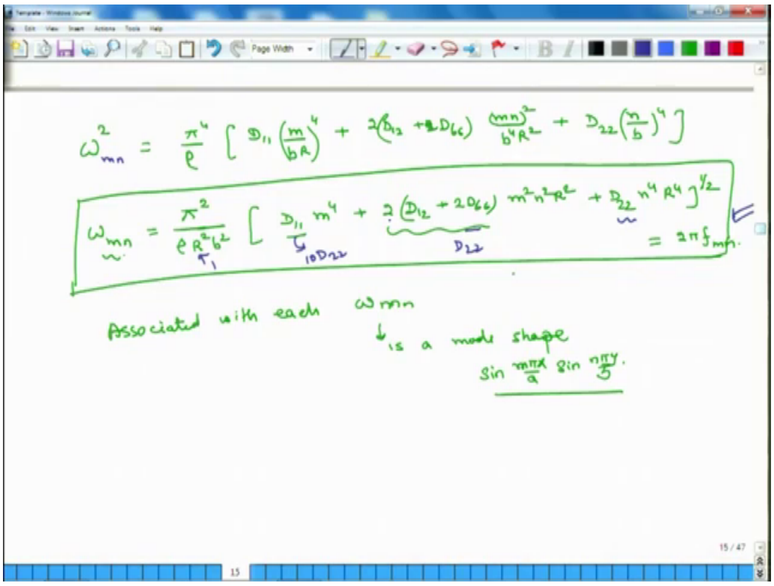
Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 60
Free Vibration in Composite Plate (Part-II)

Hello. Welcome to Advanced Composites. Today is the last day of the ongoing week, which is the 10th week of this course. And today, we will continue our discussions which we started yesterday, that was about Free Vibrations in Rectangular Composite Plates. Specifically, we had developed governing equation, which dictates the vibrational behavior of these plates.

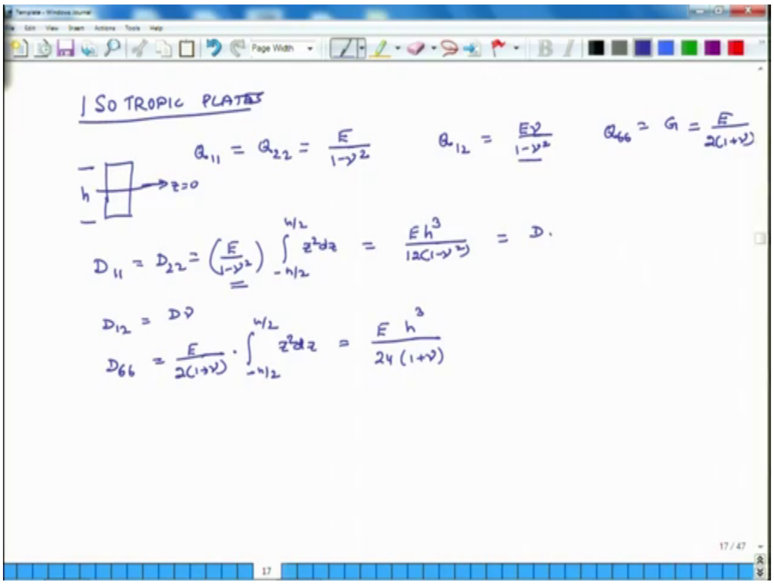
And using that governing equation, we had computed natural frequencies and associated modes for rectangular composite plate, which was simply supported on all four sides. The plate which we considered was symmetrically laminated and also each of its layer was orthotropic in nature, so that the D 1 6 and D 2 6 terms for the plate were 0.

(Refer Slide Time: 01:05)



So, for such a plate the natural frequency was computed as dictated by this relation.

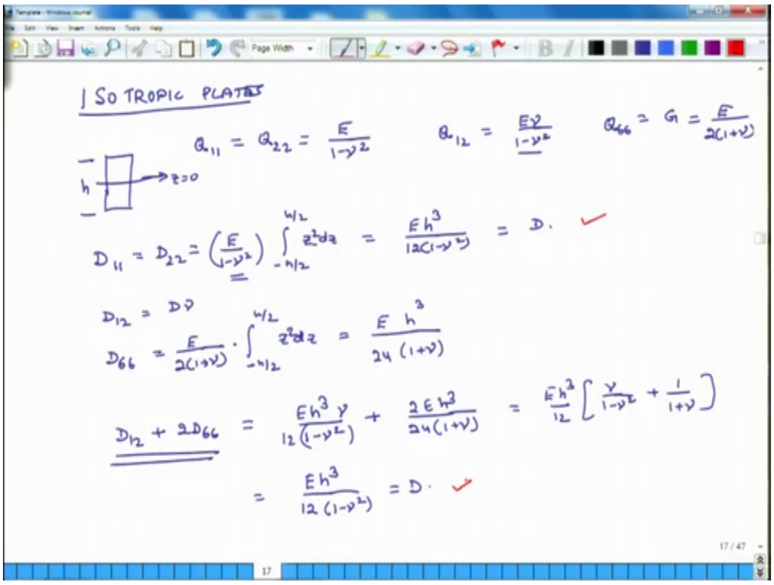
(Refer Slide Time: 01:13)



Now, we let us see how this compares to isotropic plates. So, again we will consider a plate, which is isotropic rectangular in shape than simply supported on all four sides. So, for an isotropic plate, which is h take Q 1 1 for such a plate is same as Q 2 2, and that is equal to E over 1 minus nu square. This is something which we have already discussed in one of our lectures, earlier lectures. Also Q 1 2 for such a plate is E times nu divided by 1 minus nu square of Poisson ratio. And Q 6 6 is equal to G, and that is same as E time divided by 2 into 1 plus Poisson ratio.

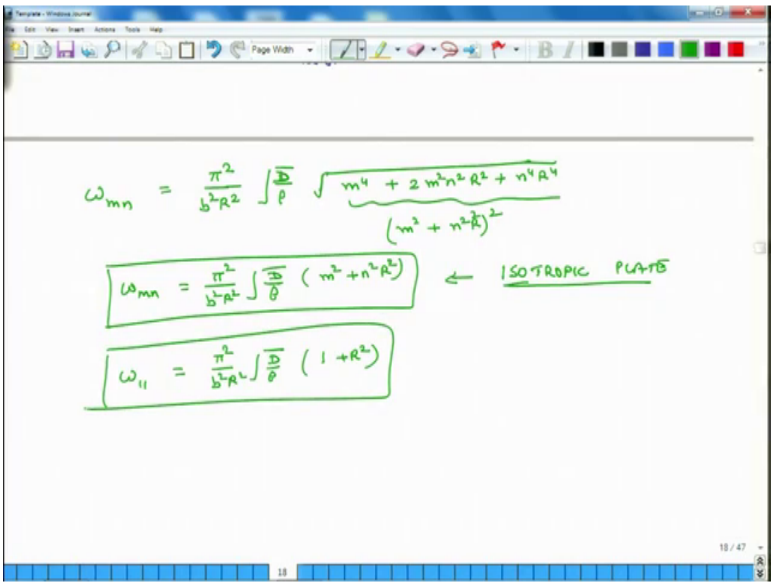
So, if the plate is h take and that is my z is equal to 0 plane mid plane, then for such a plate D 1 1 will be same as D 2 2, and that will equal Q 1 1 divided by so Q 1 1 times minus h by 2 to h by 2 z square dz. And that gives us E h cube over 12 into 1 minus nu square ok. So, this entire term I call it D. Similar, D 1 2 so D 1 2 is same thing, but instead of Q 1 1 or Q 2 2, I replace this by Q 1 2. So, it is same as D times nu. And D 6 6 is equal to E over 2 into 1 plus nu into minus h over 2 to h over 2 z square D z. So, this is equal to so it will be each cube over 24 times 1 plus nu ok.

(Refer Slide Time: 04:43)



So, let us find out the sum of D 1 2 plus 2 D 6 6. So, this is equal to E h cube. So, what is D 1 2 E h cube times nu divided by 1 minus nu square plus 2 times D 6 6. So, it is 2 E h cube over 24 into 1 plus nu ok. So, this becomes E h cube and this is nu minus 1 minus nu square. So, there is a 12 here. So, this is a 12 here plus 1 over 1 plus nu ok. So, you add these up and what you get is ultimately E h cube by 12 into 1 minus nu square. So, this is same as D ok. So, D 1 1, D 1 2 plus 2 D 6 6 same as D, and D we have defined earlier which is E h cube divided by 12 into 1 minus nu square.

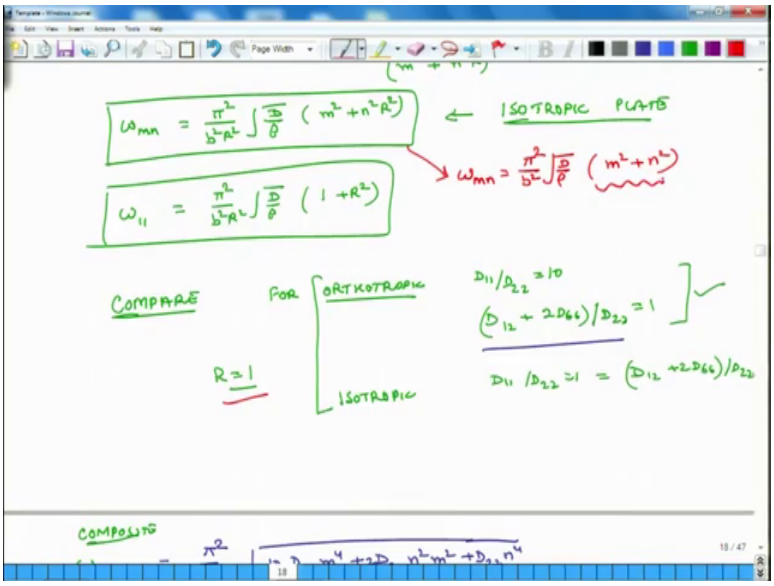
(Refer Slide Time: 06:27)



So, we plug this relation and this relation back into our equation for the composite plate. So, what we end up getting is omega m n is equal to pi square over b square R square times square root of D over rho times m 4 plus 2 m square n square R square plus n 4 R 4 is what we get. And this thing is nothing but m square plus n square R square the whole thing squared. If I square, so this so this square root of this is m square plus n square R square.

So, this is equal to pi square divided by b square R square times D over rho times m square plus n square R square that is it. So, these are the natural frequency is angular frequency natural language frequencies for isotropic plate ok. Now, what we will do is we will make some comparisons and see what it tells us. But, before we do that very quickly, we will see that, what is the first natural frequency. So, omega 1 first natural angular frequency for this case will be pi square over b square R square times D over rho and times m square is 1 plus R square that is the first natural angular frequency.

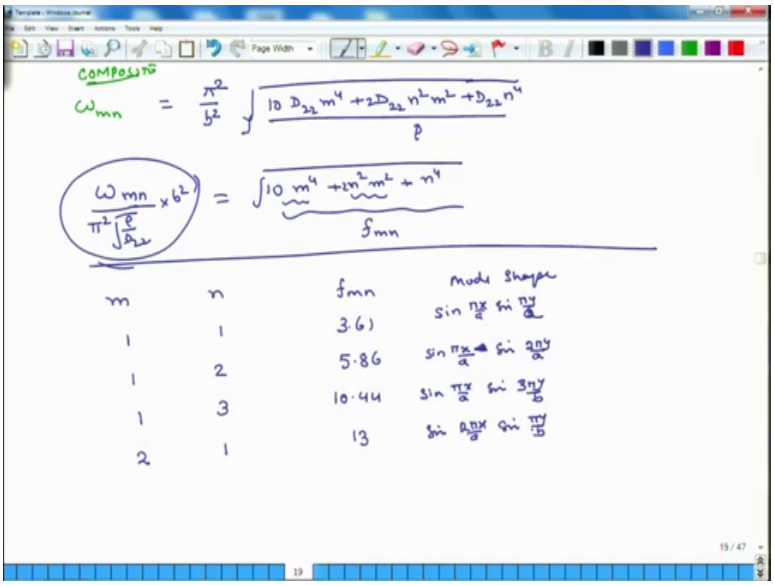
(Refer Slide Time: 08:41)



So, next thing is what we are going to do is we will compare the results from isotropic plates as well as from composites ok. So, if we have to do that comparison, we will make some assumptions for orthotropic plate. We have already developed the relation, but there we will but we also need some numbers. So, we will say D 1 1 over D 2 2 is 10. What does that mean that in the two directions, the plate is very easy to bend? In the one direction, it is very stiff ok.

And the other thing we will say is D 1 2 plus 2 D 6 6 divided by D 2 2 is 1. So, this is for orthotropic plate. And for isotropic plate, we already know that D 1 1 over D 2 2 is 1, and it is same as D 1 2 plus 2 D 6 6 divided by D 2 2. This we have already shown, we do not have to worry about it. But, both these cases we will evaluate assuming that the plate is a square, so R is 1 ok.

(Refer Slide Time: 10:25)



So, let us construct the table. So, before we construct table, we will simplify these relations. So, omega m n for composite with this simplification, if I put these ratios, back in this expression where is that. In this expression, so D 1 1 I say as D 1 1 this thing it becomes D 2 2 right. And this is also D 2 2 and this is 10 times D 1 1 D 2 2, so this is 10 D 2 2. So, we are taking a material, which is whose D 1 1 is 10 times D 2 2. This twice of D 1 2 plus 2 D 6 6 D 2 2 and so on and so forth ok and R is 1 R is 1.

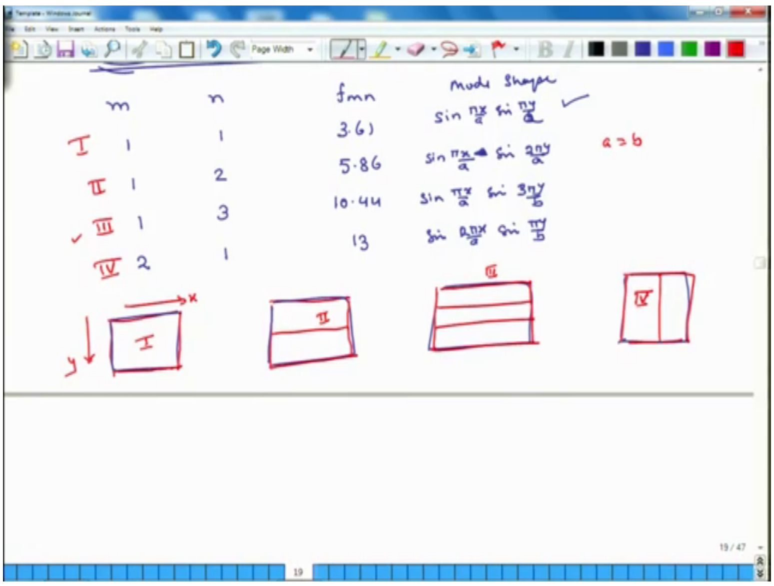
So, if we do that, this entire expression simplifies to pi square over b square R was 1 times 10 D 2 2 m 4 plus D 2 2 n square m square plus D 2 2 n 4. And then of course there is a del p in the denominator. So, I take D 2 2 out, so omega m n when I normalize it, I divide both sides by pi square and this D 2 2 to comes out rho over D 2 2 and into D square. So, this becomes just 10 m 4 plus n square m square plus n 4 this thing. And there is a 2 here, so there is a 2 here, because D 1 2 plus 2 D 6 6 is divided by D 2 2 is 1. And this entire factor gets multiplied by 2, so that is a 2 here ok.

So, let us construct a table. So, this is omega m n, and this is omega this entire thing is normalize with respect to some constant, which is b rho D 2 2 and pi square ok. So, m n and let us call this f f m n. So, this is not frequency, but some normalized frequency ok. So, we will calculate the value of this normalized frequency. So, my aim here is to find out those combinations of m and n such that f m n is list.

So, I want to find out the first four such values which are minimum. So, when will when it be minimum, when m is 1 and n is 1 that is the first thing. So, m when m is 1 and n is 1, then this is 10 plus 2 12 plus 1 13 square root of 13. So, square root of 13 is 3.61 ok. What is the next one if m is 1 and n is 2, then this becomes 1st term becomes 10, 2nd term becomes 488, 3rd terms becomes 1 note 16. So, 8 and 16, 24 plus 10, 34 so its square root of 34 5.86; if I take any other value of m and n, it will be more than this. So, this is the one which I take.

Then the 3rd lowest f m n n for this combination m is equal to 1 and n is equal to 3, and this value comes to be 10.44. And the 4th one is 2 and 1 and this comes to 13 ok, so this is there. Now, what is the mode shape, so mode shape will be sin pi x over a sin pi y over b right, and a and b are same in this case. Here, it will be sin pi x times 2 no times pi x over a sin 2 pi y over a, because a is same as b. And this is here is so these are the mode shape. And this is sin pi x over a sin 3 pi y over b, and this is sin 2 pi x over a sin pi by over b.

(Refer Slide Time: 15:55)



So, if I have to see what are so if my plate, so let us make these are four this is a square plate. So, if I the first mode if I have the first mode if this is active, then x will be 0 on this line, and x will be 0 on this line. Because, when so this is suppose x axis and this is y axis, then x will be 0 at x is equal to 0; and x will be 0 at x is equal to a ok.

This is the first mode, and of course it will also be 0 on the edges. So, this is the 1st case, this is 2nd case, this is 3rd case, this is 4th case. So, the plate will vibrate in such a way that all the 5 4 edges are fixed. And the middle of the plate will do this, but edges will remain fixed. Now, let us look at the 2nd mode, 2nd mode tells us that it is sin pi x over a times sin 2 pi by over a right.

Student: (Refer Time: 17:23).

Which means b same as a, because aspect ratio is a is equal to b ok. So, you can call it your b is does not matter, these are rectangular plates ok. So, again here the plate of course the outside edges will be fixed, and because this is 2 pi over a, it will also be fixed in at the middle line ok.

Student: 250, 250, 250.

Because, aspect ratio is 1.

Student: (Refer Time: 18:01)

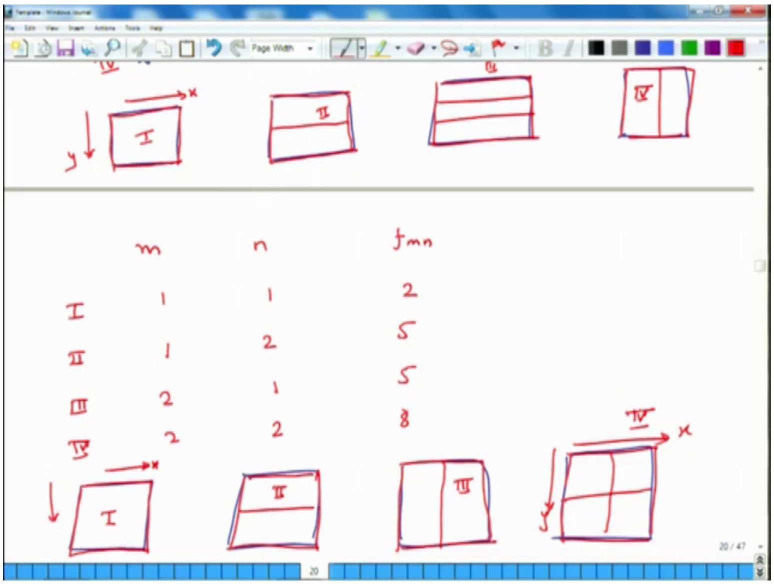
Yes, we had mentioned that aspect ratio is 1 R is equal to 1 ok.

(Refer Slide Time: 18:17)

So, what does this mean? That if this is the x axis, so the first frequency we have already termed. The second frequency will be such that if this is the x axis and this is the y axis, then the second mode will be such that the plate will tend to do like this also like this. And of course, it will do this, but it will also it actually it will do this thing ok. Then the 3rd mode is this guy. So, here all the four edges are of course going to be fixed. And the node lines for zero displacement for w will be here also. So, this is 2nd and this is 3rd.

And in the 4th case, outside edges again are going to be fixed, but an additional node line will be in the vertical, so this a 4th order. So, this is how this plate is going to vibrate. Now let us look at isotropic plate. So, in isotropic plate if R is 1, then omega m n is equal to pi square over b square times D over rho times m square plus n square that is all ok, because R is 1. So, if we plot this m square plus n square, if we create a table.

(Refer Slide Time: 19:47)



So, m, n, f m n and let us look at the first four modes. So, this is 1, 1, 2. 1, 2, 5. 2, 1, 5 and 2, 2, 8. So, how do the modes look in this case? So we will draw it. So, these are the four cases. In the first case, the fixed boundaries are all the edges are fixed. And those are the only node lines, this is case 1. In the second case, I have one half wave in the x direction. So, this is again x direction and this is y direction. 1 half wave in the x direction, and 2 half waves in the y direction. And of course, all the edges are fixed. So, it is like this is case 2.

In case 3, it is the same shape, but rotated 90 degrees. So, this is 3. And in case 4, the plate will have 2 half waves in x direction and 2 half waves in y direction, it will be like this ok. So, the point is, that even though the geometry of the plate is same. They will vibrate with different mode shapes and a different frequencies, because in isotropic plates, the plate is equally stiff in x and y direction. But, this plate is very soft in the y direction. So, it tends to bend more easily in the y direction. So, it develop more half waves in the y direction early on, and those are lesser modes.

So, this is what I wanted capture and it provide you some glimpse into how plates vibrate, when they are made up of composites. And they could be their vibration patterns could be fundamentally different, then the patterns seen in isotropic plates. So that concludes our discussion for this week and we look forward to seeing you on Monday, which is coming Monday.

Thank you, bye.