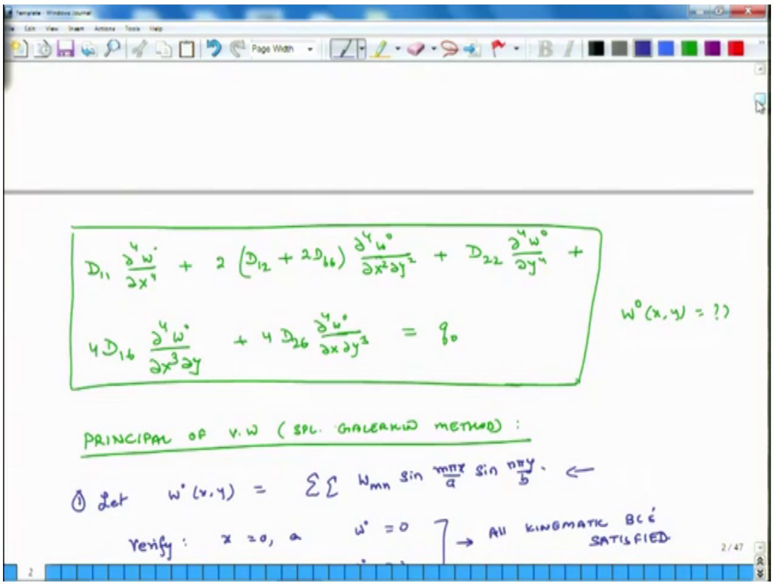
Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 56
Role of D_{16} and D_{26} Terms On Laminated Plate Response (Part –II)

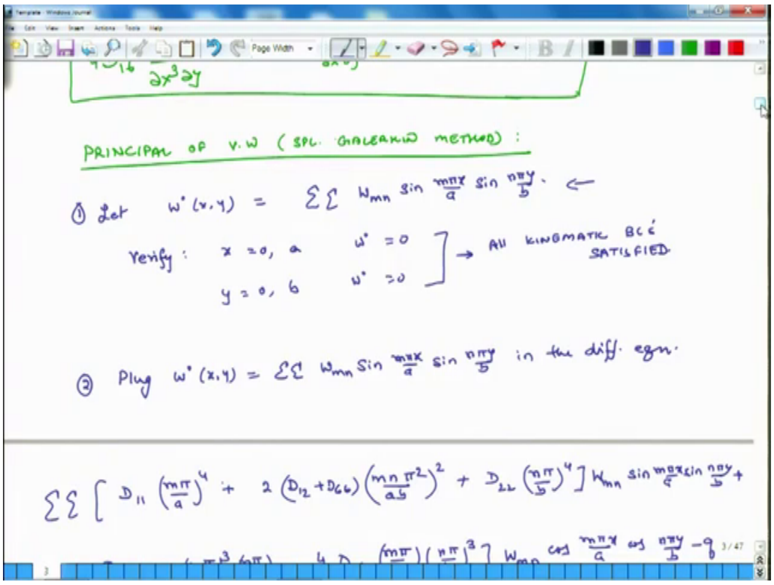
Hello, welcome to Advanced Composites. Today is the second day of the 10th week of this course; yesterday we started discussing how the terms D 1 and D 2 D 1 6 and D 2 6 influence the overall solution for laminated composite plates. And in that context we started discussing the case of a simply supported plate which is symmetrically laminated, but it is such having such a sequence that its terms D 1 6 and D 2 6 bending stiffness terms, they are not necessarily 0.

(Refer Slide Time: 00:57)



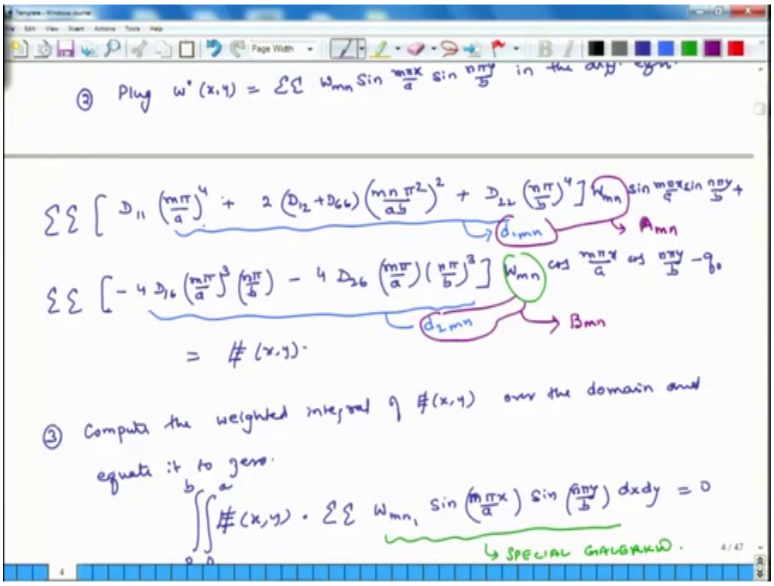
And we started looking at this problem and what we had done was that we had arrived at a formulation of the problem in terms of the special Galerkin method. And the overall governing differential equation was developed which is shown in green.

(Refer Slide Time: 01:07)



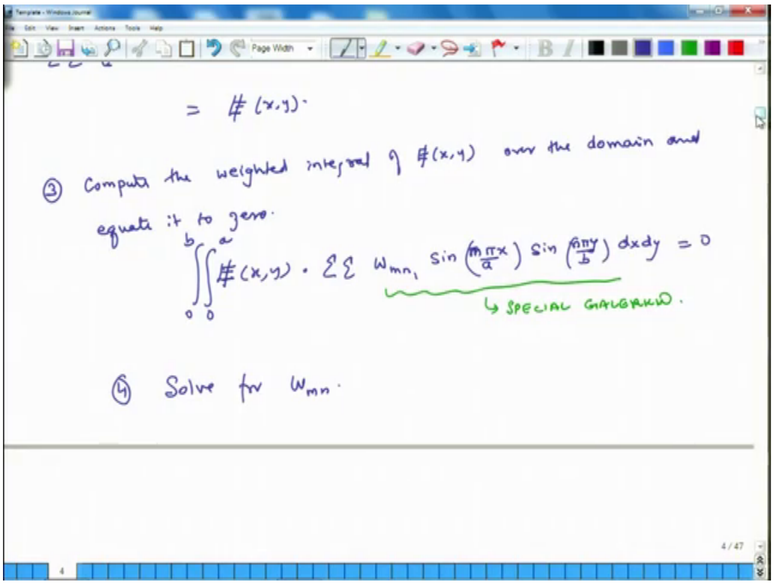
And then we started using the principle of virtual work and within virtual work we started using special Galerkin method to solve this problem. So, we assume that W is a sin series in two directions sin m x pi over a sin n pi over b. And such an assumption satisfies all the kinematic boundary conditions.

(Refer Slide Time: 01:31)



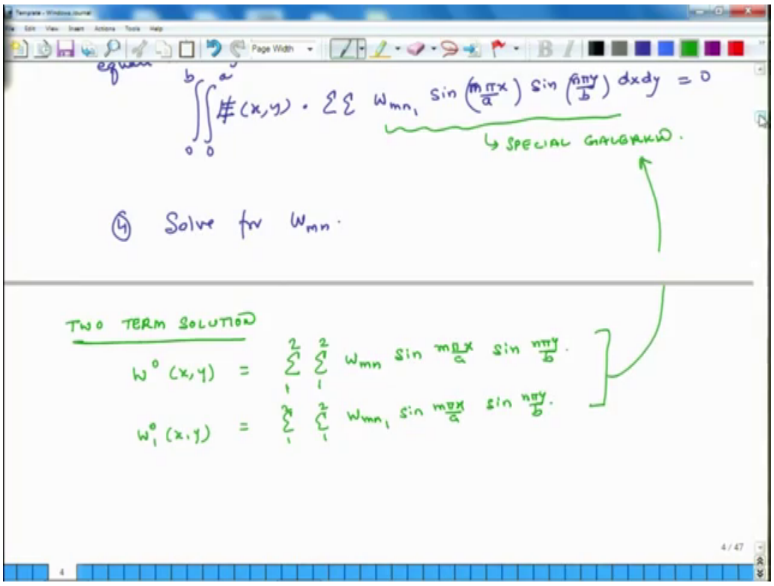
The next step was we calculated the error in the equation.

(Refer Slide Time: 01:35)



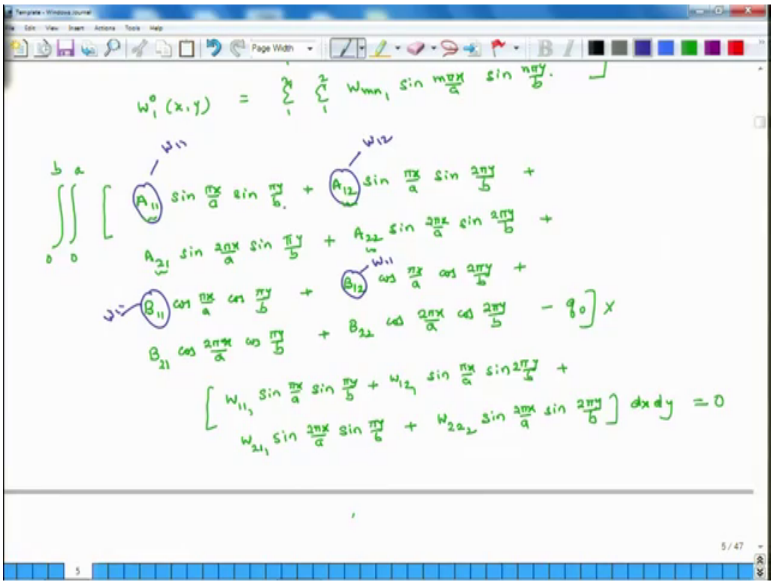
And that error got multiplied by a weight function or a trial function W m n 1 sin pi m pi x over a times sin n pi y over b; and the multiple of this function with the error is integrated over the domain and it is set to 0. And then from this integral equation, we compute the values of unknowns which are W m n, so that is what we are going to show in the next, in this lecture.

(Refer Slide Time: 02:11)



So, here we will use a two term solution ok. So, W naught x y is equal to 1 to 2, 1 to 2 W m n sin m pi x over a sin n pi y over b. And the trial function is or the weight function is similar in relation, but instead of W m n it has a coefficient one; and that also has two terms in each direction. This is also sin m pi x over a sin n pi y over b. Now, if I plug this equation back in the equation which I had developed earlier, if I had developed earlier. So, what I will get, I will still get a very long equation.

(Refer Slide Time: 03:41)



And the overall equation will look something like this integral 0 to a 0 to b and so first we will have terms related to so be just to make things a little simpler. We will call this d 1 m n this entire thing; and we will call this d 2 m n ok, this entire thing in the bracket because this is very long expression. So, d 1 m n gets multiplied by W m n and d 2 m n also gets multiplied by W m n. But d 1 m n is multiplied by sin m pi x over a and d 2 m n is multiplied by cosine n pi x over a times cosine n pi y over b. So, this is d m n and d 1 m n and d 2 m n. And then to compress things further, I call this d 1 m n times W m n as A m n. Now, this A m n is different when our a matrix term which is A 1 1, A 1 2, A 1 6; and this when this multiplies by this thing I get B m n ok.

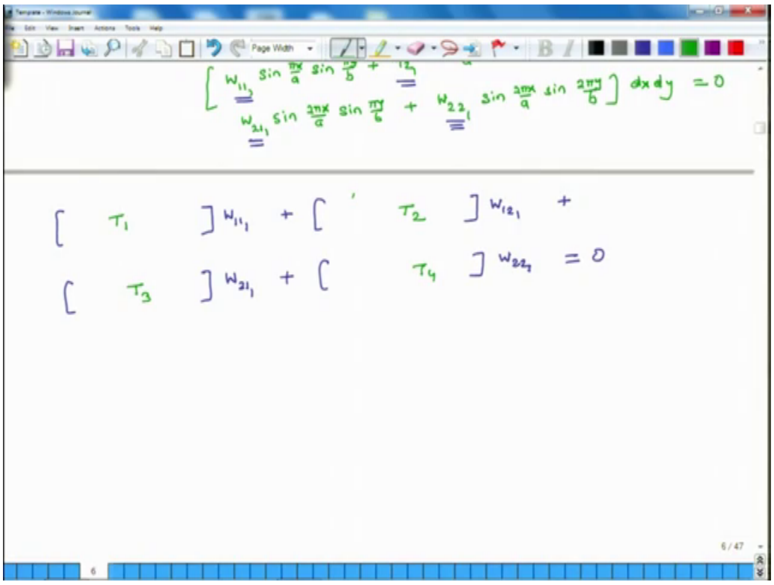
So, if I do all this then I somewhat I am able to abbreviate it and this is what I get A 1 1 sin m excuse me sin pi x over a sin pi y over b plus A 1 2 sin pi x over a sin 2 pi y over b plus A 2 1 sin 2 pi x over a sin 2 pi y over b no not 2 pi y pi y over b plus A 2 2 sin 2 pi x over a sin 2 pi y over b. So, in this A 1 1 is embedded W 1 1 times d 1 m n; and in this A 1 2 also is embedded d 1 d 1 2 times W 1 2 right.

So, so in these terms the unknown coefficients are embedded, but we are just for compressing it I have clubbed it into smaller chunks. So, these are the sin terms and then I get cosine terms. So, I get B 1 1. And then now I do not have sin cosine pi x over a cosine 2 pi y over b plus B 1 2 cosine pi x over a cosine 2 pi y over b plus B 2 1 cosine 2 pi y 2 pi x over a cosine pi y over b plus B 2 2 cosine 2 pi x over a cosine 2 pi y over b.

And then there is minus q naught B 1. So, this I had erased this, but I will make this clear. So, this is pi y over b. So, this entire thing gets multiplied by the trial function ok, which is this W m n 1 and this is also a two term thing. So, this entire thing gets multiplied by W 1 1 1 sin pi x over a sin pi y over b plus W 1 2 1 sin pi x over a sin pi y over b times 2 plus W 2 1 1 sin 2 pi x over a sin pi y over b plus W 2 2 2 sin 2 pi x over a sin 2 pi y over b. So, this is the trial function and then I am integrating over the domain, and this entire integral is 0 ok.

Now, remember that unknown coefficients, in this coefficient is embedded W 1 1; in this coefficient is embedded W 1 2 and so on and so forth. Same thing in B is also W 1 1 is embedded in B 1 2 also W 1 1 is embedded, it is same thing. So, when I do this entire multiplication, what I end up getting is essentially and once I have integrated over x and y, so these are just products of sins and cosines. So, I can integrate this, using standard integral calculus principles which is not difficult. And once I have integrated over the domain x ranging from 0 to a; and y ranging from 0 to b x and y they disappear ok, x and y disappear because I have integrated.

(Refer Slide Time: 10:55)



So, once I do that, what I will get, and then we have these coefficients W 1 1, W 1 2 1, W 2 1 1 and W 2 2 2. So, I will get four blocks of terms the first block will be multiplied by W 1 1 1 plus there will be another block of terms and this will be multiplied by W 1 2 1 plus there will be a third block of terms and this will be multiplied by W 2 1 1 plus W 2 2 1.

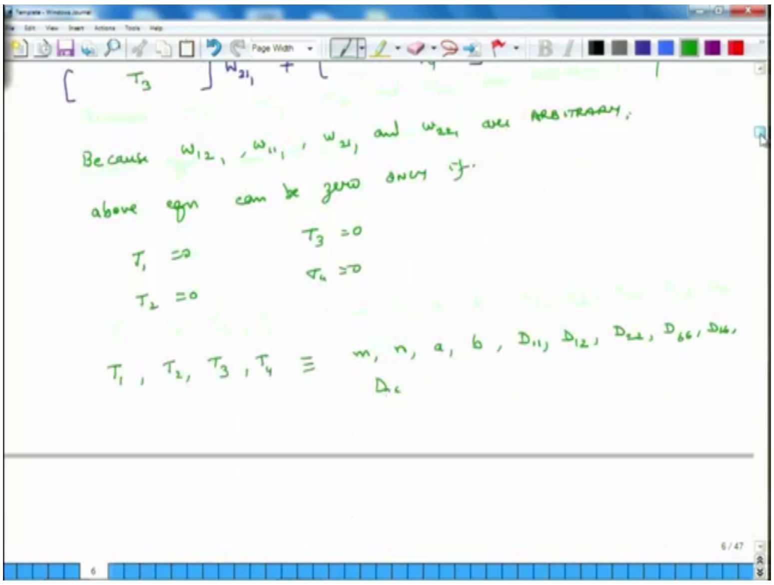
Student: (Refer Time: 11:36)

And this should be 0.

Student: W 2 2 1(Refer Time: 11:41)

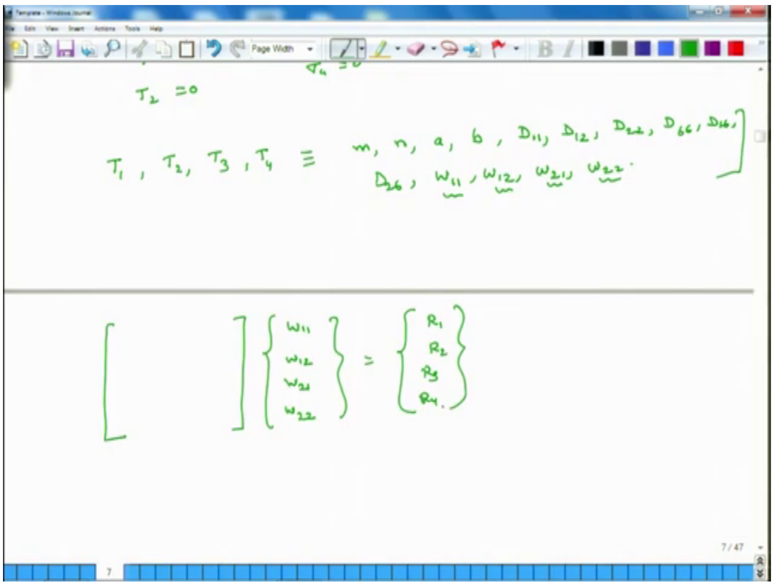
This will W 2 1 1 and this will W 2 2 1. So, this is 2 1. Now, if this entire expression has to be 0. So, we will say that these are this is T 1, this is T 2, this is T 3 and this is T 4. If this entire expression T 1 times W 1 1 1 plus T 2 times W 1 2 1 plus T 3 times W 2 1 1 plus T 4 times W 2 2 1, it has to be 0 that is going to be possible only if T 1 is equal to 0, T 2 is equal to 0, T 3 is equal to 0 and T 4 is equal to 0. Because these things because W 1 1 1, W 1 2 1, W 2 1 1 and W 2 2 1 are arbitrary because when we do the trial function these amplitudes are arbitrary.

(Refer Slide Time: 12:47)



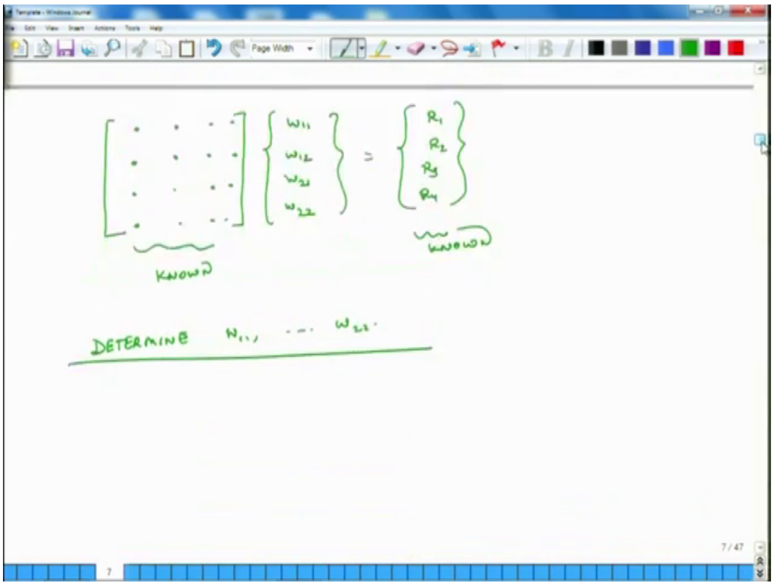
So, we state because these are virtual displacements are arbitrary above equation can be 0 only if T 1 equals 0, T 2 equals 0, T 3 equals 0 and T 4 equals 0. And what are T 1, T 2, T 3, T 4? T 1 T 2 T 3 T 4 what do they depend on they depend on m, n, a, b, D 1 1, D 1 2, D 2 2, D 6 6 D 1 6 and D 2 6. And what is their expression, if we do this integral, you will and arrange this entire integral in this format, we will exactly know what is T 1 1, T 1, T 2, T 3, T 4 ok.

(Refer Slide Time: 14:27)



And here and of course, there is m n and then of course, W m n W m n. So, actually here it will be W 1 1, W 1 2, W 2 1, W 2 2. So, we get four linear equations in W 1 1, W 1 2, W 2 1 and W 2 2 everything else in these parameters is known m, n, a, b, D 1 1 all these everything else is known, the only thing which are unknown are W 1 1, W 1 2, W 2 1, W 2 2 right.

(Refer Slide Time: 15:33)



So, we can rearrange these equations in this format ok. So, this is this is the 4 by 4 matrix. So, in this matrix, this is known and this is known. So, 4 by 4 matrix everything is known except the W s. So, using this 4 by 4 matrix, we can determine W 1 1 till W 2 2 ok. Suppose, we had had three terms solution, then we would have gotten 3 by 3 matrix, so actually I am sorry 9 by 9 matrix, 9 by 9 matrix. So, we can find out W 1 1 W 1 2 W 2 2 and so on and so forth.

So, this is how we get Ws, but what is our original goal we have to explore the role of D 1 6 and D 2 6. So, now, but we cannot explore till we know what W is going to be. So, now that we have developed the method to figure out W 1 1, W 1 2, W 2 1 and W 2 2. Now, what we will do is we will start exploring how D 1 6 and D 2 6 influence the overall solution we will explore their role, so that is something we will start exploring in the next class. So, till then have a great day ok.

Thanks.