Advanced Composites
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Lecture – 62
Force Equilibrium in z-direction for Buckling of Composite Plates

Hello. Welcome to advanced composites. Today is the second day of the eleventh week of this course. Yesterday, we were developing equations of equilibrium for a composite plate when the plate in is in the deformed position. And, in yesterday's discussion, we found that even in the deformed position the equations of equilibrium for specifically for force equilibrium, they do not change corresponding to force equilibrium in x direction and force equilibrium in the y direction. The next thing we are going to look at is force equilibrium in the z-direction.

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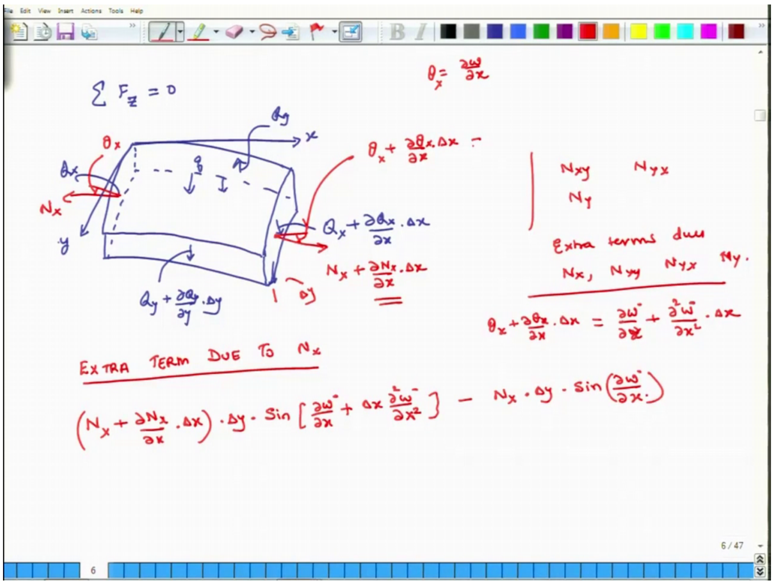

So, this is the third equilibrium equation which is sum of all the forces is equal to 0 when the body is in equilibrium; that is when the body is in the deform state.

So, I will make this picture again. So, once again, this is an over amplified view of the scheme of things. This is my x axis, this is my y axis and now what I am interested in finding out is sum of forces in the z-direction. So, what are the forces in the z-direction? One is q which is per unit area, then there are out of plane shear resultant which we had expressed as Q x and Q y. So, this is Q x plus del Q x over del x times del x and here you have Q x; likewise you have also Q y plus del Q y over del y times del y and likewise on the other face, we have Q y moving up.

Now, there is something extra on top of that because there is curvature my N x is let us say in this direction. And it will also have a vertical component ok, it will also have a vertical component. So, this N x is N x plus del N x over del x times del x and it is at an angle theta plus delta theta. So, what is this angle? It is theta plus delta theta over del x times del x and theta we have defined it as del w over del x. So, likewise here this is theta and here I have N x.

So, if I have to add up all the forces in the z direction and I have to equate them to 0 then what are the forces? So, these are the forces, but then I also have forces related to N xy and N y likewise also. So, here we are just going to illustrate the vertical components associated with N x, but we will also have vertical components associated with N xy N yx and N y. So, the only thing we are going to calculate here is the vertical component associated with N x and all other components can be calculated in the similar way which we are discussing.

So, here the plate if it is bending in the x direction, we call it theta x and if it is bending in the y direction, we call it theta y. So, this is theta x. So, here we have extra terms due to N x N xy N yx and N y and we will only compute the extra term due to N xy N x and then we will just write down the result for the overall thing. So, extra term due to n x. What is the extra term? So, this N x will have a vertical component. So, it will be what is the force total force N x into delta y Nx, it will be delta y.

So, it will be N x plus del N x over del x times del x this is force per unit length times delta y that is the total force times the sin of this angle sin of this angle. Now this angle sin of this angle is theta x. So, what is theta x plus del theta x over del x times del x delta x this is if I substitute theta x by d w d x. So, I get delta plus second derivative of w and these are mid plane displacements del x square times delta x.

So, I we have to multiply this by sin of. So, this is the force in the on the positive plane and in the negative plane it is acting on the negative direction. So, it is N x times delta y times sin of delta w over delta x. Now again, I said that these angles are very small.

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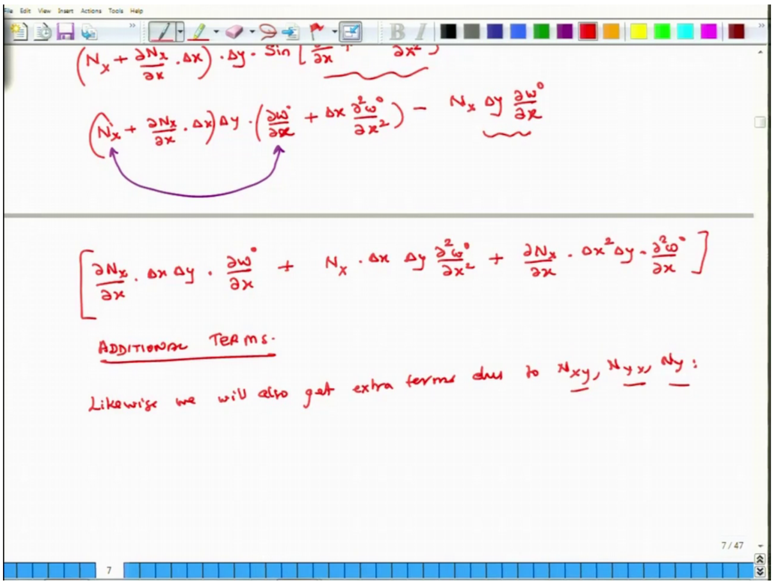

And if the angle is 0, then the sin of this entire expression in the thing will be 0, but if the angle is very small. And if I compare a small number to 0, I cannot ignore it because I am comparing it to 0. So, in that because of that reason earlier when we were taking cosine, I was taking cosine of a number very close to one and a cosine of 0 they are same, but here those differences are appreciable when I compare the sin of this angle to sin of 0.

So, if this angle is very small, then the sin of this angle is same as the angle. So, I get N x plus del N x over del x times del x del y and then I have del plus del x del 2 w naught over del x square minus N x delta y del w not over del x, ok. So, I get if I simplify this what do I get? The contribution from N x so, this term cancels out when this gets multiplied by this term. So, the terms which I have left with are 3 terms. So, what are those 3 terms.

So, the first term is if I multiplied del N x over x, so what I get is del N x over del x times delta x delta y times del w over del x this is 1 plus N x times delta x square delta y, this thing plus I get del. And then I have to multiply del N x over del x times delta x square delta y times delta w over second derivative with respect to.

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So, this is there. So, these are the residual terms. So, these are additional terms. So, these are the additional terms; likewise we will also get extra terms due to N xy N yx and N y. So, likewise we will get extra terms also due to N xy N yx and N y. So, we add; so what we have to do is we have to compute all these terms using this methodology and add up all those terms for the original equation for the z-direction was so far un deformed geometry.

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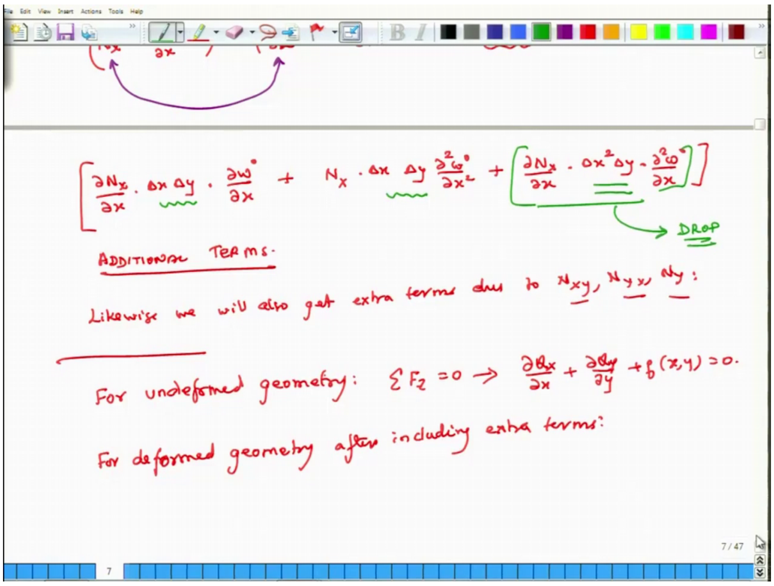

What was the original equilibrium equation? So, if I did f z is equal to 0, it gave me del Q x over del x plus del Q y over del y plus Q x y equals 0, this is what it gave me.

Now, for deformed geometry after including extra terms, so, what do we get, but before we include extra terms, I wanted to make one more comment. See here, I have a multiple of delta x delta y, here I have gained I have a multiple of delta x and delta y, but here I have a multiple of delta x square times delta y. So, if I make my delta x and delta y extremely 0, this term will very rapidly vanish to 0. So, I can drop this term and it will not make any difference to my results, because if I make everything hundred times less, this will become less by add in even faster piece.

So, this I am going to drop because it is a higher order term. So, I am going to drop. So, from each of these N x N xy N yx and N y, I get two extra terms, I get two extra terms and if I add all those terms and include them in this equation, my overall new equilibrium equation for the deformed geometry we get. So, I am just going to write down the equation.

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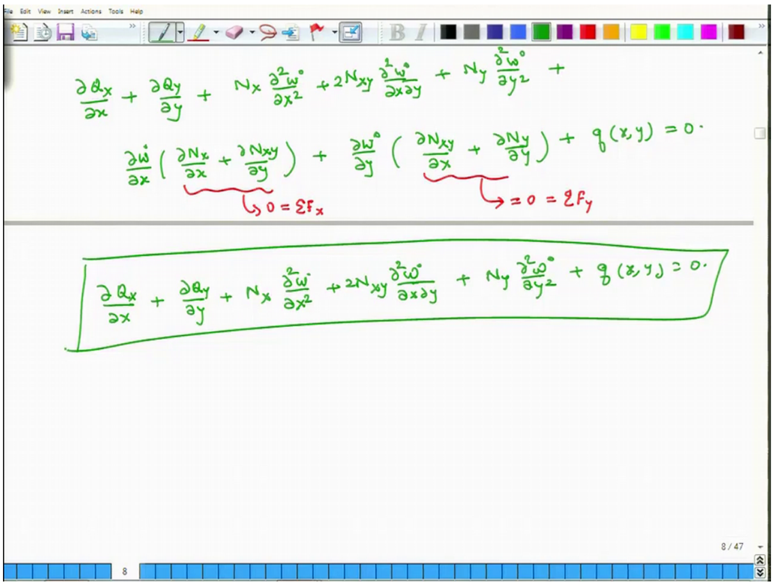

So, it is del Q x over del x plus del Q y over del y umm plus. So, the new equations which I get are N x del 2 w naught over del x square plus N xy twice del 2 w naught over del x del y plus N y del 2 w naught over del y square plus del w by del x and in the bracket; we have del N x over del x plus del N xy over del y plus del w over del y del N xy over del x plus del N y over del y plus Q x y equals 0.

Now, this term in the bracket this is 0; why is this 0 because of the first equilibrium equation; what is our first equilibrium equation? We have developed it and we have seen it several times. So, this is the first equilibrium equation ok. So, this is 0 because of the first equilibrium equation which is sum of forces in the x direction is 0 and the corresponding equation is actually this thing. So, it has to be 0. Similarly, this term should be equal to 0 because of second equilibrium equation which is sum of forces in the y direction is equal to 0.

So, my final equation for the third direction is this guy, ok. So, this is my modified equilibrium equation for the z-direction, if I consider the deformed shape of the system ok. Then these terms involving N x also appear in the third equilibrium equation. Now, we can do this analysis further and what we will do is we will complete the remaining three equations in tomorrow's class, and then we will actually start solving some of these equations and see how using these equations we can predict buckling loads for different types of plates.

So, this is what I wanted to discuss today. And I look forward to seeing you in tomorrow’s class as well.

Thank you.