Advanced Composites

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Lecture – 67
Shear Bucking in Rectangular Composite Plate (Part-I)

Hello welcome to Advanced Composites. Today is the start of the last week of this course that is the 12th week of this particular course, and what we plan to do over the span of this course is finish our discussion on bucking, which we have had in the specifically in the last week. And then in the remaining part of the week we will introduce a new topic related to short fibre composites, because these types of composites are very widely used in the industry, and we will discuss how to analyze these types of composites also.

So, the first part we will discuss little bit more about bucking phenomena and, then we will move into the area of short fibre composites and with this we will have a closer to this particular course on advanced composites. So, earlier we have discussed buckling of composite plates, when they are subjected to compressive loads and the direction of the compressive loads is normal to the plane of the, to the edge.

So, if this is the edge then the compressive force is normal; to this edge it may also be normal to the other edge. Today we are going to discuss slightly different type of bucking known as shear bucking. So, an in this the plate is subjected to in plane shear loads and, because of those shear loads also the plate may buckle, if the shear loads exceed particular threshold. So, this is type of buckling which we like to understand better especially in context of composite plates. And let us see how we analyze these types of problems.

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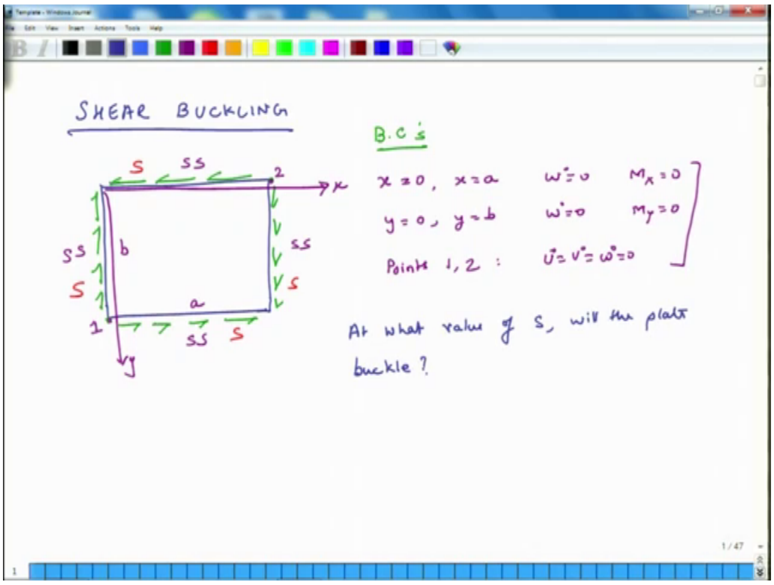

So, our theme is shear buckling; so, to give you an idea how shear buckling works is suppose this is composite plate.

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So, if I subjected to normal loads with buckles like this or if I subjected to the normal load in this direction, then it will buckle like this. But it may also buckle in shear, because if I subjected to shear forces.

So, then it tends to do this thing ok. So, in this case the deflection pattern is something like this, but when it buckles may shear, then it is deflection pattern as a little more complicated. So, this is the type of buckling we will like to discuss. So, schematically if I have a rectangular composite plate and I want to I will be subjecting it to shear and on the so, this is being subjected to shear stresses or shear forces on all of its edges ok.

Now of course, if I subject the plate to a shear, it will also tend to rotate. So, I do not want that the plate should rotate otherwise it will be free to rotate and will be just so, the way what I do is that I pin these points corners of the plate. So, what are my boundary conditions?

So, for specifying boundary conditions first I need to know what is the coordinate system. So, my coordinate system is this is my x axis and this is the y axis ok. So, at x is equal to 0 so, this is the corner x is equal to 0 and at x is equal to a w is equal to 0 and M X is equal to 0, which means that the plate is simply supported on these two edges. And similarly I am also going to simply support the plate on the other two edges as well.

So, if that is the case then on y is equal to 0 and y is equal to b. So, what is a is the length of the plate and b is the width of the plate. So, on the other two edges also the out of plane deflection w naught is 0 and M Y is 0 because it is simply supported. But the other extra condition I am going to specify so, that buckling actually happens is that I am going to pin these points. So, let us call this point 1 and let us call this point 2 so, add points 1 and add point 2 U is equal to V is equal to w is equal to 0. So, these are all met plane displacements and this is what the boundary conditions look like. And what is the shear force which is being applied?

So, let us call that so, I am applying shear force resultant and it is S on all 4 sides ok. So, it is S Newton's per meter so, that the shear force resultant. Now, so the question is at what value of S fill the plate buckle. So, once again to solve this problem, we have to solve the differential equation associated with the z direction.

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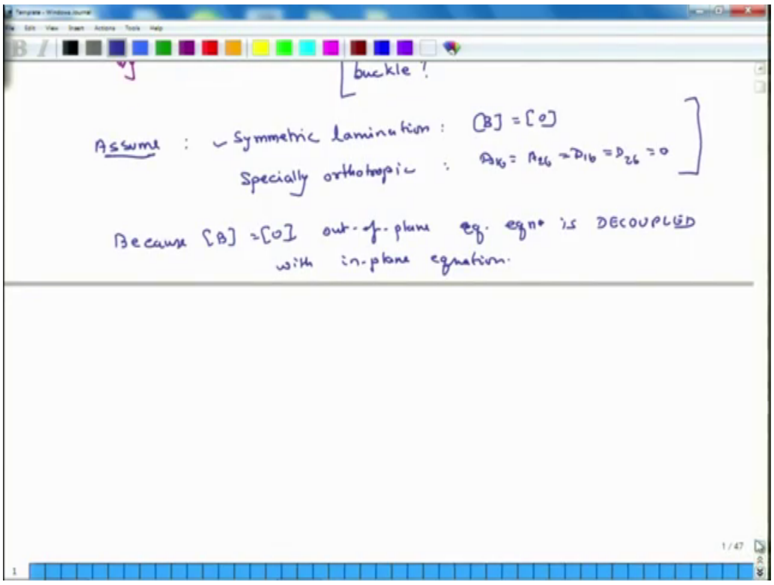

And before we do that we assume that the plate is symmetric so, lamination is symmetric and if it is symmetric, then I can say that the B matrix is a null matrix. The other thing is that the plate is especially orthotropic. And if that is the case then I can say that A 1 6 is equal to A 2 6 is equal to D 1 6 is equal to D 2 6 and this is all 0. So, because of these reasons, because of symmetry the out of plane equilibrium equation and the n plane equilibrium equations they are decoupled.

So, because B is 0 out of plane equilibrium equation is decoupled with in plane equilibrium equation ok. So, what; that means, is that I can now use this only the third equation, and if I solve the third equation, then I should be able to predict at what value of S will the system buckle.

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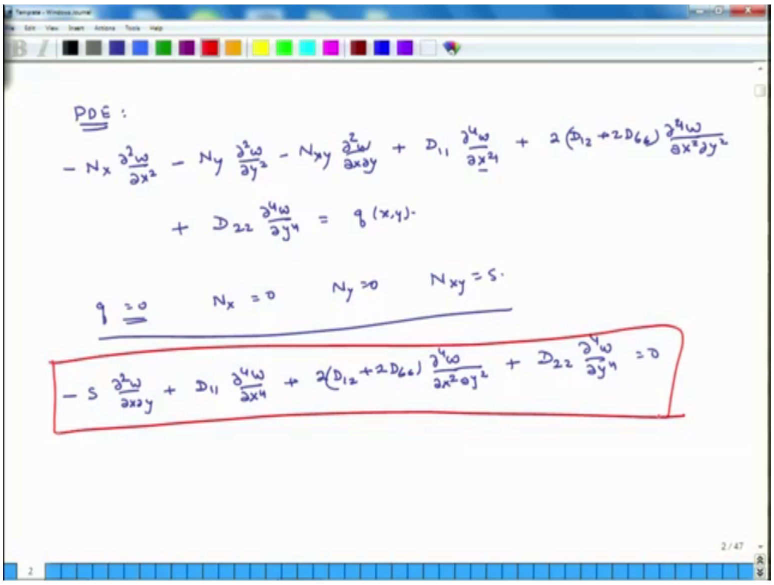

So, the partial differential equation which I am going to address is minus N X del 2 w over del x square minus N Y del 2 w over del y square minus N XY del 2 w over del x del y plus D 1 1 del 4 w over del x square plus 2 D 1 2 plus 2 D 6 6 del 4 w over del x square del y square plus D 2 2 del 4 w over del y 4 yeah.

Student: (Refer Time: 09:45) del x (Refer Time: 09:47).

So, this del should be del x 4 and all this should equal q of x y. Now, in our equation q is equal to 0 and because the plate is nice rectangular symmetric lamination, then because of the geometry and all that N X inside the plate is also 0 and Y is also 0 and N XY is same as S, because the plate is nice rectangular and it is uniformly loaded on the external edges by uniformly applied N XY whose values is S so, at every point we will assume that it is the same thing.

So, if that is the case then my simplified PDE becomes minus S del 2 o w over del x del y plus D 1 1 del 4 w over del x 4 plus 2 D 1 2 plus 2 D 6 6 del 4 w over del x square del y square plus D 2 2 del 4 w over del 5 4 equals to 0. So, this is the differential equation I want to solve and I have to also meet all the boundary conditions. And if I can do that then for some value of S I can calculate when the value of w will become a real number. So, we have to solve this equation and the way we are going to solve it is that we are going to use special Galerkin method special Galerkin method.

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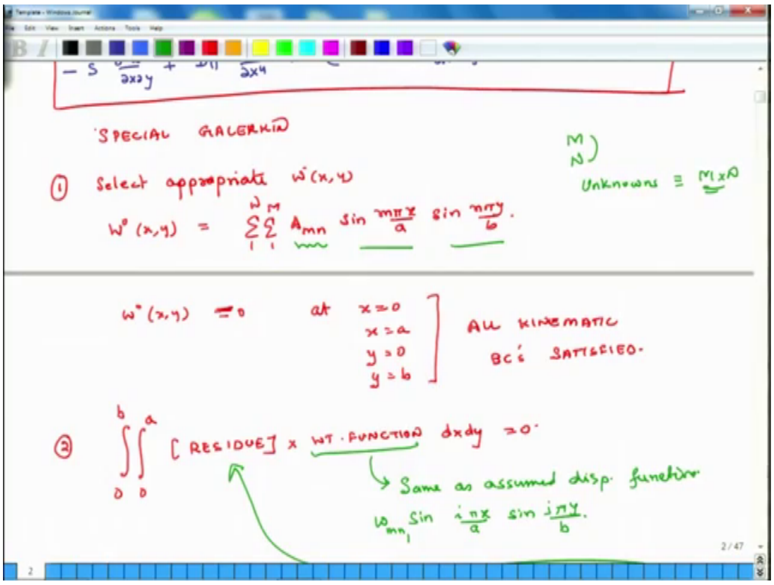

So, if that is the case, then what is the first step, the first step is that we have to choose displacement function for w. So, we have to guess a displacement function for w which meets all the kinematic boundary conditions. So, select appropriate w. So, once again these are all met plane displacements. So, we assume that w naught x y is a series and this is A m n sin m pi x over a sin n pi y over b ok. And this is being integrated from 1 2 M capital M and 1 2 capital N. So, this w x y it is equal to 0 on all the 4 edges so, it is 0 at x is equal to 0 edge x is equal to a edge y is equal to 0 edge and y is equal to b edge.

So, what does that mean? All kinematic BC's satisfy. So, this is number 1. So, once we have selected such a function then what is the next step.

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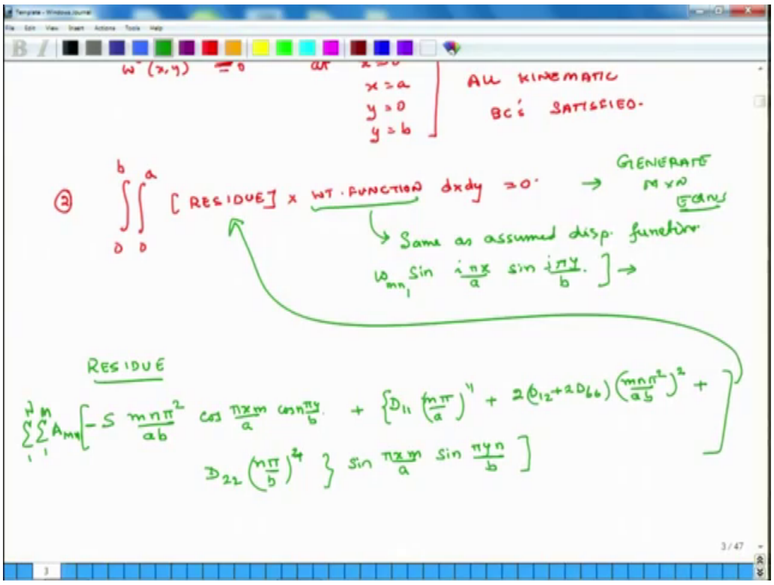

We have to calculate the residue, we have to plug this back into the equation find out the residue and integrate that residue its weighted integral over the domain of the plate. So, the second step is I integrate this thing from 0 to a, 0 to b the residue and I just do not integrate the residue I integrated after applying some appropriate weight function. And I am integrating it over the domain and I set it to 0.

Now, in the special Galerkin method this weight function is what it is same as assumed displacement function what will this be so, it will be so, what is our assumed displacement function it is sin m pi x over a times sin n pi by over b. So, this is same as sin m pi x over a times sin n pi y over b so, we actually plug this in. And it will be times w m n 1 some constant. So, what I do is I plug it all this in; so, what is the residue? First we will develop an expression for residue. So, my residue is minus S times the second derivative of w with respect to x and y.

So, it is m n pi square over a b cosine pi x m over a cosine pi y over b plus and of course, I have to multiplied by A m n. So, I will take the A m n outside the bracket that is not a problem ok. And then I get all these terms related to Ds so, D 1 1 m pi over a to the power 4 plus 2 D 1 2 plus 2 D 6 6 m n pi square over a b the whole thing squared plus D 2 2 n pi over b whole thing to the power of 4. And this entire thing in parentheses curly brackets, this is being multiplied by sin pi x m over a sin pi y n over b and I forgot and n here. So, I am going for this back there and this entire thing is multiplied by a m n and it is been in added up summed from 0 to M capital M and 0 to or 1 to capital N.

So, this is my residue. So, this is this long expression of residue I put it here and weight function also I put m pi x m pi y. So, one thing I will make a distinction, but here I will not put and this is m and n, but rather I will put i and j. And then I will just a treat so, if I so, ok. So, total number of unknowns so, if I have to so, what is the unknown what is it that we are trying to calculate? We are trying to calculate we all we are trying to calculate w naught x y; we already know these functions the thing which we do not know is A m n ok.

So, if M and N; these are the values, then total unknowns total number of unknowns is what M times N ok. So, what we are interested in finding out is m times n unknowns which are A M N so, 1 1 a 1 2 and so on and so, forth till A capital M and A capital N. So, what do I do? The way we do it is that from this we have to generate M times N equations ok. So, the way we generate it is that first I multiply the residue by sin pi x over a so, I said the value of i and j as 1 i get 1 equation, then in the second equation i put i is equal to 1 and j is equal to 2.

And like this I keep on increasing (Refer Time: 20:03) becomes capital N, then i change the value of i to 1 and then I again. So, so these are trial functions or weight functions and we can generate these m times n equations by selecting different values of i and j. And I can change this index i from 1 to capital M and j from 1 to capital N and in that way I can generate m times n equations ok.

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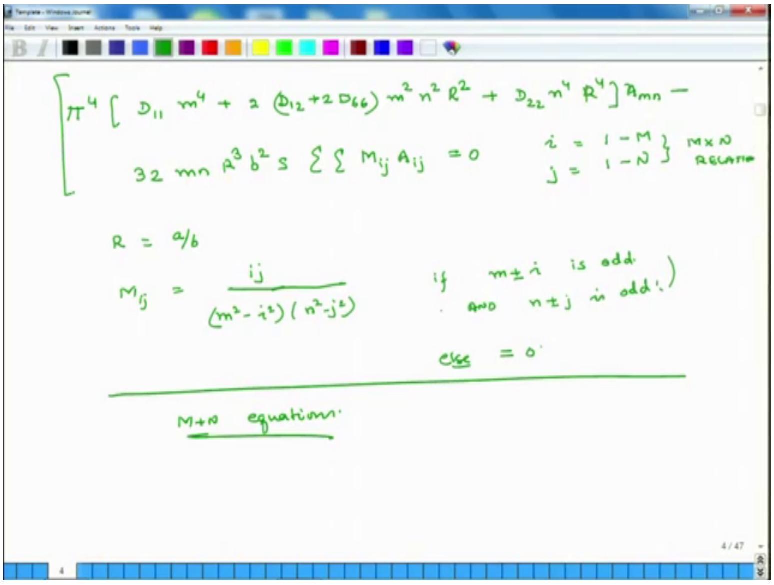

If I do all the math finally, what we get is, this is the equation we get in the general way pi to the power of 4 times D 1 1 m to the power of 4 plus 2 D 1 2 plus 2 D 6 6 m square. n square r square plus D 2 2 n 4 r 4 a m n minus 32 m n r cube D square S summation of M i j A i j equals 0 ok. So, this is the long equation and this is not just one equation. So, here I goes from 1 to M and j goes from 1 to N.

So, this single relation it represents actually M times N relations it is not 1 equation ok. So, these are different m times n equations and there is a typo here. So, here it should be R to the power of 4. And what is R? R is the aspect ratio a over b and then you will say what is S, S we already know it is the external share resultant and then what is M i j. So, M i j if we do the integral it is equal to i times j divided by m square minus i square times n square minus j square.

So, this is the value of i f i M i j if m plus or minus i is odd and n plus minus j is odd. So, both of the conditions m plus minus i should be odd and also n plus minus j should also be odd, but if that is not the case then. So, this is the case else so, if either one of these m plus minus i or n plus minus j is even, then this M i j is equal to 0 ok. So, in this way we get m plus n equations and then we solve for these equations ok, we solve for these equations. So, this is what we will actually discuss further tomorrow.

So, what we have done till. So, far is that using the equilibrium equation for the third direction, we have developed this longest expression and this one single expression represents m times n equations. And using this m times n equations we can calculate the critical value of S at which the plate will start to buckle and, how is it that we are going to calculate that is something, we will see in tomorrow's lecture. So, thank you and I will look forward to seeing you tomorrow.