Advanced Composites
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Lecture - 59
Free Vibration in Composite Plate (Part-1)

Hello, welcome to Advanced Composites. Today is the 5th day of the 10th week of this course. Over last four days, we have been discussing about the role played by D 1 6 and D 2 6 as to how they influence the behaviors of plate and specifically, we saw that they influenced the distribution of M x within the plate and also they lead to as effects, which is the difference between the computed value and the actual value of M x as per the code. So, today we are going to move into a slightly different area and that relates to, the dynamics of these plates and specifically, we will talk about Free Vibrations in Composite Plates.

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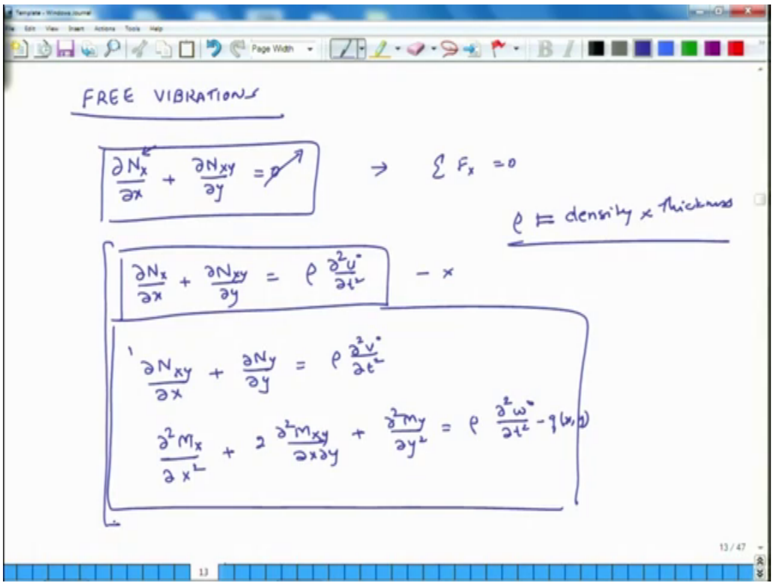

Now, when we are talking about the dynamic equilibrium plates till so far, what we had discussed was or we had developed the governing equilibrium equations that was about their static equilibrium. So, the first equation was del N x over del x plus del N x y over del y equals 0, if the plate has to be in. So, this was the equilibrium of forces.

So, we got this equation from equilibrium forces in the x direction when the plate was in static equilibrium, but if the plate is in dynamic equilibrium then this right side of the equation, this should be equal to mass times the acceleration of the plate and if the, we are talking about acceleration and this is equilibrium in the x direction then it will be the acceleration in x direction.

So and, so that was second thing and the second thing is that this is N x is, force per unit length. So, essentially all what all that means, is that this equation gets modified to del N x over del x plus del N x y over del y and that equals the area density of the plate ok. So, rho a; so, rho a is (Refer Time: 02:49) and times the acceleration. So, it is del 2 u over del t square.

So, this is not area density, it is density times the thickness of the plate rho. So, this is equal to density times thickness of plate. So, this is the equilibrium equation in the x direction. Similarly, the equilibrium equation, so, this is for x direction for y direction, we have a similar equation and for the third direction which is the combination of force and moment equilibrium we have the following equation. So, I am sorry, this should be del 2 v over del t square.

So, all these are mid plane displacements. So, these are the modified equations which have to be considered when we are talking about dynamic equilibrium of the plate. Next, what we will do is, we will develop relations for resonance frequencies of these plates, which are rectangular in shape and they are simply supported.

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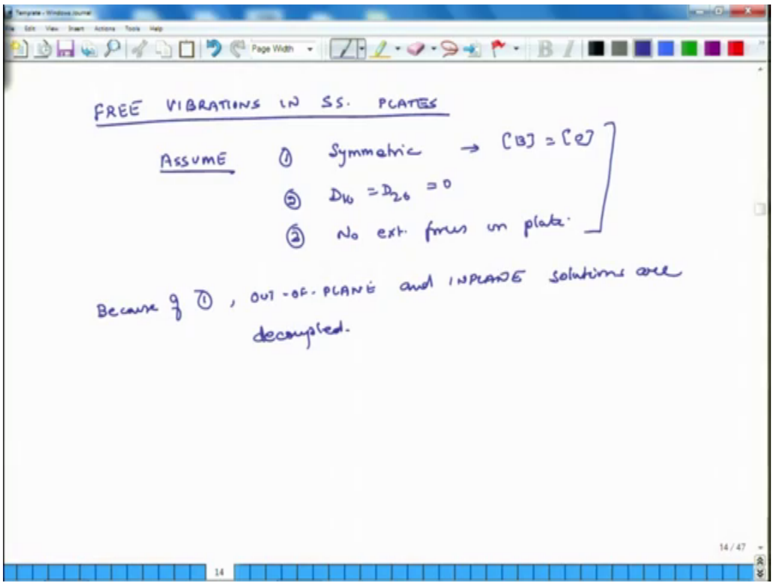

So, free vibrations in simply supported plates and here we assume couple of things. So, we assume. So, if it is free vibration, then there is no external force acting on the plate. So, we assume that the laminate is symmetric, which means the B matrix is 0; second, we assume that D 1 6 is equal to D 2 6 is equal to 0 and the third thing is so, here we missed one term, which is minus q naught or q x y. So, this is this minus q, which is a function of x y.

So, going back we do not have any external forces on the plate. So, what does that mean? That is mean that the plate has, if it vibrates, it will vibrate naturally. It is not vibrating, because of some external agent, but it has vibrating in a natural state these. These equations also do not include the effect of damping, because if damping was there then we will also have to account for damping forces in the system.

So, that is there and because of condition 1, because of 1 the out of plane solution and in plane solution these are decoupled ok. Out of plane and in plane solutions are decoupled. So, all we have to worry about is the third equation. So, the third equation is del 2 M x over del x square plus 2 del 2 M x y over del x del y plus del 2 M y over del y square is equal to rho del 2 w naught over w t square.

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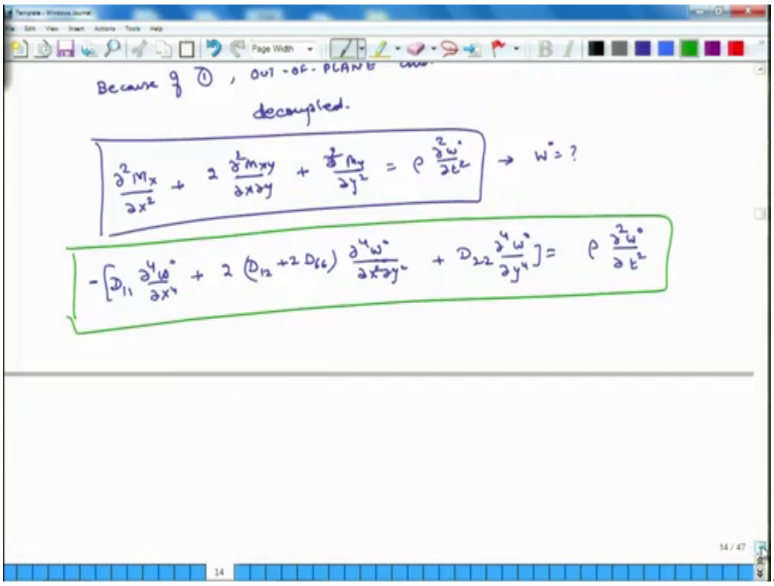

So, the first thing we do is we want; so, our aim is to find w. So, the first thing is we convert this entire equation in terms of w. We know already the expressions for M x M x y and M y and is because D 1 6 and D 2 6 are 0, so, what we get is D 1 1. So, this is the updated equation. Now, to solve this equation we have, what we will do is we will guess a function and we will guess it in such a way that it satisfies all the boundary conditions and also all the and this differential equation it satisfies.

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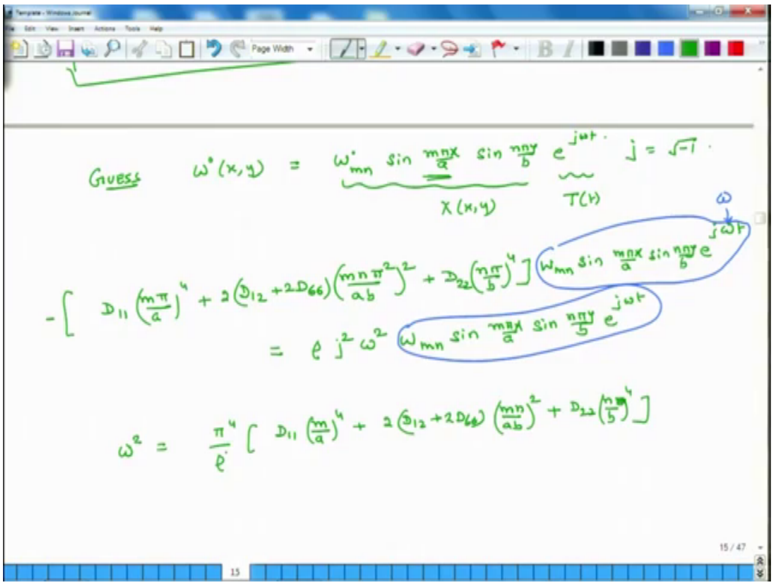

So, we guess that w naught x y equals w naught m n sin m pi x over a sin n pi y over b exponent j omega t, where j is equal to square root of minus 1 ok. So, this, this assumed solution has a function of x and a function of time. So, this is a function of x by capital X, I mean the space and this is also a function of time. So, excuse me this is this, this is a function of space and this is a function of time. So, this is a process of variable separable and now, we plug this equation, this relation back into the governing differential equation.

So, what we get is; so, this sin terms gets differentiated twice. So, I have 4 times. So, I end up with the same sign. So, I get minus D 1 1 m pi over a to the power of 4. So, I will take D 1 1 negative sign out plus 2 D 1 2 plus 2 D 6 6 m n pi square over a b the entire thing squared plus D 2 2 n pi over b whole thing to the power of 4 times w m n sin and m pi x over a sin n pi y over b e to the power of j omega t.

So, remember this is omega, this is omega, this is not w and this equals on the right side rho and then when I differentiate this term in with respect to time twice I get j square and I also get omega square w m n sin m pi x over a sin n pi y over b e j omega t. So, if you compare the left side and the right side all these terms are common. So, they get cancelled out.

So, if this equation has to be true and then j square becomes negative 1. So, if this equation has to be true then what I get is omega square is equal to pi over 4. So, I will take pi 4 out and then I get D 1 1 m over a to the power of 4 plus 2 D 1 2 plus 2 D 6 6 m n over a b whole thing square plus D 2 2 n over b whole thing to the power of 4 and on the other side I get density in the denominator.

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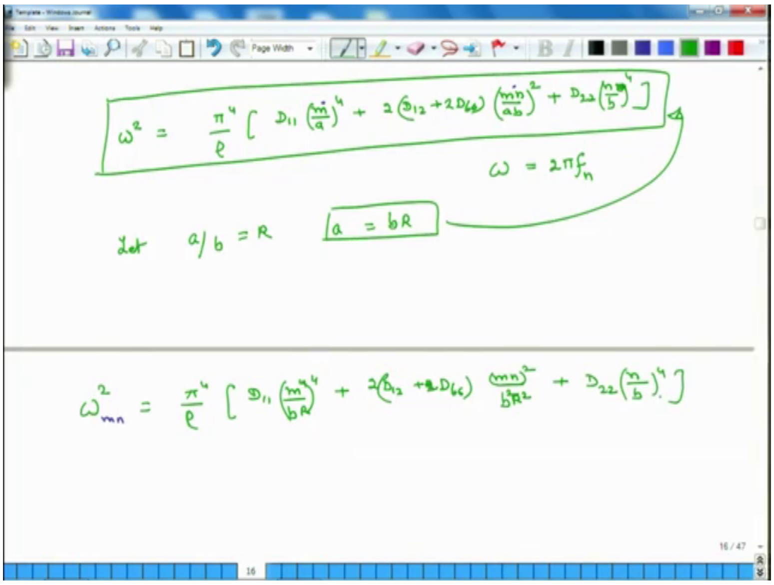

So, this is my expression for natural frequency. So, omega is equal to 2 pi f n where and this omega is again associated with m and n ok. So, this value of omega, it varies with m and n. So, this is what I get ok. Now, what I say is that let a over b be the aspect ratio of the plate a over b. So, I can say that a is equal to b times R. So, if I substitute back this into this equation I get omega and what I will do is I will apply a subscript m n, because each natural frequency is associated with this m n n ok.

So, I just called it the m n natural, angular frequency. So, that equals pi over 4 times rho D 1 1 m 4 over b R to the power of 4 plus 2 D 1 2 plus 2 2 D 6 6, this is m n over b R square b square R square and this is m n whole square plus D 2 2 n by b to the power of 4. So; so, this is b 4 R square and just to make this clear, this is m the whole thing ok. So, I get omega m n is equal to if I take the square root of this entire thing and bring out the outside.

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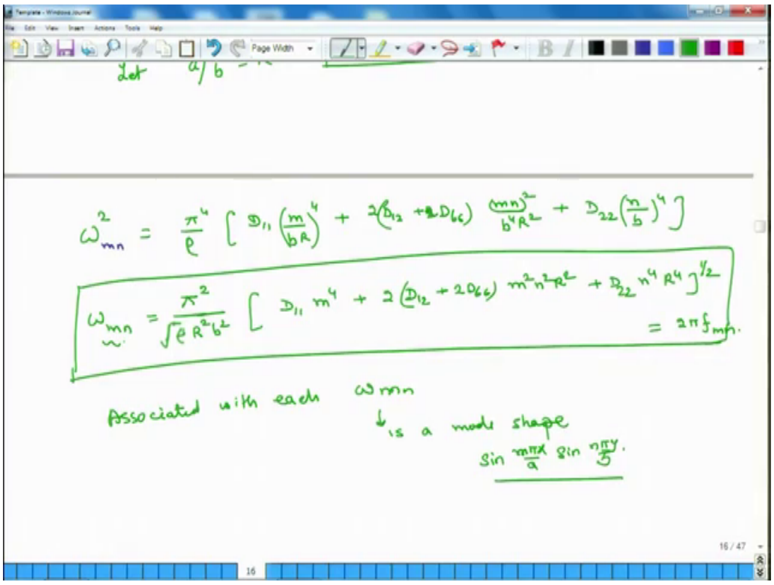

So, I get pi square over rho R square b square, I take everything b square as well as R square out then this is what I end up with D 1 1 m to the power of 4 plus 2 D 1 2 plus 2 D 6 6 m square n square R square plus D 2 2 n to the power of 4 R to the power of 4 the entire thing. So, this is my natural frequency of the system and so, this is the m nth frequency and this is the angular frequency.

We have to calculate the cyclic frequency, I just you know, divide this omega m n by 2 pi and I will get the frequency and associated with each of this natural frequency, so, associated with each omega n m n. So, this is the frequency is a mode shape and what is that mode shape, it is sin m pi x over a sin n pi y over b.

So, what it means is that if the thing is going to vibrate at angular frequency of omega 1 1 then the shape of the plate will be such that it has 1 curve in the x direction and another curve in the y direction and so on and so forth. So, this is the natural frequency of the system and this we can also call it as 2 pi f m n. So, this is another one.

So, this is what I wanted to cover today, tomorrow we will extend this discussion a little further. We will also see how this behavior of the plates gets applied to isotropic plates and how do vibrations in isotropic plates as well as in orthotropic plates how do they differ based on some of the results, which we have discussed till so far. So, that concludes our discussion and, let us come let us see what gets extended in context of this discussion in the tomorrow's lecture.

Thank you.