Advanced Composites
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Lecture - 07
Concept of Tensor

Hello, welcome to Advanced Composites. Today is the start of the second week of this course and over the span of this week we will primarily cover the topic, which helps us understand how do stresses change when we are also transforming the coordinate system that is one and also the stress strain relationship for an isotropic materials, orthotropic materials; especially orthotropic materials, materials with planer isotropy and so on and so forth.

So, that is pretty much the agenda for this week. A lot of this stuff which I am going to cover in today's; today as well as this week lectures. It has being described in great detail in the introductory course that is introduction to composite materials, but just for purposes of completeness, I will go through all these topics over this period of this week.

A little bit fast and in case you wish to learn more about some of this specify topics and their details please go back and look at the lectures for introduction to composite materials which was the introductory course for composites.

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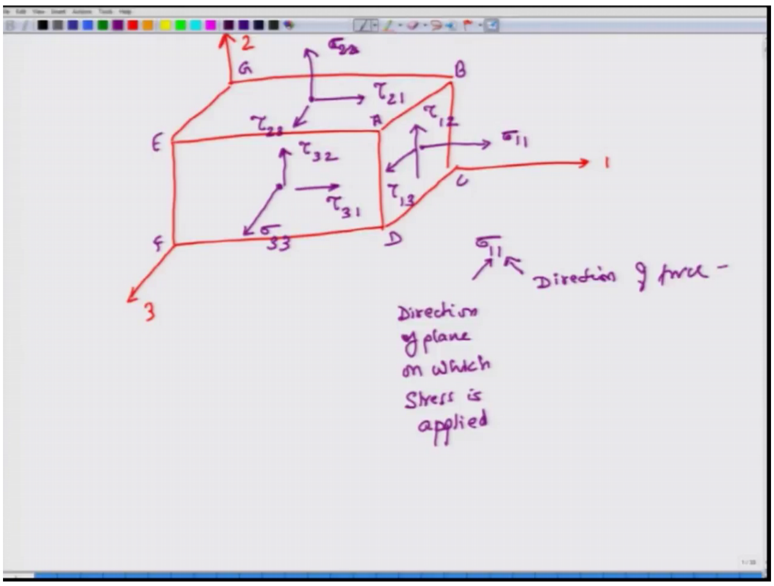

So, having said that, what we will start discussing today is stress convention. So, let us explain that. So, suppose I have an I have a piece of material and this is important because we will follow this convention. So, it is important to understand this correctly. So, this is a piece of material and let say I have three axis; this is axis 1, this is axis 3 and this is axis number 2. So, all these three different axis they are mutually perpendicular to each other and this block of material which is rectangular block, it is experiencing several stresses. So, let us set look at what kind of stresses this material is seen.

So, if I apply a tensile stress in one direction and it is applied on this plane, I call it sigma 11 ok. So, why do I call it sigma 11? Because the first subscript it refers to the direction of plane on which a stress is applied and the second subscript, which is again 1 is the direction of force. So, to specify any stress we have to specify two directions one is the direction force, which is there in the second subscript and the first one is the direction of plane. So, the direction of plane is this plane A B C D and what is its direction typically whenever we talk about direction of a plane we refer to the direction of the normal which is to that plane.

So, the normal which is to that plane is direction 1. So, the first subscript refers to one direction because that is the direction of plane on which force is being applied and the second subscript is directional force in which the force is getting applied so that is why it is sigma 11.

I can also apply another stress on it. So, sigma 11 is tensile stress and the other stress could be shear stress and once again, because it is a shear stress, I designated by letter tau and the first subscript is 1 because that is the direction of plane on which the stress is being applied and the second subscript is 2, which is which relates to the direction of the force in which this is stress is been applied.

Similarly, I can also apply another shear stress which is in this direction, which is again parallel to this plane and once again it is a shear stress. So, I designate it with a letter tau first subscript relates to the direction of the plane. So, that is still 1 and the second subscript relates to the direction of the force and this force is getting exerted in 3 direction. So, it is tau 13.

So, let us do it couple of couple of times more. So, let us consider this plane and the normal stress will be in this direction and because this is the normal stress, the I call it designated by letter sigma and then it will have again 2 subscripts, the first subscript will be the direction of the plane on which it is getting exerted. So, it is getting exerted on plane, plane E A D F and the direction of normal is 3. So, the first subscript is 3 and the second subscript tells us the direction of the force. So, that is also 3, so, sigma 3.

Let us look at shear stresses on this plane. So, this is one shear stress. So, what is this shear stress? How do we designate it? We designate it by letter tau and it will have 2 subscripts; first subscript relates to the direction of the flame, second subscript relates to the direction of the force, which is 2 and then there is another shear force which can get exerted on this plane and that is tau because it is a shear stress. First subscript relates to direction of plane and second subscript relates to direction of force which is 1 and then we will do this one last time. So, we are talking about plane E A B G and the normal stress will be sigma 22 and then there could be a shear stress and that shear stress is tau 2; 2 is the direction of plane and direction of force is 1, so, tau 21.

So, and then we have another shear stress and we designate it as tau because it is shear first subscript is 2, which is directional plane and the second subscript is 3 which is the direction of the force. So, this is the convention. So, this is how we designate a stresses in the whole course. The next thing I wanted to talk about is this concept called tensors; tensors.

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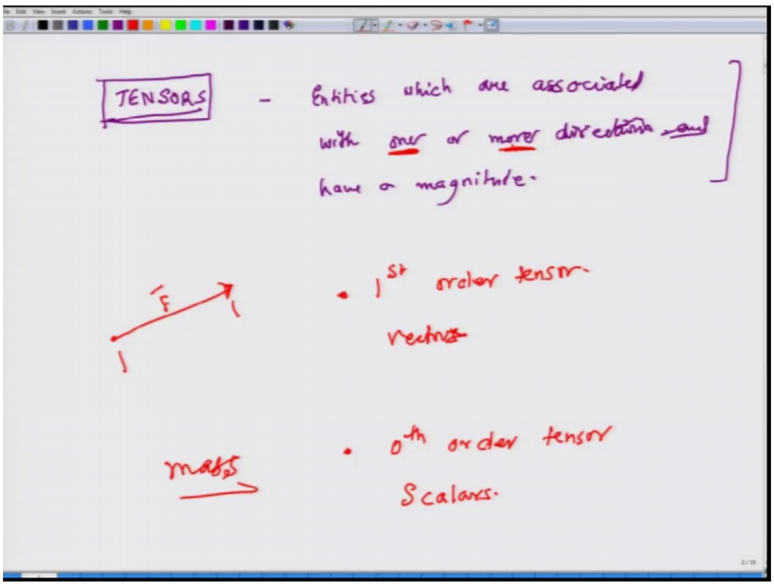

So, in general tensors are entities, which are associated with one or more directions and have a magnitude also.

So, this not exactly the mathematical definition, but this will you help you understand the concept. Now you may be wondering that, if this is what tensors are then how are they different from vectors? So, that is what we are going to discuss. So, the key thing is that they may be associated with either one direction or another more sets of directions. Now if we consider force, right? So, let us say this is force the length of this vector is the magnitude of the force and the direction of this arrow designates the direction of force. So, this force is associated only with one particular direction.

So, in that sense force is also a tensor, in that sense force is also tensor. We can also call it a vector because vectors also are associated with direction, but in case of vectors the dependence on directions is only 1. So, force depends. So, when we define force we only have to specify one set of directions we do not have to specify 2 sets of directions or 3 sets of directions 4 sets of directions. So, in direct sense force is a tensor and it is a special tensor because it is associated with only one set of directions and because it is associated with one set of directions, it is called first order tensor; it is called first order tensor. Further, it is first order tensor and first order tensors are also known as vectors they are also known as vectors.

Let us consider mass, the mass is not associated with any set of directions. So, it iszeroth order tensor and zeroth order tensors are also called scalars. So, in that sense tensors are general entities. They incorporate vectors scalars and higher order tensors. Now we will look at couple of tensors which are associated with not one just one set directions, but two or more than two or more than one set of directions.

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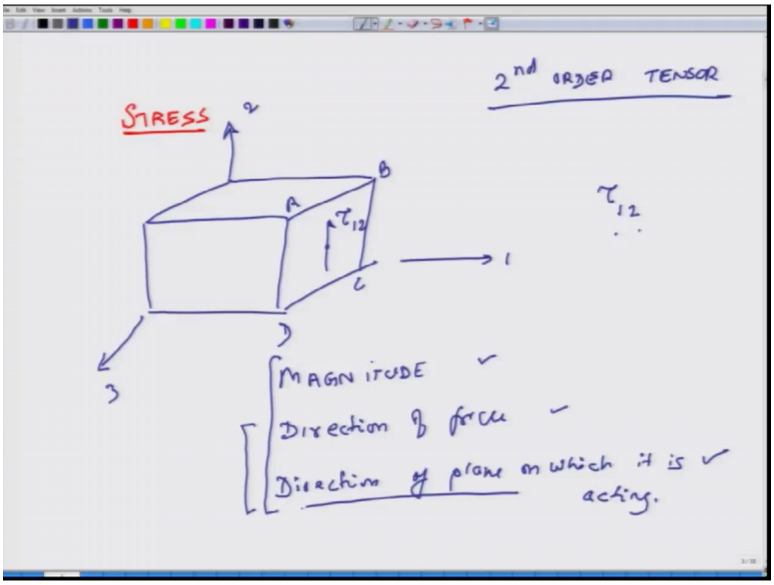

So, consider stress consider stress. Now we have shown earlier that whenever we talk about the stress as a tensor, what do we have to do? So, suppose we want we are saying that it is shear force on this plane and let say this is my axis system 1 2 and 3. So, whenever we specify a stress, we specify 2 things. We specify the direction of force.

So, whenever we specify a stress for instance in this case shear stress, what do we do? We specify its magnitude, we also say that it is you know what is it is what is the direction of the force. So, this is one thing we specify and we also specify the direction of plane on which it is acting ok. We have to specify all these 3 things; we have to specify magnitude, we say it is so many mega Pascal's; 30 mega Pascal's. So, 30 is the magnitude we say that it is shear stress on plane a B C D and the moment we say it is shear stress on A plane A B C D we are specifying that what is it we are specifying?

We are specifying the direction of the plane. So, we are saying that it is 30 mega Pascal's, its shear stress and the plane on which it is acting is plane 1 and then we also have to specify in which direction the force is acting. So, in this case the force is acting in; so, this is 3 direction and 2 direction force is acting in 2 direction. So, it is tau 1 2 ok.

So, stress because of this we have to specify 2 independent set of directions. The direction of the force does not get influenced by direction of plane on which it is acting and vice versa. So, this 1 and 2 are independent set of directions, they do not influence each other ok. So, when we specify stress we have to specify the magnitude, we have to specify the direction of the force and we have to specify the direction of the plane on which the stress is getting acted upon and because we have to specify 2 independent sets of directions associated with shear stress tau and 2 we say that is stress is a second order tensor. It is a second order tensor. So, whenever we specify stress, it is a second order tensor.

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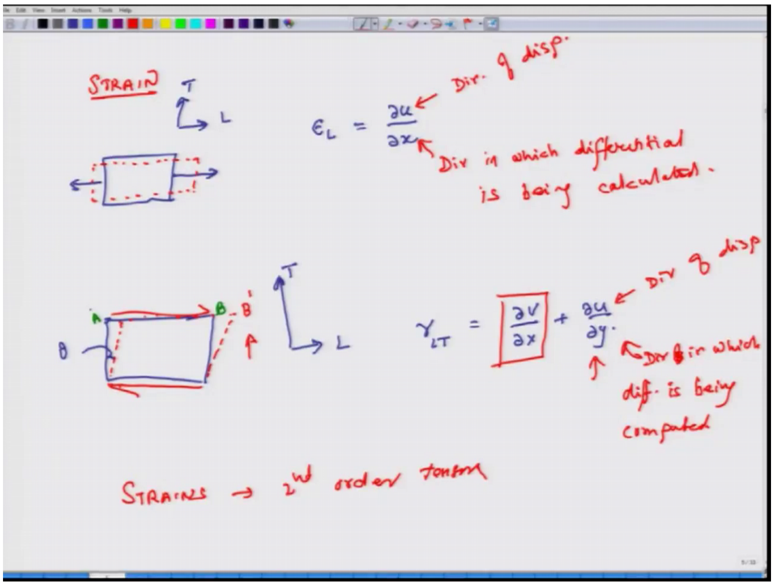

Unlike a stress pressure; pressure it is 0th order tensor. And why do we say pressure is the zeroth order tensor? Suppose, there is a complicated cavity and the moment we say that the pressure inside the cavity is let us say 50 Mpa, it is implied the moment we say that at every point the force per unit area in the direction normal to the surface whatever the normal is in the direction normal to the surface is 50 Mpa. So, here we do not have to specify, it is by default; it is already known the moment we know the surface the directional force is also defined. So, they so that is why it is a zeroth order tensor, but stress is a second order tensor.

So, stress is an example of second order tensor. Another example of second order tensor is strain and we will explain that in couple of minutes why that is the case. So, suppose consider the case that I have a block of material and I pull it and I as I pull it will become little bit longer and also a little bit narrower, if this material is isotropic.

So, if I pull it then extensionally strain will stress will only produce extensionally strain as we discussed last week. So, when I say that this body is going to experience a strain, what is the definition of strain? It is extensional strain, for instance if this is my L direction, this is my T direction and if it is pulling getting stretched in the longitudinal direction then epsilon L is defined as del u over del x ok.

So, there are 2 directions in this relationship; one is the direction of displacement and this direction has to be in the direction of x, only then it will be defined as epsilon L. If it was del v which is in the transverse direction, then it would not be longitudinal strain. So, this is the so u is associated with the direction u displacement is associated with direction and also the direction in which I am computing this differential is also the other directions.

So, I am computing the differential of u with respect to x direction. So, they are 2 directions embedded in this formula; first is direction of displacement and second is direction in which differential is being calculated ok.

So, these are 2 directions another example of strain would be suppose I have a piece of material and let us say just I apply some force on it, only on top and bottom surfaces; just assume although it is physically not possible that there is no force on these vertical sides. So, what will happen and if it is isotropic material what will happen? It will become something like this. So, if it is rectangle, it will some become something like a parallelogram and when it becomes a parallelogram this angle initially was 90 degrees or 0 degrees, this becomes some non zero angle, right.

Now if this is my L direction, this is my T direction, then the displacement let us say the top surface let say the top edge is A B, A B is getting is moving out ok, A B is moving out and if I want to compute shear stress shear strain then the relation for shear strain gamma L T is del v and v is the displacement in y direction over del x plus del u over del y ok.

Now, just think. So, just for a moment do not think about this one. Let us just focus on this the top surface is moving in L direction. So, point B moves to B prime location right and what is y? Y is this direction. So, I am computing the differential of displacement u with respect to y axis ok. So, the this is direction of displacement and this is direction of direction in which this gradient or differential is being computed.

So, once again gamma L T, if you look at the second component it is associated with 2 directions similarly del v over del x is also associated with 2 directions. So, strains are also having associated with 2 mutually independent set of directions and that is why strains are also second order tensor, they are also second order tensors and then finally, will look at E.

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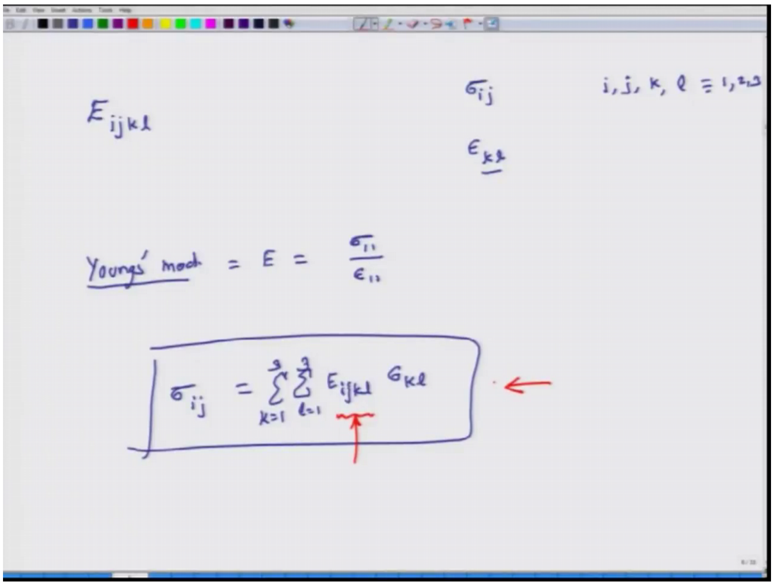

So, if a strain stress is a second order tensor in general I can express it as sigma i j right. If a strain is a second order tensor then I can express it as epsilon k l, where i and j all of these could assume values of 1 2 3; 1 could be x axis 2 could be y axis 3 could be z axis and so on and so, forth right.

So, this is a second order tensor and that's why it has 2 subscripts; epsilon k l is also a second order tensor, so it has 2 subscripts. Now turns out that Young's modulus or elasticity constant elasticity constant is a forth order tensor. Example: What is Young's modulus? Young's modulus is what? It is for isotropic materials we have this Young's modulus and we call it E; and what is it? It is sigma 11 divided by epsilon 11 right.

So, a strain is a second order tensor, stress is a second order tensor and in this case Young's modulus is function of 2 second order tensors and all these directions are mutually independent. The first subscript in sigma relates to the direction of plane which is nothing to do with all the other 3 directions, second subscript in sigma is direction of the force which is nothing to do with all other 3 directions, third subscript that is epsilon in the first subscript of epsilon is the directions of displacement and the other second subscript is the direction in which we are calculating the differential. So, all these 4 directions are mutually independent ok. It is not that if you know one you can figure out the other one, we have to know all 4 of these.

So, E or elasticity constants they are fourth order tensors. So, I am just explaining this in this way ok. So, E is fourth order tensor. So, we give it 4 subscripts; E i j k l in general and the relationship between stress and the strain for a; for this E i j k l is sigma i j equals summation of E i j k l epsilon k l and you sum it twice.

So, first time you sum it on index l; l is equal to 1 2 3 and second time you sum it on k; k is equal to 1 2 3. So, this is the stress strain relationship and this is stress strain relationship is linked through this elasticity tensor. A stress is a second order elasticity tensor, strain is a second order elasticity tensor; stress is second order tensor strain is a second order tensor and strain times elasticity which is a fourth order tensor, when you multiply and add up all the individual products you get sigma i j.

So, this is important especially in context of composite materials because in case of isotropic materials if you try to find the Young's modulus in x direction or y direction or z direction or any other direction, it just turns out because of mathematics and physics involved that E 11, 11 E in all the directions same, but the same may not be true for an isotropic materials. So, we have a much more complicated and sophisticated model for relating stresses and strains in context of an isotropic materials.

So, in the next class we will start discussing how do we handle these elasticity constants and so on and so forth. So, that is all and we will meet tomorrow at the same time.

Thank you.