Advanced Composites
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Lecture – 65
Buckling of a Simply Supported Finite Plate

Hello, welcome to Advanced Composites. Today is the 5th day of the ongoing week which is the 11th week of this course. And over this entire week, in last 4 days, we have been discussing the phenomena of Buckling as it gets seen in composite plates. We have developed a different set of equations which govern the buckling phenomena. And we have also solved the problem of buckling in context of plates which are extremely long or infinitely long.

And what we found in such place is that the buckling load is as each buckling load is associated with the characteristic wavelength of the in the plate. And as I keep on increasing my buckling load, this wavelength keeps on decreasing. So, I for given buckling load I can compute lambda, or for given lambda I can compute corresponding in the value of N. Now, what we are going to do is we are going to extend this discussion in context of finite plates, which are rectangular in shape. And we will start discussing in context of simply supported plates.

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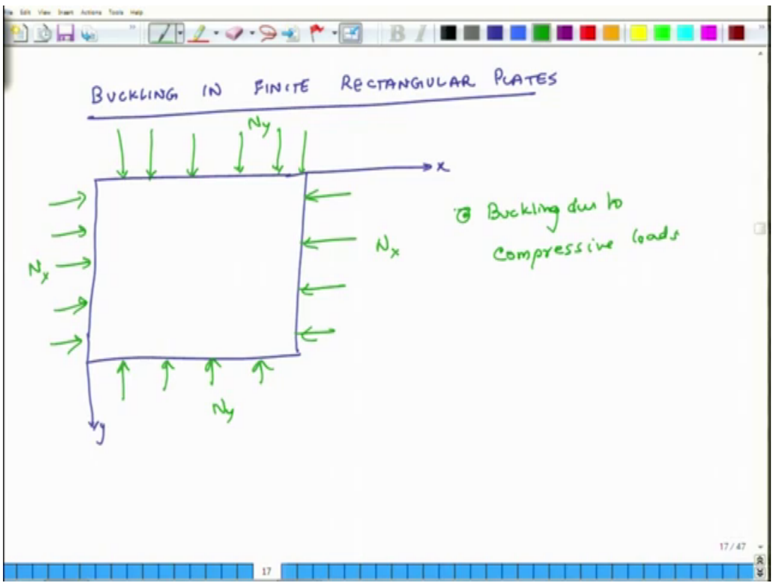

So, suppose so this we will we will discuss about buckling in finite rectangular plates. So, suppose there we have a plate of some dimension. Now, this plate, suppose this is my x-axis, this is y-axis, and the z-axis is perpendicular plane of the screen. This plate can buckle, because of three different sets of forces or force resultants.

So, first one is N x. If I apply some compressive force in the x direction, it can buckle. It can also buckle, because of application of N y, so this is N y. So, if it buckles because of N x or N y, this is because of application of compression loads. So, here buckling due to compressive loads.

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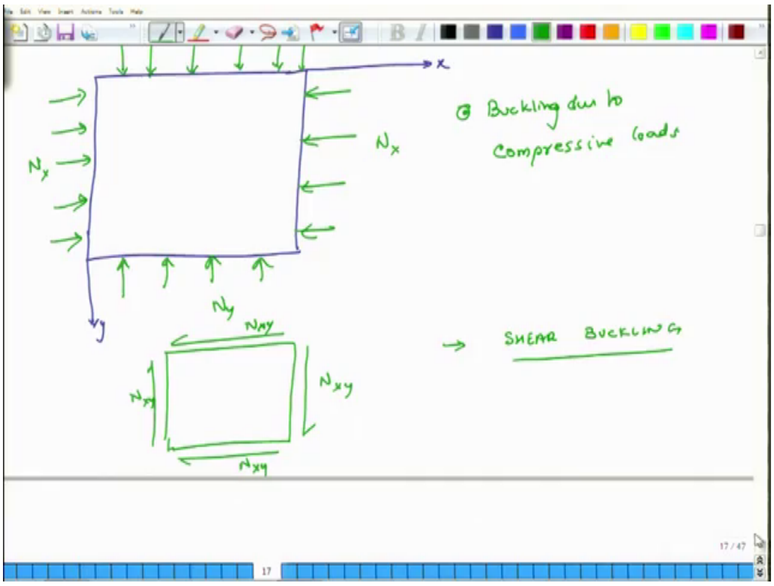

But, the plate can also buckle, if I apply shear loads on it. So, suppose I apply N x y, then also it can buckle. This is called shear buckling, this is called shear buckling. So, the plate will exhibit some shear strain in plane shear strain up to a certain point, but if the external shear stresses shear force resultant N x y exceeds a certain threshold it can buckle, so that is called shear buckling.

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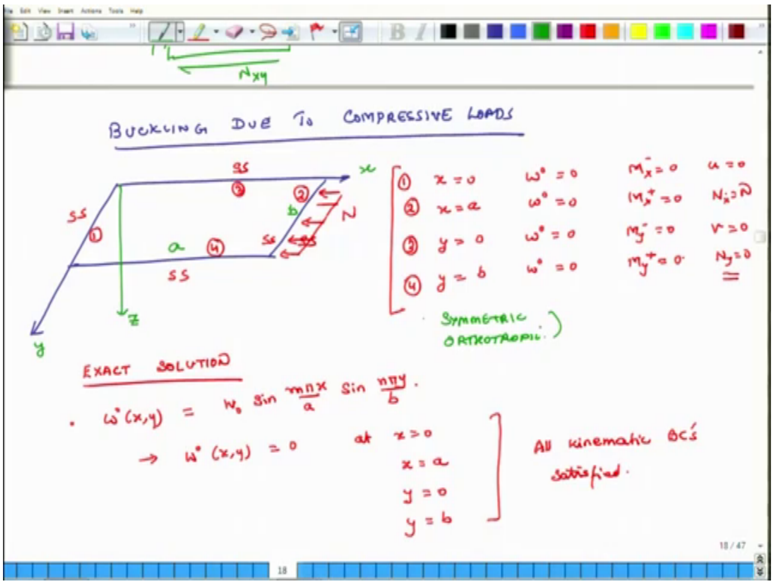

So, what we will discuss right now is buckling due to compressive loads ok. This is what we are going to discuss, so buckling due to compressive loads. So, once again we consider a plate. So, the dimensions of the plate are b and a this is my x axis, y axis, this is the z axis. And the plate is simply supported on all 4 sides. It is a rectangular plate. And it simply supported on all four sides. And I have to explicitly say what the simple supports mean, the simple supports ensure, so an on this side it is also seeing some, so it is simply supported on this, and also it is seeing some compressive stress. Let us call that N.

So, what do these simply supports do? So, what the simple supports ensure is that x is equal to 0, which is s 1, w naught equals to 0 and M x equals to 0 ok, and we will call it M x minus. Then s 2 x is equal to a and of course, u is equal to 0, because if u was not 0, then if I apply N then the plate will just run in it will just move so it will never buckle so, u is 0. On x is equal to a w naught equal 0 M x plus is equal to 0, but u is not constraint so, but N is equal to N x is equal to N.

So, here the simple support is such that it facilitates the motion in x direction at x is equal to 0 it constraint the motion in x direction. So, we have to be very clear about that. And at y equals 0, w equals 0 M y minus equals 0, and y equals b w naught equals 0, and M y naught M y plus equals 0. And what about u and what about v actually it really does not matter really does not matter.

Actually it will matter I am sorry so I will say that on y is equal to 0 v is equal to 0, and here n y equals 0. So, v is constraint to move in the y direction. So, the plate is free to move in the y direction on this s 1 s 4 on s 3 it is not free to move on s 3 it is not free to move. So, this is s 1 s 2 s 3, s 4. If it was not free to move in the y direction on both the s s 3 and 4, then there would be a poison compression.

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There would be a because, basically if I am pressing a plate in the x direction, because of poison effect it will try to expand. If I restrict the motion and y axis in the on these two s s, then essentially I am also applying N y in this. But, all I am trying to do is I just want to apply force in the x direction that is all I am trying to I am applying force in the x direction. I am not applying any force in the y direction, which means I am allowing the plate to move freely in the y direction to have the poison expansion I am not constraining it so that is the problem definition.

So, so N y is 0 which means I am not having any external forces. If V was 0, and s 4, then N y would be applied and that will create some other issue. So, these are the 4 conditions. So, we again develop an exact solution it develop an exact solution. So, we assume. So, once again when we look at the governing differential equations, we are interested in finding out the out of plane deflection pattern for this thing, and at what load this thing is starts to buckle.

So, first two equations are not going to play a role, because the terms involving D matrix are all 0. So, first two equations do not involve do not involve w s. So, the only equation we are going to various about the third equation. And for exact solution we have to gets a solution which satisfies all the boundary conditions, and it also satisfies all the governing differential equation.

It also should satisfy the differential equation, if I am looking for an exact solution. So, this is w naught x y is equal to w naught sin m pi x over a sin n pi y over b. So, when we check this. So, w naught x y is equal to 0 at x is equal to 0 x is equal to a y is equal to 0 and y is equal to b. So, all kinematic BC's satisfied.

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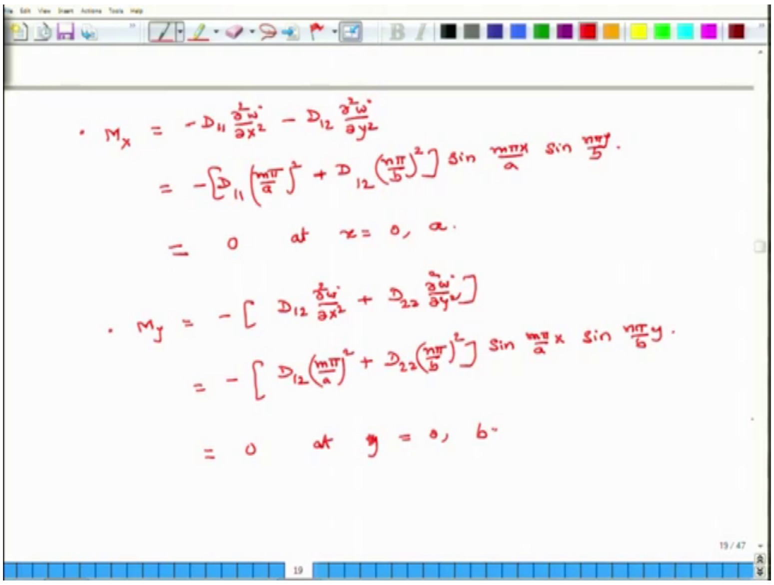

The next thing is M x and M y, M x should be 0 on x is equal to a, and x is equal to 0, and M y should be 0 on y is equal to 0 and y is equal to a. So, what is the definition for M x. So, before we create expressions for M x I should state very clearly that this plate is symmetric an orthotropic ok. It is symmetric as well as orthotropic.

So, if that is the case, then M x is equal to minus D 11 del 2 w naught over del x square minus D 12 del 2 w naught over del y square. And this is equal to minus D 12 m pi over a whole square minus plus D 22 n pi over b whole square times sin m pi x over a sin n pi y over b it a mistake here. So, this is D 11 and D 12. So, this is equal to 0 at x is equal to 0, and also at x is equal to a. So, moment the M x condition is satisfied.

Similarly, M y is equal to minus D 12 del 2 w naught over del x square plus D 22 del 2 w naught over del y square. And this is equal minus D 12 m pi over a square plus D 22 n pi over b square. Sin m pi over a x sin n pi over b times y. So, this is also equal to 0 at y equals 0 and b. So, the boundary conditions are moment are also satisfied.

So, those are the boundary conditions relevant in context of the third equation. So, we do not have to worry about boundary conditions related to u and v, which we have discussed here, because right now what we are wondering is the boundary conditions related to the third equation ok, where only w s and slope related conditions are involved.

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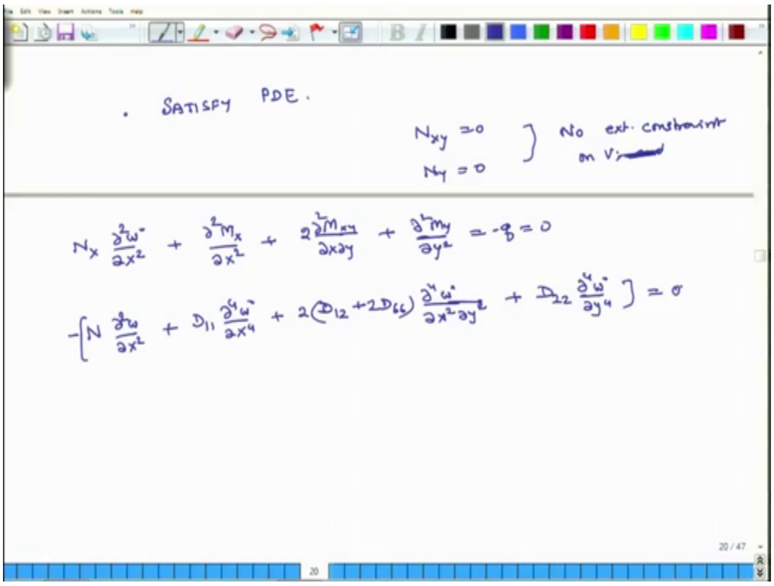

So, if that is the case, then the third thing is satisfy the PDE partial differential equation. So, what is the partial differential equation. So, partial differential equation is N x del 2 w naught over del x square N x y and N y are 0 they are 0, because no external constraint on v and right.

So, if V had been constraint, then these would have gotten generated, so because this is the case, so because of the way we have defined the problem N x y, and N y are going to be 0. So, only N x term is going to be there. So, this is there plus del 2 m x over del x square plus 2, and this q is 0 so I put it there. And if I substitute M x M x y, and M y in terms of w what I get is, and N x is minus N x is minus N minus 0.

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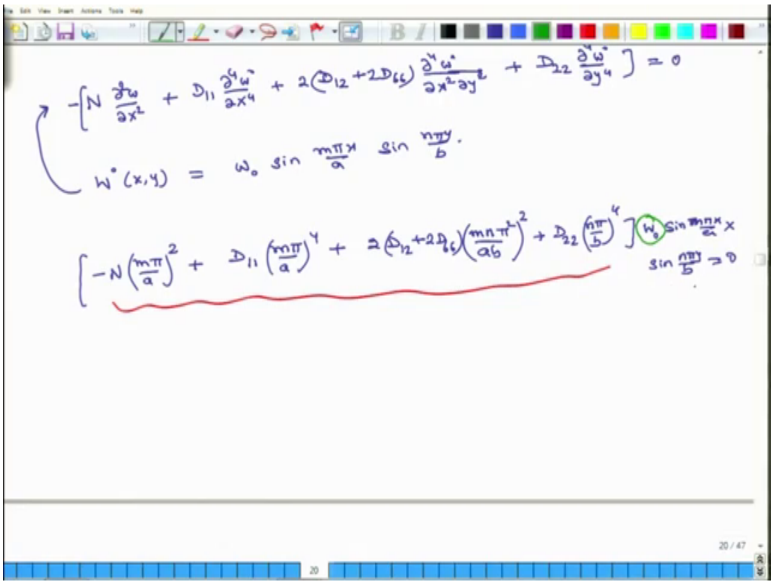

Now, what we do is W naught x, y is equal to w naught sin m pi x over a sin n pi y over b. So, we substitute this in this equation. Then what we get is, minus N m pi over a whole square plus D11 m pi over a 4 to the power of 4 plus 2 D12 plus 2 D66 m n pi square over a b whole square plus D22 n pi over b to the power of 4 times, W naught sin m pi x over a into sin n pi y over b equals 0. So, this is the term in bracket and there is w naught here. Now, this is this equation is an outcome of the differential equation.

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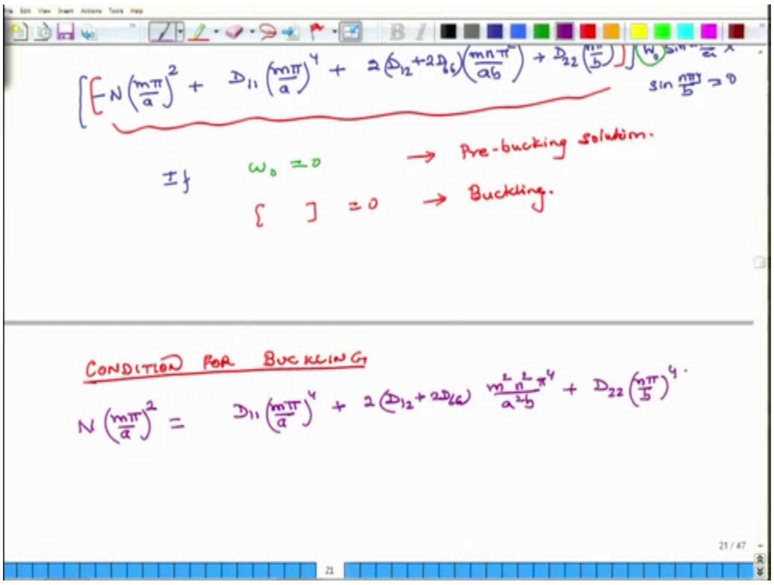

And if the body is going to be in equilibrium, then this differential equation which reflects equilibrium condition based on Newton's laws. It has to be valid at all times and that is possible only. If so it is going to be possible only if w naught is equal to 0, which means that the plate will not buckle all the term in red brackets all the term in red brackets is going to be 0, then also this term this equation will be identically satisfied.

So, this w naught equals 0 is for the condition when for the pre buckling configuration pre buckling solution. And this is the solution once the plate has buckled. So, this plate will buckle, when n becomes equal to becomes such large that it equals to other terms. So, the condition for buckling what is the condition for buckling condition for buckling is N m pi over a whole square equals D11 m pi over a to the power of 4 plus 2 D 1 2 plus 2 D66 m square n square pi to the power of 4 divided by a square b square plus D 22 n pi over b to the power of 4 or n n n is the load at which it will buckle.

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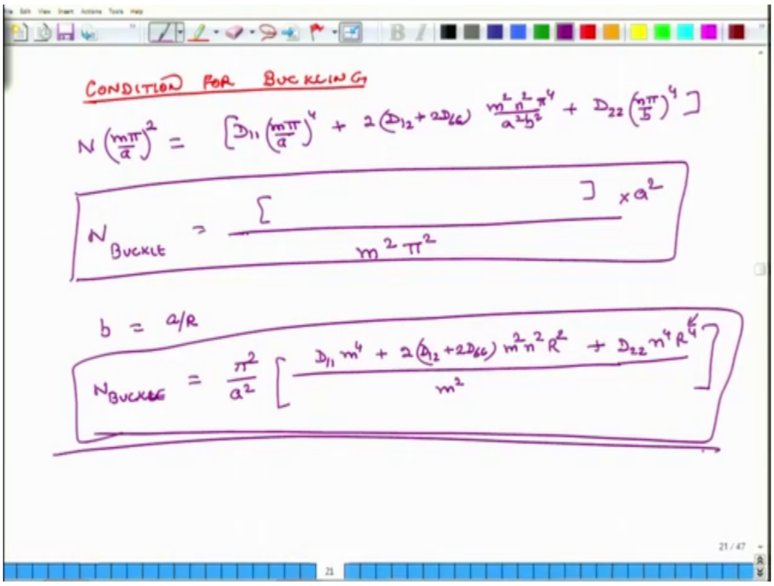

When N equals this right hand side n times m pi divided by a whole square it equals the right hand side, then the condition for buckling have arrived. So, N so N buckle will be what it will be this entire thing divided by m square pi square into a square.

Now, this depends on the value of M and N. So, a plate will have infinite buckling loads depending on what M N and you are talking about at different buckling. So, it will have infinite buckling loads corresponding to different values of M and N. So, what is the least value of M and N, what is the least value of M buckling load ok, so that will depend on the specific value of this N, when the denominator is denominator divided by m square is as small as possible. So, we will just simplify this one more time.

So, if we say that b is equal to a over some number R you know, so R is your aspect ratio, then if we put in this, then N buckle N buckle is, and if I simplify it then I get this thing. So, pi square divided by a square times D11 m to the power of 4 plus 2 D12 plus 2D66 m square n square R square plus D 22 n 4 R 4 divided by m square plus n square R square no I am sorry. So, this is what we get.

And using this expression we can compute, once we know actual values of D11 D12 D 66, and D22, and also the aspect ratio what is the first buckling load, because then we have to play with different values of m n n n c. What is the minimum buckling load at which the plate is going to buckle, so that is going to be the actual buckling load of the plate, because that is when the plate will buckle.

So, this is what I wanted to discuss today. Tomorrow, we will extend this discussion for another plate which will still be simply supported, but the plate will be loaded in two directions. So, it will be loaded in two different directions. So, this is all what we wanted to discuss today. And we will meet once again tomorrow at the same time, till then a bye to you all of you.

Thanks.