Advanced Composites
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Lecture – 63
Moment Equilibrium Around x, y and z-directions for Buckling of Composites Plates

Hello. Welcome to Advanced Composites. Today is the third day of the ongoing week which is the second last week of this course that is the eleventh week. In last two days, we have been developing equilibrium equations for composite plates as pertinent to the deformed configuration of the plate, not necessarily which is same as the undeformed position we have been doing this, so that we are able to capture the buckling phenomena in composite plates. What we have till we have seen still so far is that the first two equations related to force equilibrium in the x and y z, y directions they remain unchanged when we considered the deformed position of the plate.

However, the equilibrium equation related to sum of forces in the z-direction does undergo changes and it does require some additional terms. So, this is one difference we have noticed.

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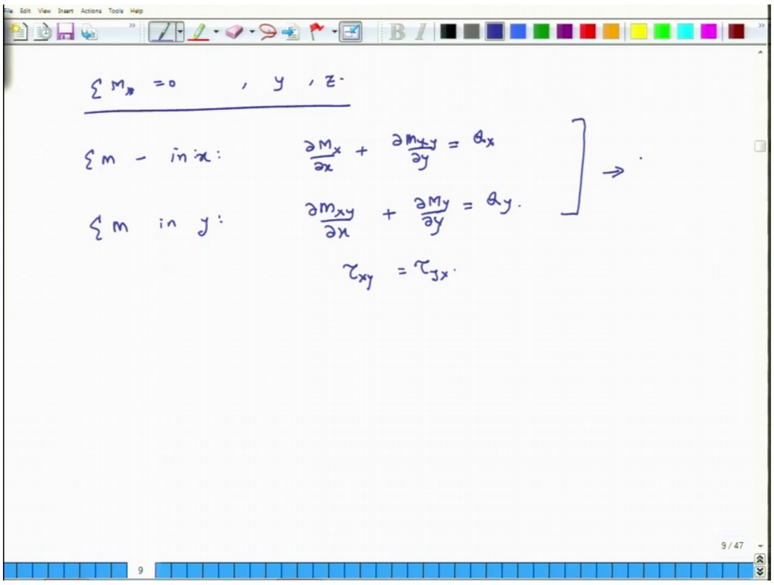

Now, the other three equations for the deformed for equilibrium are sum of forces for M x to they should be 0 and then the same thing should be true for sum of forces in the y direction and in the z direction some of I am sorry. So, this is moment equilibrium around x axis, y axis and z axis. So, for these three directions if you do the analysis in the way exactly as we have discussed what we will see is that we do encounter some additional terms, but those additional terms are so small and of higher orders that they can be neglected.

So, the only equation with changes is the third equilibrium equation which relates to the condition sum of forces in the z direction as 0. So, sum of moments in x direction it remains unchanged and it is del M x over del x plus del M xy over del y and that equals Q x and if we do some of moments in around y axis then that equation also does not change. So, that is del M xy over del x plus del M y over del y and that equals Q y and then we have shown that the sum of moments in the z direction it gets idently satisfied because power x y is equal to tau y x because of this condition.

So, if I plug these equations and I put them back into the equilibrium equation for the z-direction force equilibrium equation because here it involves Q x, Q y and the Q x is and Q y's they are describe in terms of M x and M xy, then my overall equation which is a combined equation for sum of moments around x-axis, around y-axis and sum of forces in the z-direction all these are individually 0. The overall equation which governs these requirement it comes to this.

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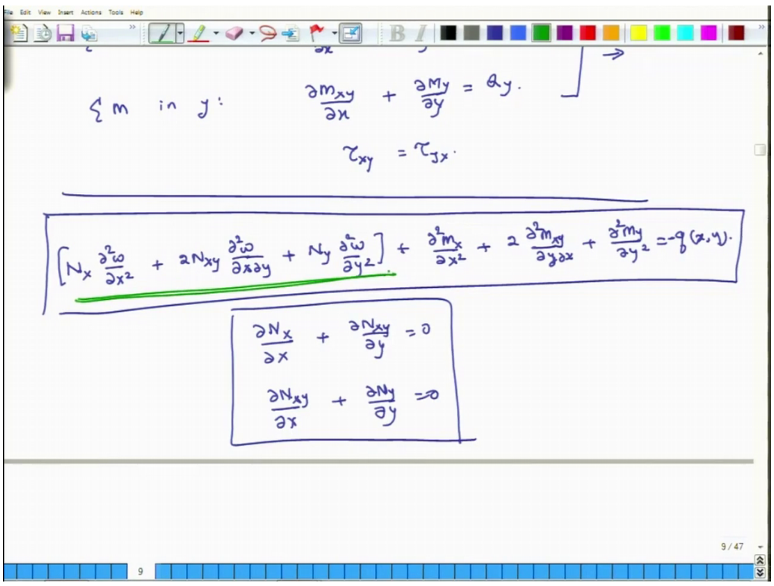

So, it is N x. So, this is the overall governing equation. What we will do is, we will actually use this equation.

So, this is one governing equation and what are the other two governing equation for composites plates if I seek their equilibrium and deformed configuration. So, this is so, these are the other two equilibrium equation. So, if I have to capture effect like buckling I have to consider these three equations rather than the other equations which we have developed earlier. And, we what we find is that these terms are the additional terms which come into picture as I am considering equilibrium of a plate whether it is a composite plate on anisotropic plate. So, for such types of plates if I am considering their buckling phenomena then these three terms related to N x, N xy and N y appear in the third equilibrium equation. And, once we consider that these terms we will be able to capture the buckling phenomena.

What we will do next is actually in today's as well as the remaining classes we will actually solve this equation for different situations and see how this helps us predict the overall buckling phenomena.

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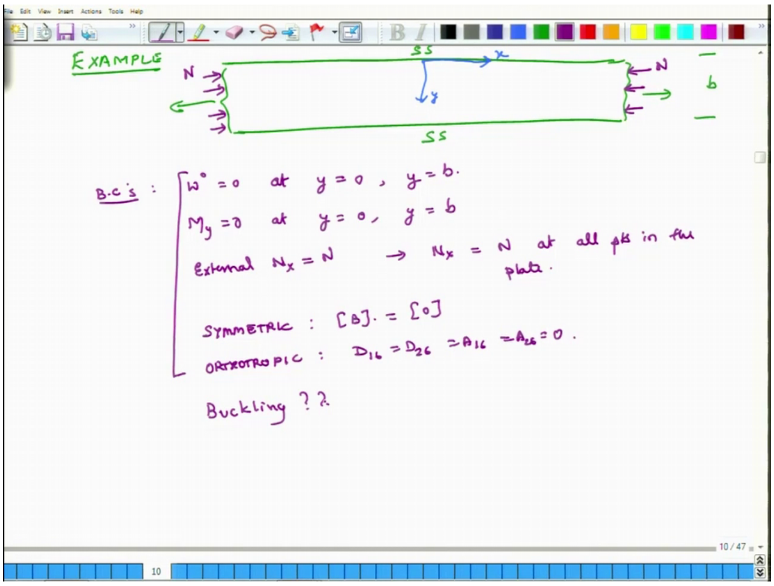

So, the first example we will consider is of a very long infinitely long plate. So, this is a plate and this is infinitely long, it has a finite length. So, this dimension is b it has a finite length and the plate are is getting. So, what is so, especially about the plate the plate is simply supported on this entire length in this entire length.

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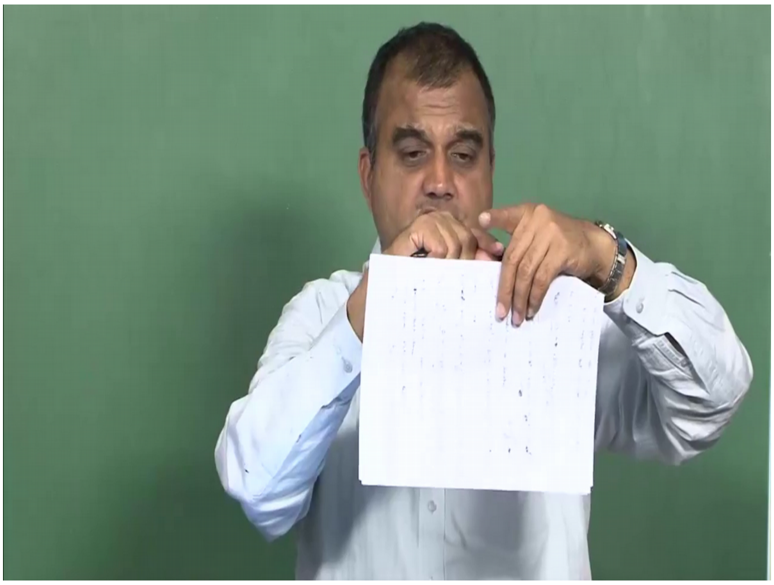

So, you have a long plate. So, the plate is extremely long and this plate is entirely simply supported on this edge and on this edge on these two edges. So, this is one edge, this is another edge, this is the length direction. The plate is very long. So, it simply supported on this edge and simply supported on this edge. So, this is one so, which means that what are the boundary condition. So, if this is my x-axis, and this is my y-axis and the thickness is the z-axis, so, I am looking at the plate in the x-y plane not in the x-z plane which we have been to typically doing.

So, in the x-y plane on this, so, I can position my axis system somewhere at end. So, this is my x-axis let us say this is my y-axis. So, what are the boundary conditions? The boundary condition is that w naught is equal to 0 at x is equal to, no at y y is equal to 0 and y is equal to b. Also because the plate is simply supported along it is length moment and what moment will it be M x M y and M x y? It will be M y. So, M y this is also 0 at y equals 0 and y equals b. The other thing I am doing is that somehow the plate is very long. So, somehow I am subjecting it to an external load uniformly distributed load and this is some N.

So, the other boundary also is that external N x equals N and because the plate is just rectangular and very long N x will be equal to N at every point in the plate. So, because yeah so, this implies N x is equal to N at all points in the plate. Some other extra conditions we will say that the plate is symmetric and because the plate is symmetric what does it mean? The B matrix is 0 and also the plate is orthotropic. So, it means D 1 6 is equal to D 2 6 is equal to A 1 6 is equal to A 2 6 is equal to 0.

So, these are the conditions and now, what we have to do is, we have to figure out whether this plate when it is subjected to this compressive load, if it will it buckle or not and how will it buckle that is what we are interested in.

So, we have to understand the buckling of this plate, ok. So, now, we look at these three equations because we have to solve these equations to understand the buckling phenomena and what we find is that the equations for equilibrium x and y direction. We do not have to worry about these because N x depends only on u and v. And, a y only depends on u and v only and N x y also depends only on u and v because the b matrix is 0 because b matrix is 0. So, what is N x? N x is equal to A 1 1 times epsilon x plus B 1 1 times epsilon y naught plus A 1 6 times gamma x y naught plus B 1 1 times curvatures, where W comes into picture. But, because B terms are 0, these two equations do not influence the solution for W.

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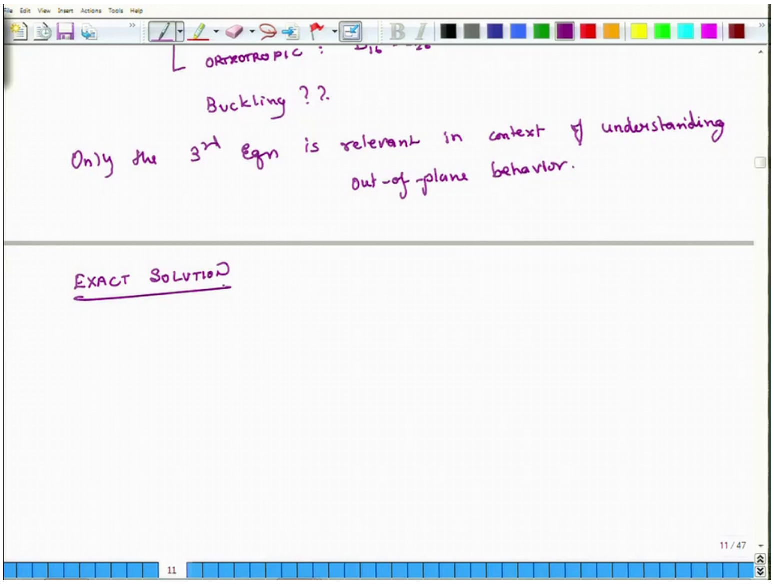

So, only the third equation is relevant and because we are thinking about buckling we want to see how the thing is going to deform and at what load. So, we are interested in finding out W. So, only third equation is relevant in context of determining W naught x y, ok. So, we are not interested in can to context of out I will not call it determining W naught x, y, but in context of understanding out of plane response out of plane behavior.

So, what do we do; we have to solve this differential equation. We have to solve this differential equation and the way we will do it is remember, how do we solve these term? One is if we are interested in finding the exact solution is that we gets a displacement function and that displacement function has to do three things. It has to satisfy all the kinematic boundary conditions, it has to satisfy all the secondary boundary condition related to forces and moments and the displacement function also has to satisfy the differential equation.

So, this is if you want to develop an exact solution. If we wanted some approximate solutions using valerate or Galerkin method, then we only satisfy some boundary conditions. In valerates we are we only solve we only ensure that displacement based are kinematic boundary conditions are satisfied and an Galerkin we satisfy all the boundary conditions. But, we do not worry about the accuracy of the other for we do not worry too much about satisfying the governing differential equation. But, here we are going to develop an exact solution. So, will develop an exact solution.

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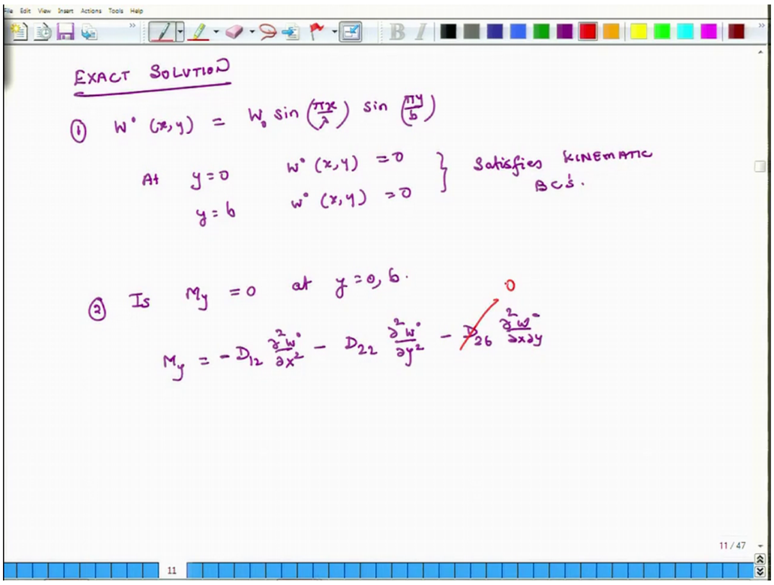

So, we assume that w naught x, y is some constant times sin pi x over lambda sin pi y over b, and lambda is a parameter and will understand what this means later. And, then now we check whether this satisfies the boundary conditions.

So, at x is equal to, I am sorry; at y equal 0, w naught x, y is 0 and at y equals b, w naught x, y is again 0. So, it is satisfies kinematic boundary conditions, ok. The next thing we check is is M y equal to 0 equal to 0 at y is equal to 0 and b. So, what is M y? M y is equal to it is D 1 2 del 2 1 naught over del x square and there is a negative minus D 2 2 del to w naught over del y square minus D 2 6 del 2 w naught over del x del y. Now, D 2 6 is 0 because of the material.

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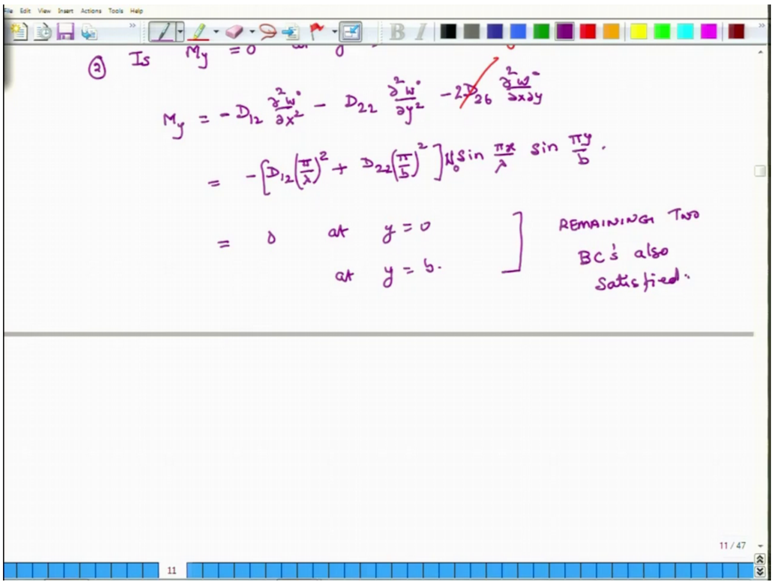

And, if I plug in the value of w which we have assumed what I get is minus D 1 2 and this is pi over lambda whole square plus D 2 2 pi over b whole square sin pi x over lambda sin pi y over b. So, this is the function we have chosen and times this constant w naught. So, I will write 2 naught here, ok.

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Yeah. So, this is there and now we check whether this M y is 0 at so, this value is equal to 0, at y equals 0 and also at y equals b. So, what this means is that remaining two BC's also satisfied they are also satisfied.

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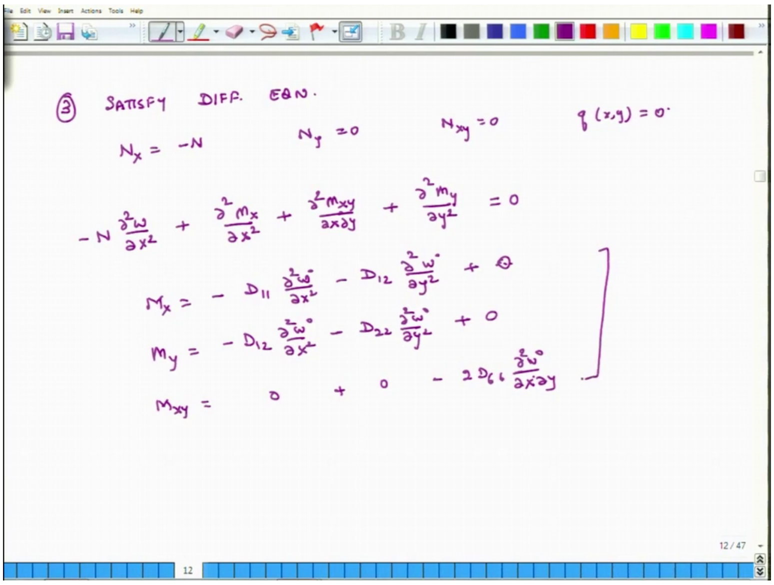

So, the third thing is we have to ensure that it should satisfy the differential equation, ok. Now, in the differential equation the value of N x is what minus N. The value of N y is 0 because it is a long plate it has no external loads, it has no external loads in the y direction. So, N y 0 and N x y is also 0 at all points in the so, in the system. So, my overall modified differential equation becomes minus N del 2 w naught del x square plus del 2 M x over del x square plus del 2 M xy over del x del y plus del 2 M y over del y square and this equal to 0, because there is no external transfers load also on the plates Q is also 0.

So, M x is equal to what minus D 1 1 del 2 w naught over del x square minus D 1 2 del 2 w naught over del y square plus other terms 0 D 1 6 and D 2 6 term is 0. Similarly, M y is equal to minus D 1 2 del 2 w naught over del x square minus D 2 2 del 2 w naught over del y square plus other terms involving D 2 6 and D 2 6. And, M x y is equal to 0 plus 0 plus 2 D 6 6 del 2 w naught over del x del y and there is a minus here.

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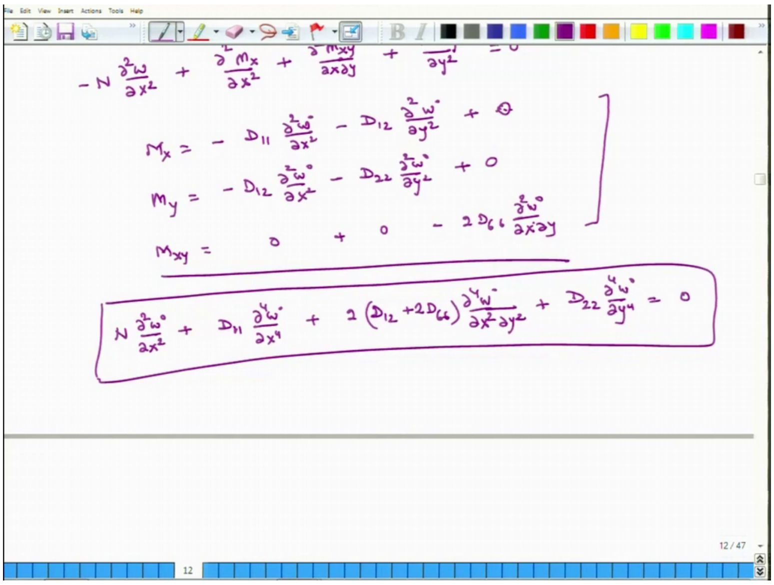

So, what I get is, if I plug in all these numbers what I end up getting is N del 2 w naught over del x square plus D 1 1 this is a fourth derivative 0, ok.

So, what we will do is now, what we will do is that we will plug in the value of w naught the assumed function and we are assume that w naught is this. This function and we will plug in this function in this simplified differential equation which will also help us understand buckling and using this then we will compute what kind of buckling loads and wavelengths are involved in this buckling phenomena. So, that is precisely what we plan to do tomorrow and then till then have a great time.

Thank you.