third way of looking at things of this Galerkin method or this whole approach which we have been discussing is from the standpoint of least errors and this becomes particularly significant in context of the special Galerkin method.

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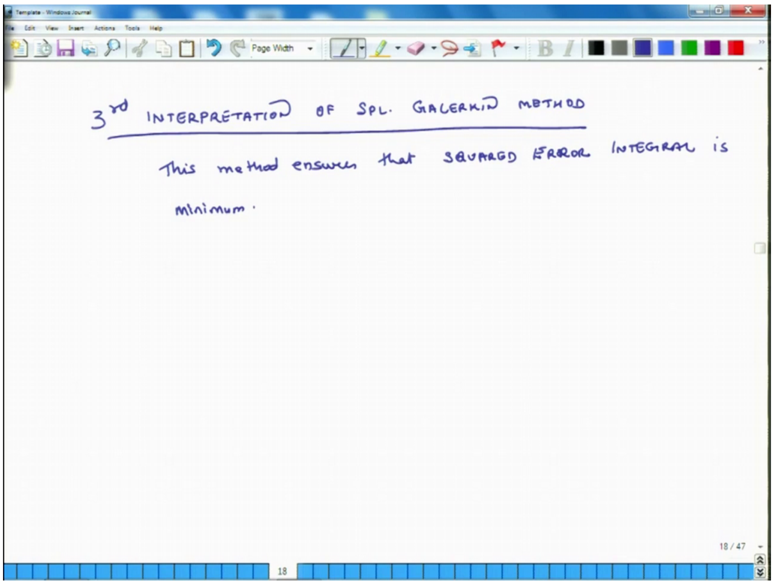

So, we will say that this is the 3rd interpretation of special Galerkin method. So, what does it mean? It is so, what I will first make the statement and then we will see what it means it says that, this method ensures that square of squared error integral is minimum. Such make the statement and now we will look at it that what it means and why is it like that and we will explain it using an example.

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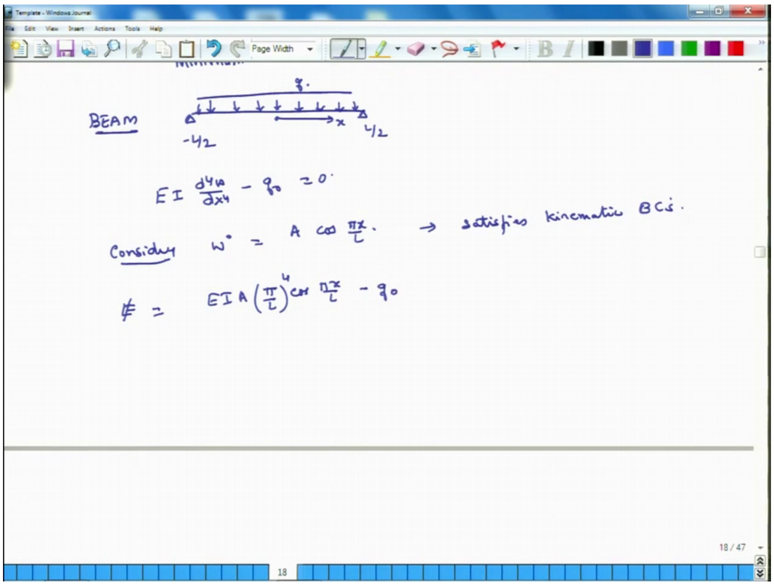

So, again we will consider the example of a beam. So, beam is simply supported this is x this is L over 2 this is minus L over 2 it is simply supported. So, w 0 at both ends moment is 0 at both ends uniformly loaded. So, all the good stuff which we had discussed earlier so, this is q naught. So, for this the governing equation is E I d 4 w over d x 4 minus q naught is equal to 0. Now if w was exact, then this error in the equation force error will be 0. So, we assume that approximate function consider w naught as A cosine pi x over L ok.

So, this why did we choose this because this function satisfies the value of w as equal to 0 at both ends and. So, it satisfies kinematic boundary conditions. So what is the error? Error is equal to E I A pi over L cosine pi x over L. So, this pi over L is to the power of 4 minus q naught this is the error this is the error in force. Now, what we do is we square this error.

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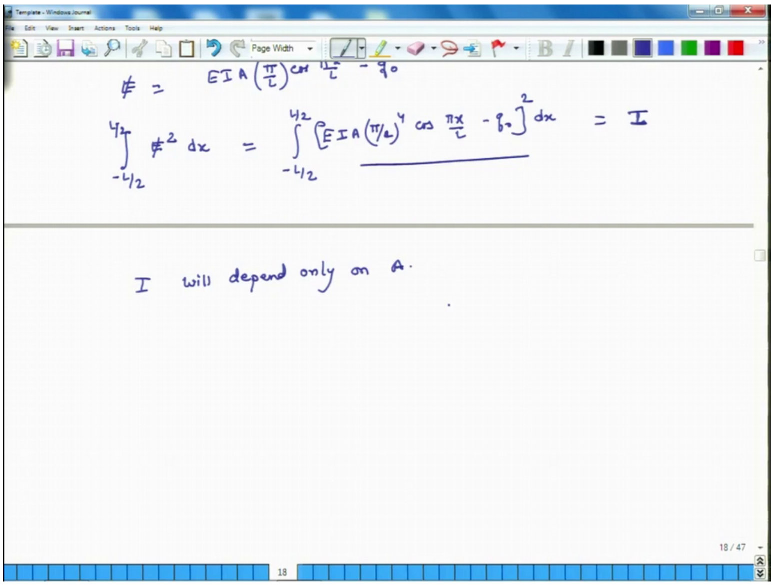

We take the, this is error and square it and then we integrate the square of this error over the domain. So, what we do? Minus so, this is and let say what it gets. So, what we are doing is this error I am squaring it and integrating it over the domain ok. So, what do? So, this is minus L over 2 to L over 2 and the error is E I A pi over L to the power of 4 cosine pi x over L minus q naught dx. So, this is the square of error.

So, let us call it so, let us call this I some integral I ok. So, what is this integral also this is this is the square of the error integrated over the domain and that I call it I. Now I want to minimise this integral I. Suppose I want to minimise this integral because if I want a good solution, then this integral this square of error should be less. So, for minimum I what should be the case? Some derivative of I should be equal to 0 for a minimum or extreme maximum.

Now, once I have for minimum I. So, before I do this let us explain something. So, once I evaluate this I, what will happen? I will depend only on A. It does not depend on I and E because they are constants. A is E a is unknown it can change, but I will depend only on A, it does not depend on x why does it not. Because once I have done the integral x goes away it does not depend on E, it does not depend on I because these are constants and it does not also dependent on q naught and L. So, it depends on A only.

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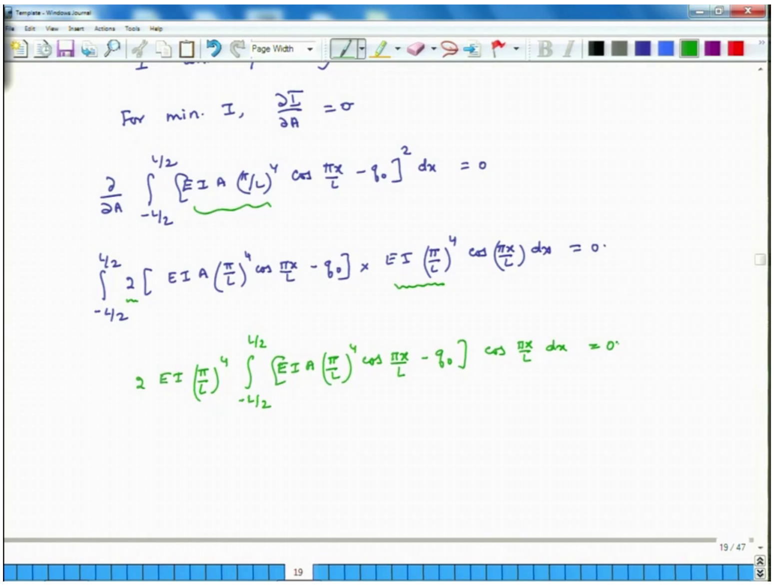

So, if that is the case then for minimum I del of I over actually del of should be equal to 0 if I have to minimise I. So, I can say that del of this integral this has to be 0 and because this involves only a I can move this derivative inside the integral. If it was involving x and other thing then I cannot, but it. So, I can say so, when I differentiate this with respect to A, this is what I get. So, this when I am differentiating it, I am differentiating this with respect to A. So, this is 1 plus into the derivative of this term.

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Yeah in to the derivative of this term I, A goes away pi over L to the power of 4 cosine pi x over L dx is equal to 0, this is what I get now what I do next is I take this and this constant away. So, I get 2 EI pi over l to the power of 4 minus l over 2 l over 2 EI a pi over L to the power of x cosine pi x over L minus q naught times cosine by x over L dx is equal to 0.

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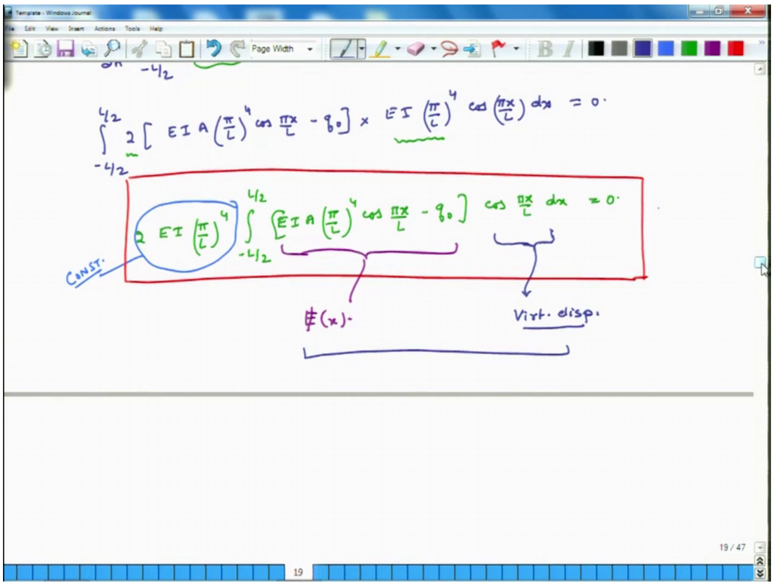

Now, look at this relation here this term is just a constant number the one which is in blue. So, this does not influence A basically the aim is to find out the value of A and how do you find out the value of a because we said that I should be minimum and I will be minimum if this condition holds ok, this is a constant the one term in blue. So, that does not affect the solution of A.

The second term you look at is this term and this term is nothing, but the error which is the function of x. Remember this is exactly the same term which is this thing ok. So, this is the error and the third term when we look at this term this is like your virtual displacement except that its amplitude is 1, but in this case the virtual displacement function is the same as the displacement function of the actual value of the actual displacement function.

So, what; that means, is that if I try to minimise the square of the error in and the integral of the square of the error over the domain essentially I will get the same equation as the principle of virtual work ok, with the condition that the virtual displacement field will be same as the will be similar to the actual displacement field. So, this means that this. So, what this means is that, special Galerkin method minimises integral of square error right.

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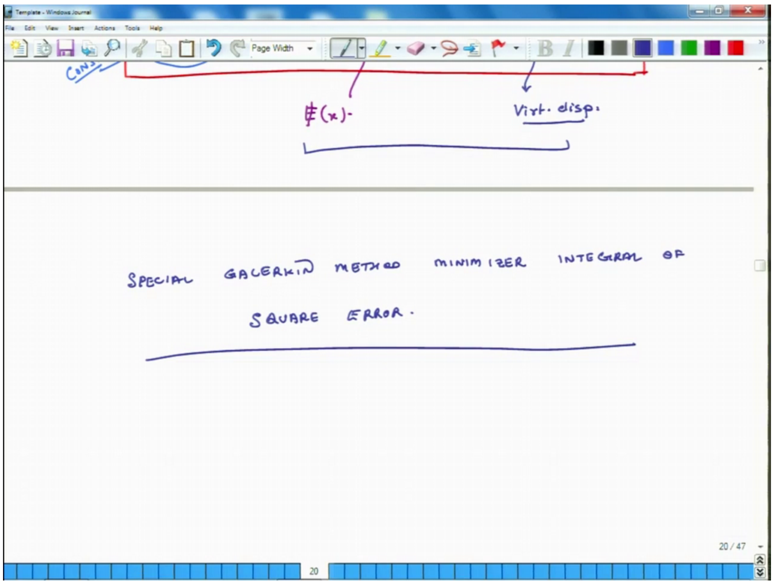

So, the special Galerkin method if we are trying to use solve the problem by using a special Galerkin method essentially another way of looking at it is that we are trying to minimise the square error integral. So, that is what I wanted to talk about.

So, in that case this special Galerkin method is more special is special not only because of the choice of the displacement fields, but also because it reduces the square error. So, that is why it gives even better result than regular Galerkin method. So, this is what I wanted to discuss today we will continue our discussion on virtual work methods even in part of the next week and hopefully we will close it may be in two more lectures, but once again I want to emphasise the importance of these this particular method this is the powerful method where we have to only worry about the kinematic boundary conditions and once we get some functions which satisfy the kinematic boundary conditions, then when then we can use those functions to find the solution of the problem these solutions are approximate.

So, they may not give us the exact solutions, but if we choose a sufficient number of terms in the problem statement then we can very rapidly converge to accurate solutions. So, that is pretty much what I wanted to discuss over this week I forward to seeing you next week as well.

Thank you.