Advanced Composites
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Lecture – 68
Shear Bucking in Rectangular Composite Plate (Part-II)

Hello, welcome to Advanced Composites; today is the second day of this course and yesterday we started discussing this phenomena of Shear Buckling. And specifically we started discussing the shear differing buckling phenomena in case of a rectangular laminated plate. And in that context we developed an expression which will help us understand and compute the value of critical shear load which will cause the plate to exhibit shear buckling. And in that context this is the equation which we had developed.

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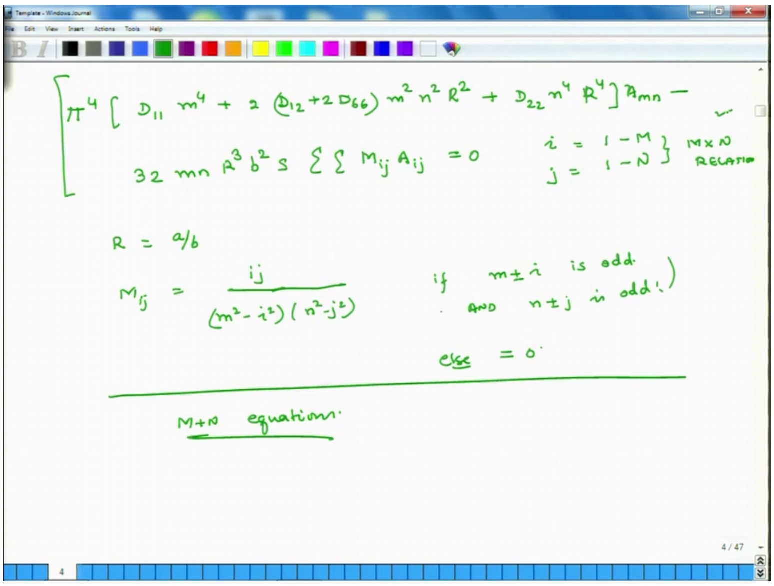

And what we will do is we will see how to use this particular equation and calculate the value of shear buckling ok.

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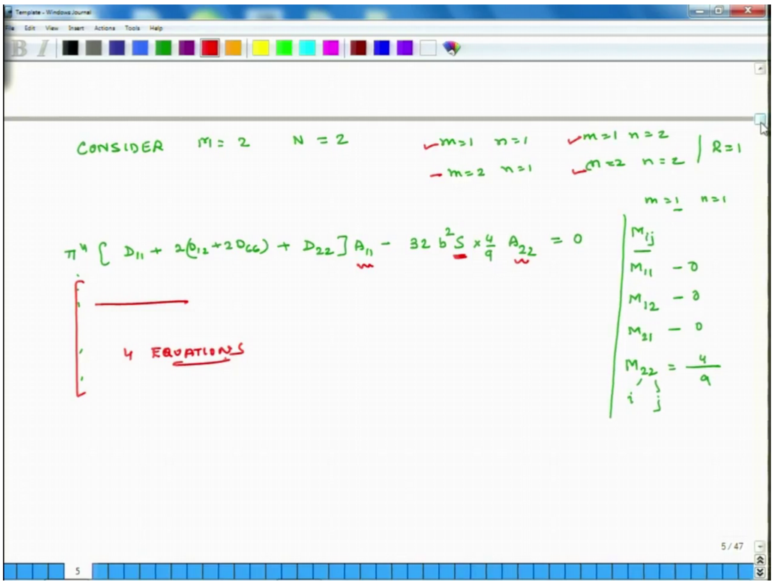

So, what we will do is for illustration purposes; just for illustration purposes what we will do is we will consider M to be capital 2 and N to be capital 2. So, in that case we will get how many equations? Total 4 equations; so, let us see how we generate those equations. So, what are the 4 cases? m is equal to 1, n is equal to 1, m is equal to 1, n is equal to 2 the third case is m is equal to 2 n is equal to 1 and the fourth case is n is equal to m is equal to 2 and n is equal to 2.

So, we will just generate first equation; so it is D 11 m 4 and let us for a moment assume that the plate is square. So, we also just for purposes of simplicity we consider that R is equal to 1. So, if that is the case then my first expression first equation becomes pi to the power of 4 times D 11 plus 2 D 12 plus 2 D 66. So, in this case the first so I am generating the first equation using m is equal to 1 and n is equal to 1 plus D 22.

And this will be multiplied by A 11 minus and then I have this summation over this entire series M ij and A ij. So, let us look at what is M ij? What is M 11? In case of M 11 ok; so our m is equal to 1 and n is equal to 1. And we are going to calculate for this m is equal to 1, n is equal to 1 I am going to calculate M ij am I am going to calculate M ij right.

So, what is the thing? It says that when m is equal to; so when i; m plus i is odd, then it is a nonzero this m plus i is nonzero or odd as well as n plus j is nonzero then M ij will be nonzero; again I will I will repeat that if m plus i. So, what is m plus m plus i? That is small i; m plus i is odd and n that is small n plus j is odd, then M ij will be nonzero otherwise it will be 0. So, in this case there will be 4 values of M; M 11 in this case what is M plus i? 2 M is 1, i is 1; so, it is 2 right. So, it is not odd it is even.

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So, it is 0; M 12 using the same logic; it will be 0, M 21 it will be 0 and what will be M 22? M 22 will be in this case the first 2 is i; first 2 is i. So, this is i and the second 2 is j what is the sum of i plus M? 3. So, it is odd and what is sum of j plus n? It is again 3; so, this is odd.

So, in this case M ij will be i times j. So that is 4 divided by m square minus i square; so, m m is 1 and i is 2. So, 1 square minus 2 square is minus 3 and same thing also for n square minus j. So, it is basically 9 it is 4 by 9. So, the only term in this whole expression I will get will be 32 m is 1, n is 1, R is already 1 b square S times M ij and the only component of m ij which is nonzero is when m is equal to 2 and n is equal to 2.

So, that is equal to 4 by 9 and times A 2 2 and this is equal to 0 understood? So, likewise I can generate; so this was the first equation this is how I got the first equation. Now I go to the next equation there what do I do? I put m is equal to 1 and n is equal to 2 and I generate the second equation. Then I generate the third equation and I generate the fourth equation.

So, in this way I generate 4 equations because capital M equals 2 and capital N equals 2. So, I get generate 4 equations now when we I am looking at these 4 equations in every equation I will find that there will be a term which will involve minus S, because it is there in the original general equation; there is a minus S here; there is a minus S here ok.

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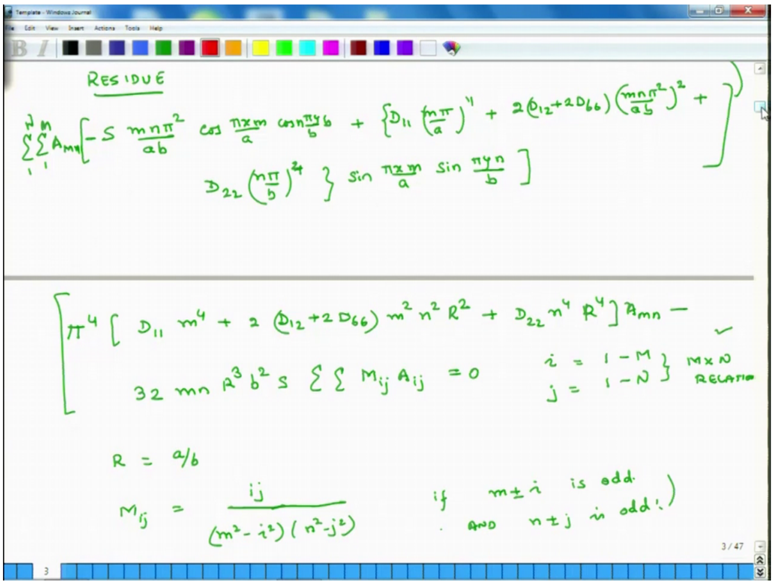

And of course, there will be also these unknowns A 11, A 22, A 33 and so on and so forth.

So, if I reorganize these 4 equations or if I have M times N equations then total number of equations will be M times N; if I reorganize all these things what I will get is an overall system of equations.

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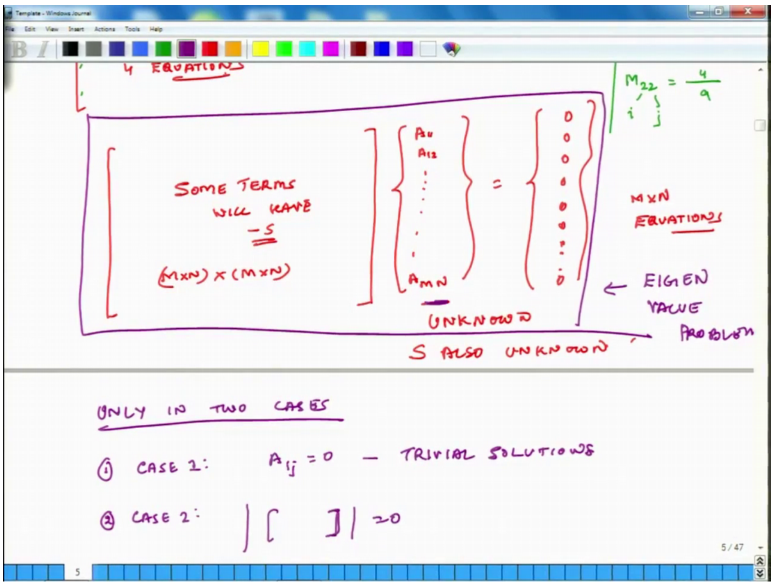

So, this will be what ok. So, here it will be unknowns A 11, A 12 like this and it will come down to a capital M, capital N. So, this is 1 and on the right side what will I have what did I get in the right side? 0s.

So, I will have 0s M times N times ok. So, these are total M by N equations total MN times N equations and in this; so, what is the order of this matrix?

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It will be M by N; it will, how many elements it will have M by N times M by N ok. So, this matrix order is M by N it will have M M N times or M square N square elements ok. So, this is going to have M square N square matrix; the other thing is that in some terms there will be minus S S like here. So, some terms will have minus S S minus S and S is also unknown, S is also unknown ok. So, this is unknown and S is also unknown.

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S is the applied load and we want to know that at what value of S the plate is going to buckle; so we want to calculate that S. Now when will this set of equations be satisfied? The left hand side of this equation is the right hand side of this equation is a vector with all 0s; now this will happen. So, this set of equations will be satisfied only in 2 cases ok; it will be satisfied only in 2 cases.

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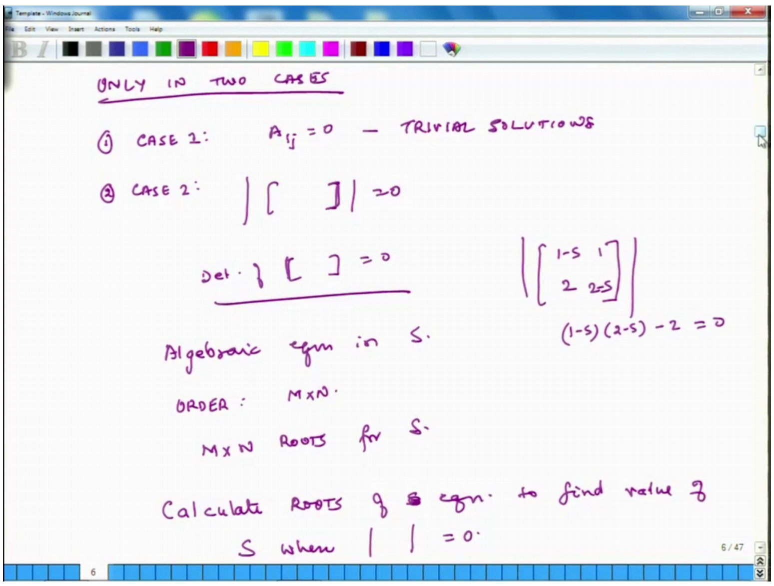

What are those 2 cases? The first one is; so case one when A ij is equal to 0. What does that mean? That if this particular vector is this particular A 11, A 12 all these terms are 0; then it will be satisfied.

So, that is a trivial solution and what does that mean? That the plate has not buckled ok. So, if I apply very small shear loads the plate is not going to buckle and A ij will be 0. So that is a trivial solution we are not interested the second situation will be case 2 and what when. So, the other case will be when the determinant of this matrix is going to be 0 when the determinant of this matrix is equal to 0.

And remember in this matrix every row will have S terms and also some other constants. So, this is a constant known constant and there will be an S term also ok. So, in every row there will be some constant terms and some negative S terms. So, as I keep on changing the value of S for some value of S the matrix will become 0 ok. So, it means that determinant of this matrix is 0 and what will be the; so when I.

So, for example, the determinant of this is going to be 0 how we add for how many values of S this is going to be possible? So, in every row there will be some SS and if I compute the overall determinant I will get a algebraic equation in S and what will be the order of that algebraic equation? It will be M times N; you see every row there will be S and as I keep on taking the determinants for instance suppose I have 1 minus S, 1, 2, 2 minus S.

So, if I take its determinant what is the determinant? Its determinant will be 1 minus S times 2 minus S minus 2 and this will be 0. So, this is a quadratic if I have a 3 by 3 matrix it will be a cubic equation if I 4 by 5 and so on and so forth. So, the order of matrix is M by N, and because every row has only linear terms in S; the order of the algebraic equation in S will be M times N.

So, order will be M times N it means that it will have M times N roots for S ok. It will have M times N roots for S and I use by solving this algebraic equation I can calculate. So, we calculate the roots of equation to find value of S value of S when the determinant goes to 0 ok. So, at those values of S the plate will exhibit also buckling.

Now, this is my standard problem and if you go back and refer to your notes and mathematics; this is a typical eigenvalue problem ok, it is a typical eigenvalue problem.So, you calculate M times N roots of S and for each value of S you can calculate the values of A 11, A 12 and all these things. So, you will have M times N eigenvalues and for each eigenvalue there will be an associated eigenvector.

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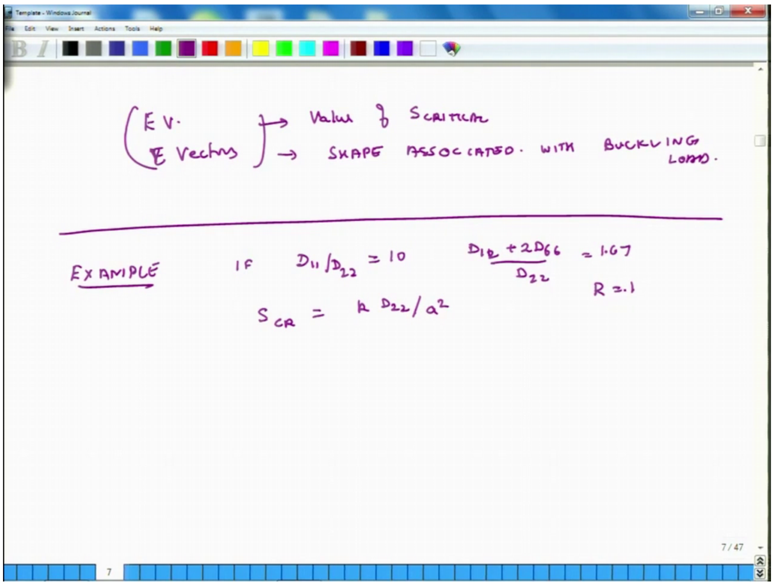

So, we calculate eigenvalues and we calculate eigenvectors. So, what do eigenvalues tell us? The eigenvalues will tell us that at what value of S the plate is going to become unstable. So, it may become unstable at different loads and whatever is the minimum load that will be the buckling load of the systems. What do eigenvectors tell us? That for that value of eigen this shear load; what are the corresponding relative values of A M N. So, what is going to be the shape of the plate?

So, this tells us the value of S critical and this tells us the shape associated with buckling load. So, in this way we can methodically compute the buckling load for the plate. So, I will also like to share some example or some actual numbers. So, we will just share some example. So, what we have said is that if D 11 over D 22 is equal to 10 and D 12 plus 2 D 66 divided by D 22 is equal to 1.67.

And we define that S critical is equal to k times D 22 divided by a square and we assume that the plate is rectangular square. So, R is 1 then we will construct a table.

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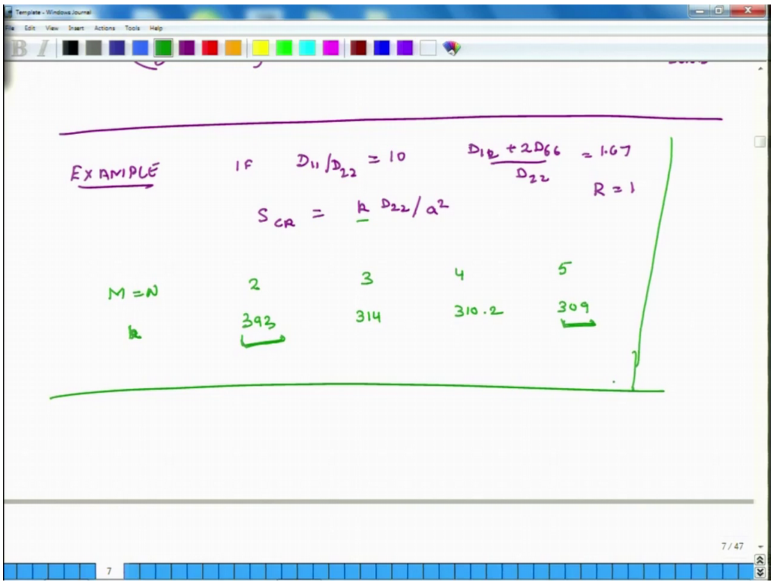

So, M is equal to N and in the second row we are going to plot write down the value of k. So, if there are if M is equal to 2 and capital N is equal to 2; then the total number of solutions in the term will be 2 times 2; 4.

So, this is a 4 term solution, this buckling normalized buckling load k it comes out to be 393; normalized buckling load. If I increase the number of terms my solution becomes a little more accurate and this buckling load drops down and it becomes 314. If I increase my number of terms further; so, it is 4 by 4; so 16 term solution.

Then this buckling solution becomes even more accurate. So, it becomes about 310 and now 0.2, and if I go to 5 term solution or 5 M equals N equals 5; it is a 25; term solution , then it becomes 309 ok. So, as I keep on increasing the number of terms the solution converges to a particular thing and what we see is that the 4 term the 5 or the 25 term solution is about 25 percent less or 26 percent less than a 4 term solution ok. So, this is something I wanted to share. So, as we are trying to solve buckling problem we have to include sufficient number of terms, so that we are sure that the solution has fairly converged.

So, this concludes our discussion on buckling starting tomorrow we will switch gears. And we will start discussing about composites which use short fibers, and we will develop some mathematical models which will help us to understand the behavior of short fiber composites.

Thank you very much. Bye.