Advanced Composites
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Lecture - 61
Buckling of Composite Plates

Hello, welcome to Advanced Composites. Today is the start of the 11th week of this course. And over this entire period of this particular course, we have looked at different aspects of composite plates; starting with equilibrium equations, then how to solve these equilibrium equations in context of semi-infinite plates, finite plates.

And for finite plates we approach the solution using different methods, exact methods and as well as approximate methods. And then we also looked at thermal stresses. And in the last week we covered the area of vibrations as it relates to composite plates. Today we will discuss a different topic which relates to buckling of composite plates. Now, in the so, that is what we plan to do today.

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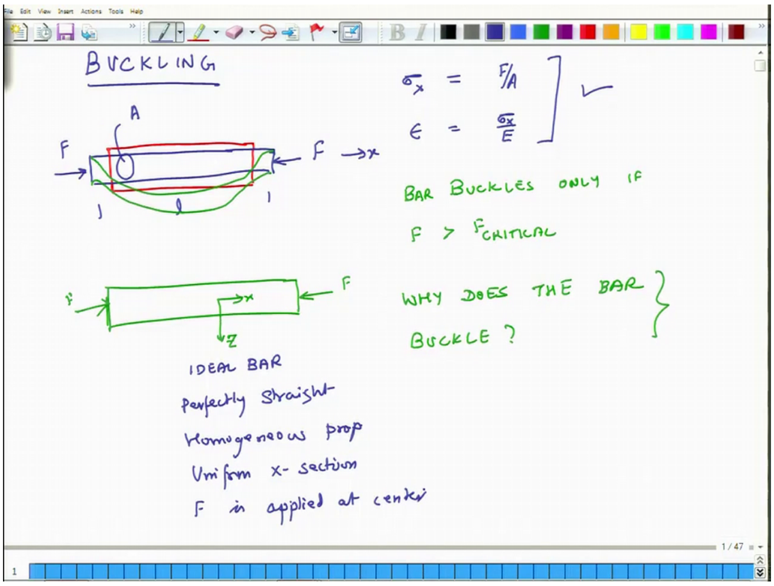

So, this is the overall theme for this week. And this is an extremely important topic because, here the way we look at differential equations which govern composite plates, they have to be developed in a somewhat different way. But before we start doing that I wanted to give you some basic idea about buckling. So, suppose I have just a regular metallic bar. And it is a perfectly straight bar and I apply a compressive force on it, I apply a compressive force on it, then what happens? Essentially the bar keeps on getting smaller and smaller.

So, if the force is F and the cross sectional area is A, and it is, let us assume that the cross sectional area is A and it is uniform across the whole length. Then the stress in the bar will in the x direction; so, if this is x direction. Then the sigma x will be F over A. And the strain in the bar will be sigma x divided by Young's modulus of the bar.

And as long as I keep on increasing, as I keep on increasing the force, the overall length of the bar, it will keep on becoming smaller and smaller. So, this is what we expect would happen, but in reality that does not happen. What happens in reality is that, if I exceed the external stress on the bar or external force on the bar beyond a certain threshold, then this bar deforms.

So, we would expect using these relations that the deformation would be just of this type. So, it will become a little fatter, and it will become a little shorter. But in reality what happens is that the bar if it. So, it deforms if the force exceeds a particular threshold and the bar becomes something like this, something like this. So, here the bar buckles, but it does not buckle at any load, it buckles only if F is greater than some critical value, let us call it F critical. If F is less than F critical, then it does not buckle if it exceeds or equals this critical value it buckles.

So, the question is why does it buckle? Because as long as I if I look at the geometry of the bar and if it is perfectly straight, and as if I apply the forces, the only displacement I should expect should be in the x direction. It should not have any displacement. So, if this is my x axis, and this is my z axis. The only displacement because of F which is in x direction should be in the x direction. So, it should only deform in the x direction it should not deform in the z direction. But still we see that in reality the bar buckles and it deflects out of plane. So, it develops and it develops displacements in the z direction or w displacements.

So, the question is why does why does the bar buckle? Why does the bar the buckle? Ok so, this is the question we want to address. At a conceptual level and then we will use the same thought process for composites. So, the answer to that is; that if the bar was a strictly perfectly straight. And it had homogeneous properties, and the force which was being applied was perfectly at the center of the bar and the bar was not getting disturbed by air or anything external, then the bar would never buckled, it would never buckle.

But in reality what happens is that so this is an ideal bar. So, what is an ideal bar? Perfectly straight homogeneous properties, uniform cross section, it is uniform cross section, and F is applied at center so, it is actually applied.

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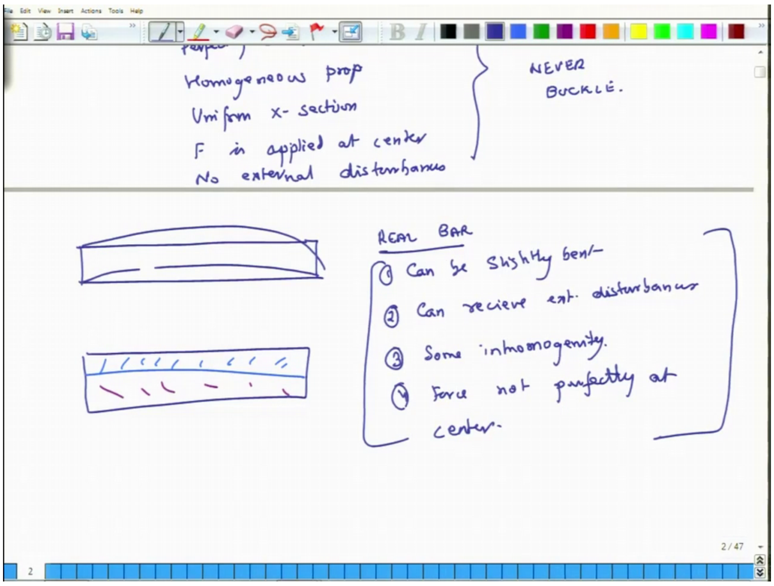

And no external disturbances, no external disturbances. So, if the bar if this was the ideal situation, it will never buckle. But in reality this is not the case. In reality what happens is that either the bar is somewhat like this, instead of being a perfectly straight bar, the bar may be in reality, it may be something like this, it may be slightly bent.

So, it may be so real bars can be slightly bent, or can receive external disturbances. What do I mean by external disturbances? That as I am pressing it may be seeing some external load because of some vibrations from the ground or there may be air pushing it, or you know whatever. So, the bar may be straight, but it may be getting out of plane loads because of air or some vibrations or whatever.

So, or it could be slightly bent, or it could be that the properties of the bar may not be homogeneous. So, maybe on the top half, the properties of the bar may be something and on the bottom half they may be slightly different. And if they are slightly different then we have seen in composites, this generates a b matrix.

So, third thing in reality could be some in homogeneity ok, or force not perfectly at center. So, due to all these reasons buckling gets triggered; buckling gets triggered. And what do we do if we have to account, if we have to predict buckling, if we have to predict buckling, what do we have to do? We have to account for these types of variations ok, we have to account for these types of variations, or a better approach could be, so to predict buckling to predict buckling what do we have to do?

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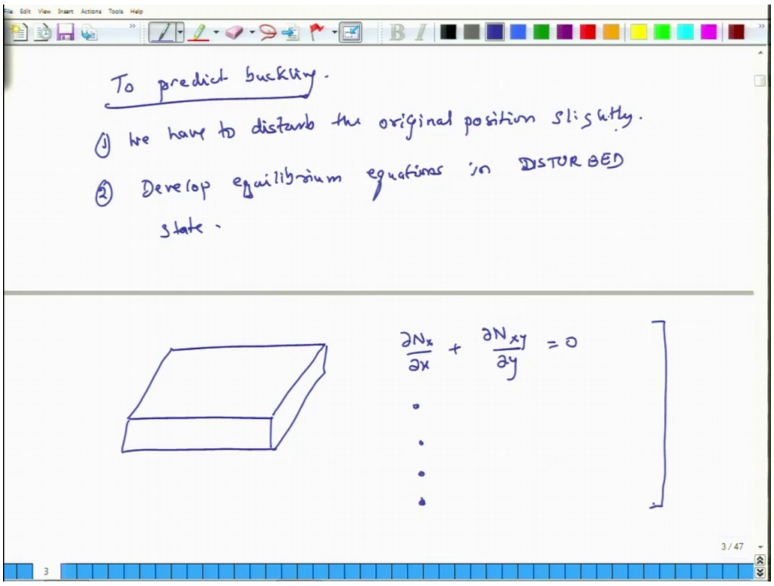

First is, we have to disturb the original position slightly. We have to disturb the original position slightly. In mathematical terms we say that we have to perturb the original configuration slightly. And then develop equilibrium equations in disturbed state. So, what do I mean by develop equilibrium in the disturbed state? When we were looking at developed the differential governing differential equations for the plate, we said, that the plate is perfectly flat and straight. So, our original equations which we developed were these del N x over del x plus del N x y over del y is equal to 0. And this we developed 5 equations, and in all these 5 equations, we assumed or we developed the equilibrium equations by considering the undeformed position of the plate.

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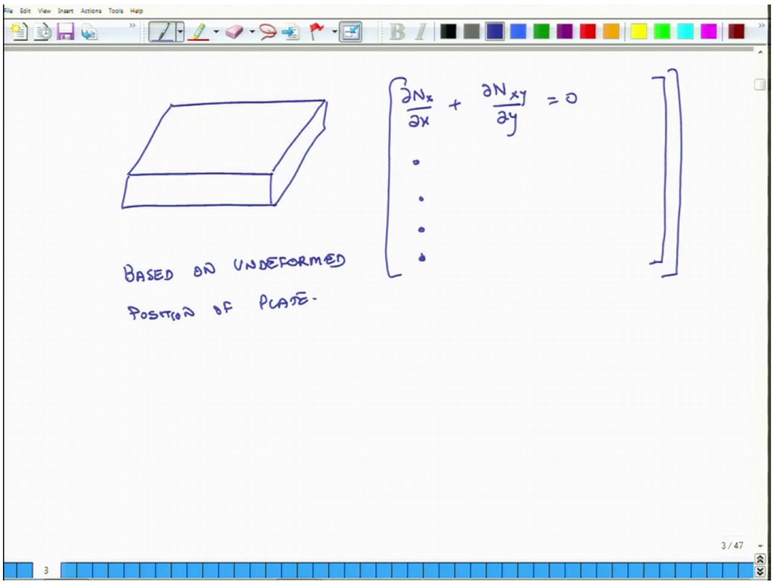

So, all these equations were developed based on undeformed position of plate, ok. We assumed that the plate is undeformed, and then we assume we develop the equilibrium equations.

But in reality when a plate is in equilibrium any structure is in equilibrium, it has already slightly deformed it; may be slightly or some more highly deformed, but the equilibrium equations strictly speaking have to be developed relative to with respect to the deformed position. Because that is the position in which the plate is in equilibrium.

So, mathematically speaking it is more accurate or it is accurate to develop equilibrium equations which account for the deform position of the plate. When we develop these 5 equations, we did not account that we assumed that the deform position and the undeformed position are virtually the same because we said that; because the displacements are so small that the undefined position and the different position are virtually the same. So, we said we thought internally and I had not explained it.

But this is what was implied that the governing differential equations for equilibrium, for deformed position and undeformed position would be almost identical. Now all that works, but that does not work in when we consider phenomena like buckling. Because in buckling, buckling will happen only if we consider either the presence of external displacement or slight imperfections in geometry and so on and so forth. So, if we have to predict buckling, we have to redevelop these equilibrium equations, and we have to redevelop these equilibrium equations based on the deform position of the plate.

So, we deform we consider the deform geometry of the plate, and then use that deform geometry and then develop new set of equilibrium equations. So, that is what we will do in the next several minutes.

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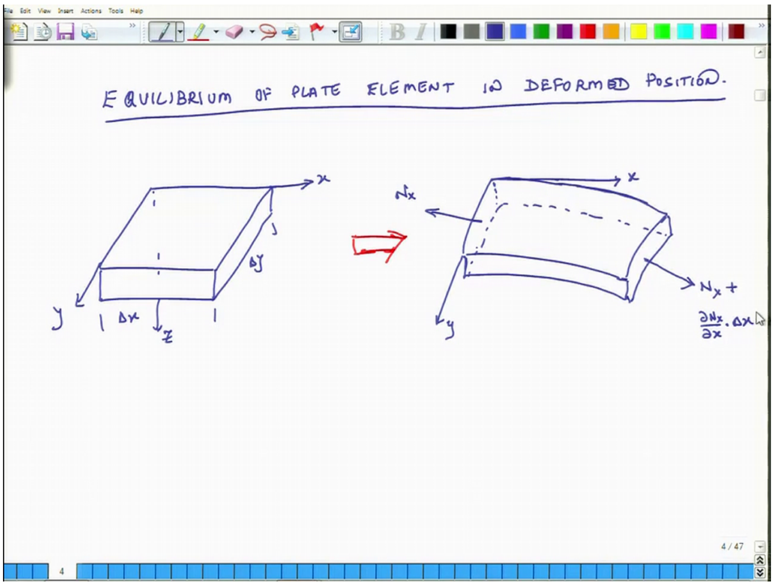

So, equilibrium of plate element in deformed position, in deformed position. So, this is a small plate element. And when this plate element so, let us consider that this is x, this is y and this is z. And when this plate element deforms what does it become? It becomes something like this.

So, it develops curvatures right. So, this is a deformed position of the plate. So, it is this plate is seeing some external loads, it is seeing N x N y N x y and because of it is deforming. Now my x coordinate is still with respect to our original system. So, this is y and the z is the vertical direction. Now in this deform position, this is N x. So, we will just look at one term. So, on this so, it is N x here, and what is the value of N x on the other side, it is N x plus. So, if the overall length of the plate element is delta x and this is delta y, then it is N x plus del N x over del x times delta x.

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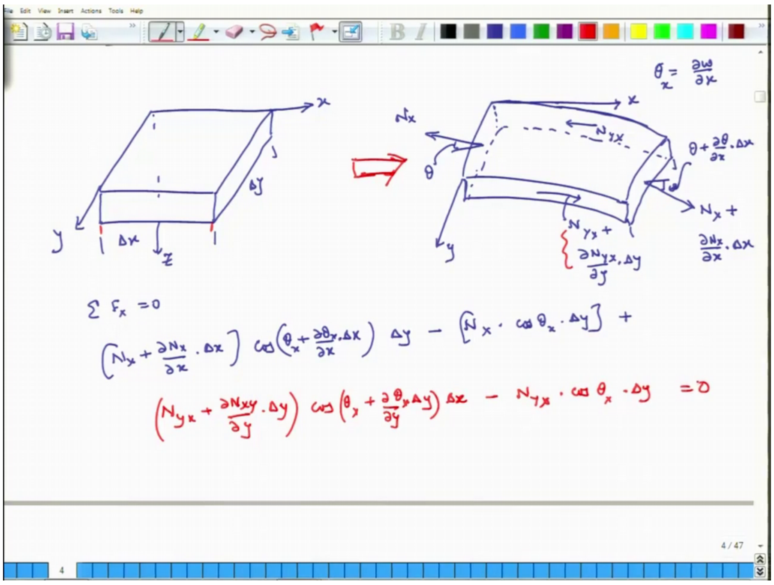

Now, if I have to develop the equilibrium equation for the x direction for the x direction. So, it is sigma F x is equal to 0, right. So, this is N x, likewise I can also put on this, this is N y x plus del N y x over del y times delta y, and here it is N y x right. So, if I have to sum up all the forces in the x direction, and that sum of forces should be equal to 0. So, remember now, see you see that this N x is not exactly aligned with the x axis.

So, I have to take the component of this N x with respect to the x axis. So, how do I do that? So, if this angle is theta. So, suppose excuse me suppose this angle is theta, then in the x direction what will it is component be? And x cosine theta, and here this angle will no longer be theta, it will be this angle, it will be theta plus del theta over del x times delta x, right. And what is theta?

So, what is theta? Theta so, suppose the plate is bending like this. So, this is my x axis, this is my x axis and plate is bending like this. So, that angle is theta. So, theta I can say that theta equals del w over del x right. So, this is theta, theta in the x direction, if the plate is bending like this, this is theta in the x direction. But the plate will also bend like, the plate is also going to bend like this, and plate is also going to bend like this. So, I can call this theta x the angle which is develops in the x direction, and similarly there will be theta in the y direction. So, we have to consider all these things.

So, just where we will what, we will do is we will just look at the x direction forces. So, it will be N x plus del N x over del x times delta x. And I have to multiply it by the cosine of the angle. So, it is cosine theta plus delta theta over delta x times delta x. And N x is force per unit length. So, what is the overall length on which N x is being applied; is delta y. So, this is delta y and then on the other phase I have N x times cosine theta times delta y ok.

Plus now, I have to see forces related to N y x, forces related to N y x. So, we consider this force. So, I get N y x plus del N y x over del y times del y and cosine. So, this was theta x right, this was theta x. Similarly, I will have a theta y, which is the rotation in the other direction.

So, it will be cosine theta y plus del theta y over del y times del y and this entire thing is multiplied by delta x, because this is the length of the element, this is the length over which N y x is being applied. And if I look at the other phase it is N y x times cosine theta y times. So, theta y and actually this should not be, theta y it should be theta x because we are considering the variation only in the x direction. So, this will be theta x and this will also be theta x.

Student: (Refer Time: 22:16).

Theta x and there will be also delta y.

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And this cosine is of this entire angle. So, this entire thing should be 0, this is what it means. Now we assume that the deformations are small, which means that these rotations also are not very large.

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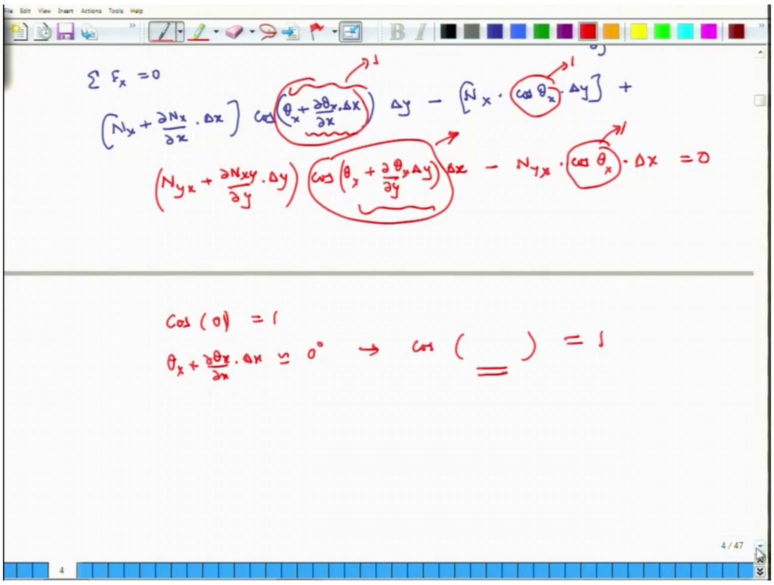

And if that is the case, then this entire angle is roughly equal to 0 degrees. It is roughly equal to 0 degrees. So, cosine of 0 is equal to 1, and theta x plus del theta x over del x times del x is approximately equal to 0 degrees. So, the cosine of this angle so, cosine of this entire angle will still be pretty close to 1. So, compared to 1, the change in the cosine of the angle, yeah.

Student: So, there should be a theta (Refer Time: 23:33) x.

So, this is delta x. So, compared to the value of cosine at 0 degrees, the cosine of this angle which is this entire angle or this angle, it is still going to be close to 1. And if it is close to 1, then this equation, then this so, this equation this thing this term, I can approximate as 1, I can approximate it as 1, I can approximate this term as 1, and I can approximate this entire term as 1. So, I end up and if I do all these approximations and I add up all the terms.

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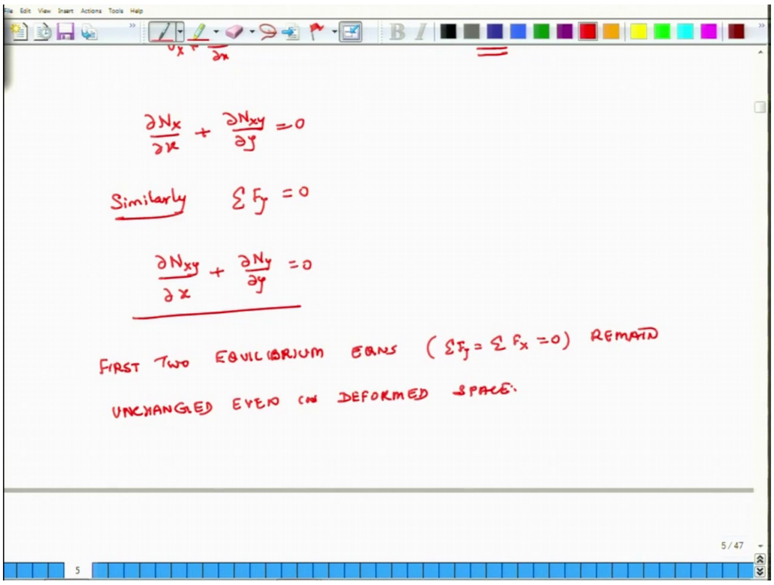

I still get my equilibrium equation for the x axis for the x direction is same. So, del N x over del x plus del N x y over del y equals 0.

So, the equilibrium equation even in the disturbed state related to x direction does not change. Similarly, if I add up all the forces in the y direction and I equate them to 0 I still get the same equilibrium equation which I had developed when I had considered equilibrium with respect to the undeformed state of the plate. So, which is del N x del N x y over del x plus del and y over del y equals 0.

So, what we see is that first 2 equilibrium equations and what are these first 2 equilibrium equations? They correspond to the condition F summation of F y is equal to summation of F x is equal to 0. So, these 2 equilibrium equations remain unchanged, even in deformed space ok. Even deformed space, this is assuming, what did we assume here?

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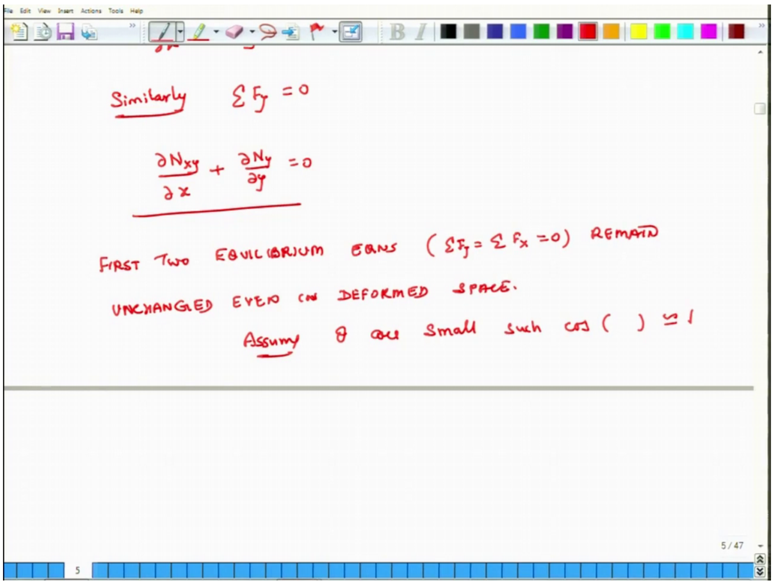

We have assumed that thetas are small such that cosine of thetas are still approximately equal to 1. This is the assumption we are making.

So, these equations remain unchanged, the first 2 equations. But we have a total of 6 equations, sigma F x equal 0, sigma F y equal 0, sigma F z equals 0, summation of moments in x y and z direction they are 0 6 equations. So, we will see which equations change their equation do not. So, we have seen these 2 equations, in the next class we will continue this discussion and we will develop equations for the other directions also. So, that is what I wanted to discuss, and I look forward to seeing you tomorrow.

Thank you.