Advanced Composites
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Lecture - 66
Composite Plate Under Bidirectional Compression

Hello, welcome to Advanced Composites. Today is the last day of this course and today we will be continue our discussion on Buckling of Composite Plates. And in this case we will have a composite plate which will be simply supported on all 4 sides, but it will be loaded Bidirectionally. So, it will experience load and the x direction as well as in the y direction.

Now, these kinds of loads can be generated in 2 types of situations; either we are actually putting external load in both the directions. So, the external force is being applied in both the directions or these forces could also be developed. That suppose I have a plate which is being compressed in the x direction by some value n, and if I am constraining the v displacement in the y direction then the plate naturally will try to have an expansion as it is getting compressed in the x direction it will try to expand in the y direction due to Poisson effect.

But if I am putting constraint in the y direction so that the plate cannot expand then it will generate N y automatically. And because of that value of N y the plate will get compressed in bidirectionally and that will generate some different type of buckling pattern. So, in either of these cases we can solve the problem of plate buckling especially for simple geometry such as rectangular plates which are simply supported or claimed or in these kind of situations.

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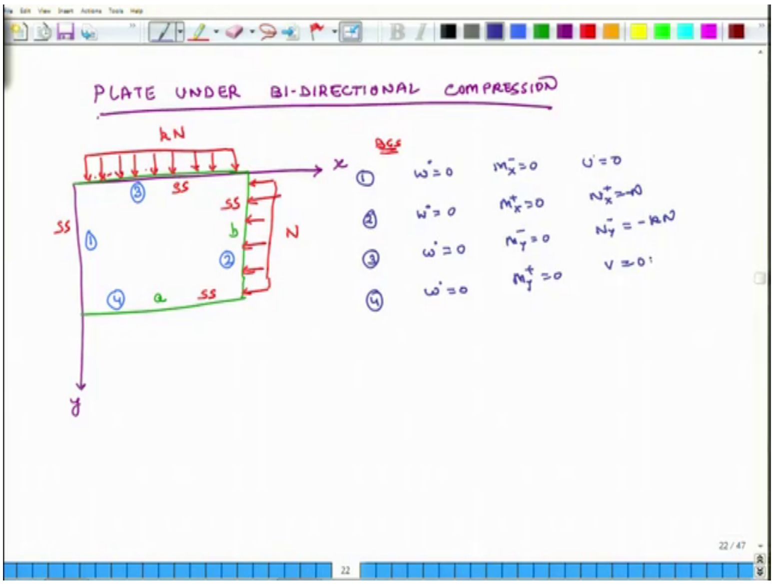

So, we will look at plate under bidirectional compression. So, this is x, this is y, that is my plate; this dimension is b, this dimension is a, I mean having the external load. So, it is uniformly we getting compressed in the x direction. And here it is simply, so plate is simply supported on all 4 sides. But once again we will explicitly straight; what do the simple supports support mean.

So, it is getting compressed in this direction and also it is getting compressed in the other direction. So, let us say this is N in the x direction and the compression force resultant in the vertical direction or y direction is some constant k time's N, now we look at the boundary conditions. So, when this kind of loading will happen when the v displacement is being constraints. So, if I double N, the value of N y in the other direction we will also get doubled. So, this is a type of simulation we situation we are trying to simulate.

So, this is are so let us call these edges. So, this is edge 1; edge 2, edge 3, edge 4. So, on edge 1 we have w equals 0 M x equals to 0, u equals to 0. On edge 2; w equals 0, M x equals to 0, and N x is equal to, N x plus. On edge 3; w equals to 0, M y equals to 0, and N y equals k times N, and that is actually negative both are compression. And on 4th w is so these are all mid plane displacements; so w equals to 0, and y plus equals to 0, and v equals to 0.

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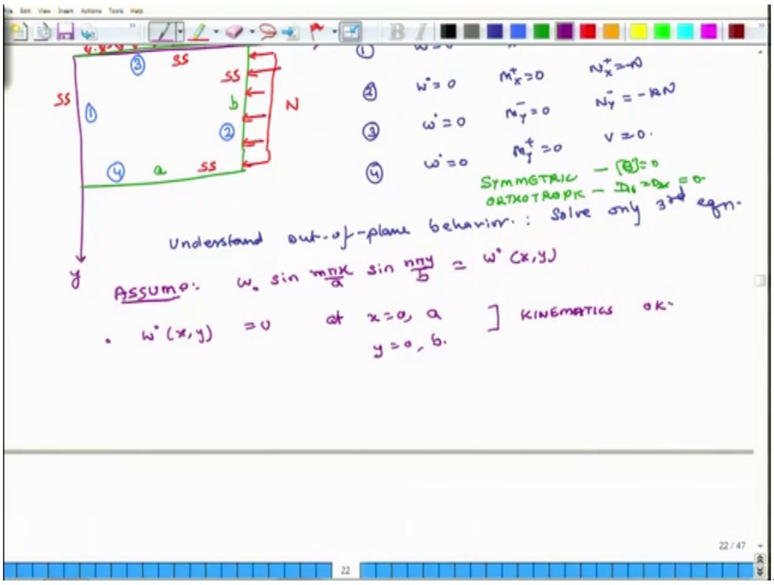

So, once again we are interested in understanding out of plane behavior. So, we solve only the third equation, why is that the case? Because we again assume that plate is symmetric and it is also orthotropic. So, because of symmetry b is equal to 0, b matrix and because of orthotropy d 1 6 is equal to d 2 6 is equal to 0 ok. So, we assume a solution, so again we are going to solve this problem in an exact way.

So, we assume that the solution is w naught sin m pi x over a, times sin n pi y over b. So, this is equal to the mid plane deflection in the z direction. So, then we see that w naught x y is equal to 0 at x is equal to 0 and a and y is equal to 0 and b. So, kinematic disease satisfied. Then we have to ensure that moment related boundary conditions on a four simply supported edges are also 0.

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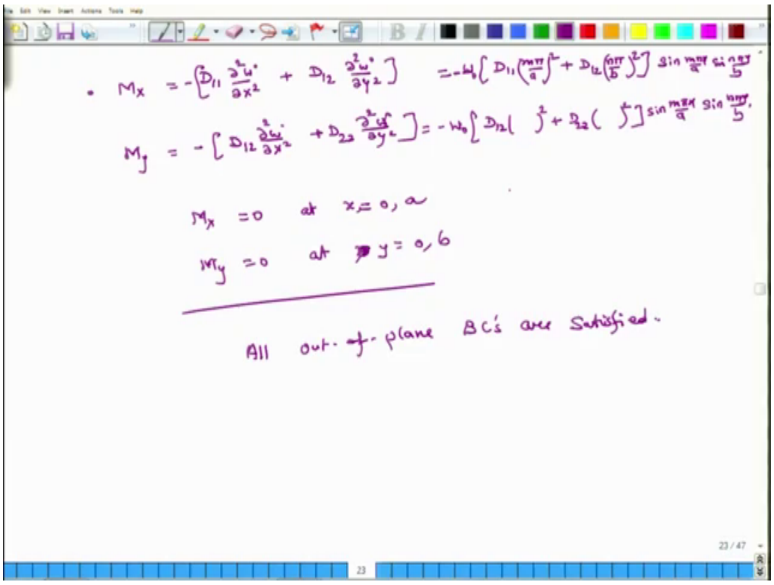

So, M x is equal to for this plate D 1 1, del 2 w naught over del x square plus, D 1 2 del 2 w naught over del y square and this entire thing is negative. And M y is equal to minus D 1 2 del 2 w naught over del x square plus D 2 2 del 2 w naught over del y square. So, this is us minus D 1 1 m pi over a whole square plus D 1 2 n pi over b whole square sin m pi x over a sin N pi over b.

And then of course, there is a w naught here and minus and this is minus w naught D 1 2 this is square plus D 2 2 the other parameter in bracket whole squared times again, sin m pi x over a sin N pi y over b. So, if I plug in the values of x and y for different edges I see that M x is equal to 0 at x is equal to 0 and a and M y is equal to 0 at x no at y equals to 0 and b. So, all BC's are satisfied, out of plane BC's are satisfied.

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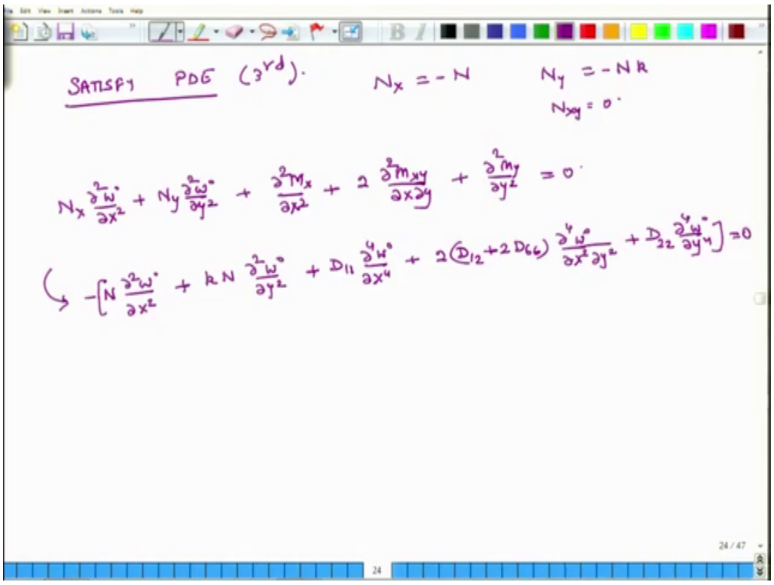

So, the last thing is that we satisfy the PDE, Partial Differential Equation and which partial differential equation, which is the third. So, now here we have both N x and N y and we know that N x is constant in the entire plate because uniformly loaded. So, N x does not change with respect to x and y and N y is also constant and that is equal to k and both of them are negative.

So, and then of course, N x y is 0. So, are overall differential equation is N x del 2 w over del x square plus N y del 2 w over del y square plus del 2 M x over del x square plus 2 del 2 M x y over del x del y plus del 2 M y over del y square is equal to 0. Now, we have already so this gives us minus N del 2 w naught so this is all mid plane displacement. So, del 2 w naught with respect to x, minus k times N del 2 w naught over del y square minus.

So, I am going to take minus out and everything inside the will be the positive. So, this becomes D11 del 4 w naught over del x 4 plus, 2 D12 plus 2, D66 del 4 w naught over del x square del y square plus, D22 del 4 w naught over del y 4 equals to 0 ok.

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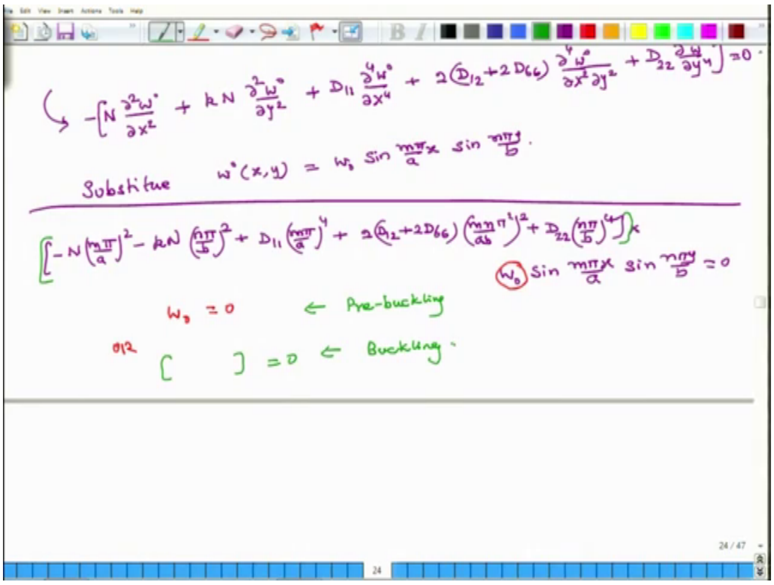

So, this is the differential equation and now what I do is substitute; w naught x y as what, w naught sin m pi over a times x sin n pi y over b. So, I get minus N m pi over a whole square minus k, n pi over b whole square plus D 11 m pi over a to the power of 4 plus, 2 D 12 plus 2, D 66 m n over a b pi square entire thing squared plus 2 D 22 N pi over b. Whole to the power 4 into w naught sin m pi x over a sin n pi y over b is equal to 0.

So, again for this equation to be true which means right left side should be is 0. So, suppose I have this entire thing in green and this is w naught. So, far this equation to whole either w naught should be 0 or this entire expression in green should be 0. So, that is my pre bulking solution and this is buckling solution pre bulking (Refer Time: 14:51).

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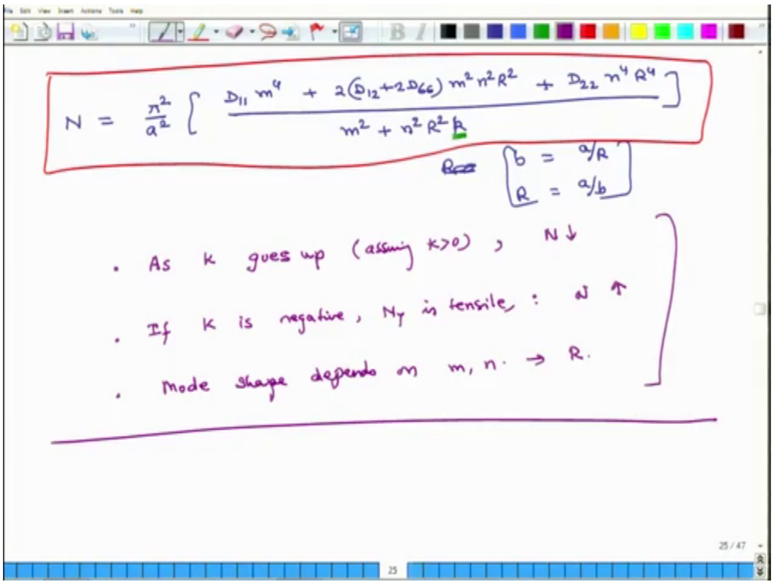

So, we look at the buckling solution so, what do you get. So, if I have to solve for N, then what I get is pi square over a square times D 11 m 4 plus 2, D 12 plus 2, D 66 m square n square R square plus, D 22 n 4 R 4 divided by m square plus n square R square times k ok. Where; R is equal to b is equal to a over r or R equals a over b so, that the aspect ratio the plate.

So, this is my this is how I can calculate the value of N at which it is going to fail, not fail it is going to buckle. Now there are two important things one is this parameter k. So, as k goes up assuming k is more than 0 which means the compression load on the y axis also is increasing, so as k goes up N decreases and vice versa ok. Second if k is negative which means that I am pressing the plate like this, but on these side I am not pressing the plate, but I am puling it ok.

If k is negative which means; in y direction so which means that N y is tensile. So, what does that mean that N goes up, N goes up so plate tends to become more stable so this is something important to understand ok. And the mode shape depends on M and N and which intern is related to aspect ratio with the plate related to aspect ratio with the plate.

So, that is all I wanted to discuss for today; and this concludes are discussion on Buckling. Tomorrow we will introduce a new topic and we will switch gears is starting next week. And we will start talking about Short Fibre Composites. So, that is what I plan to do today and I look forward to seeing all of you next week as well.

Thank you.