Advanced Composites
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Lecture – 70
Theories of Stress Transfer (Part - I)

Hello. Welcome to Advanced Composites. Today is the fourth day of the ongoing week that is the twelfth week of this course. Yesterday, we started a new topic in this particular course which is about short fiber composites and we started our journey by started discussing the idea of or the concept of stress transfer in context of short fiber composites where fibers are aligned in the direction of the load.

So, we will continue that discussion today as well.

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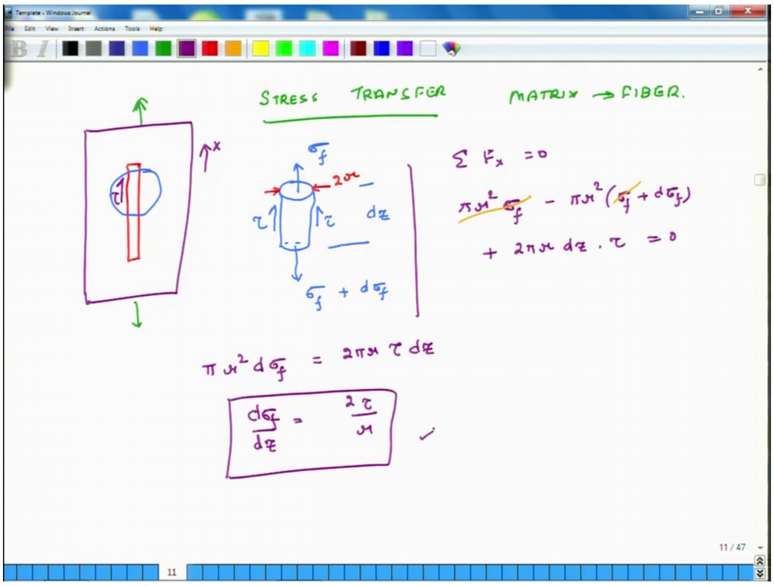

So, what we had shown was; that if I have a piece of composite and there is just one fiber embedded in it, and if I apply some force here, stress transfer occurs and how does it occur? It occurs from matrix to fiber. Now, if there is no adhesive bonding between the fiber and the matrix, then the fibers will not experience any stress.

So, the stronger this bond is between the fiber and the matrix the stronger, the better the stress transfer is going to happen. So, one thing is that whenever we choose a matrix one obvious conclusion is we have to make sure that it bonds extremely well with the fibers which are being used. So, if you have graphite and some matrix system, you have to make sure that the compatibility adhesive forces between graphite fiber and matrix are extremely strong.

So, you have to consider whenever you are selecting a matrix these important criteria that the adhesive bonding between the matrix, and the fiber of choice it should be very strong, and it should be very strong at all operational temperatures. So, suppose your system is working at 100 degree centigrade or minus 40 degree centigrade. So in that entire range of temperatures it should have good strength and also in context of moisture and other operating parameters.

So, what we had shown yesterday was that there is some shear force, there is some shear force which gets developed and let us call this shear not shear force shear stress on the edge of the fiber, ok. So, what I is that I will take a small curve small amount of this fiber and look at it in detail. So, this is the fiber, a small piece of that fiber and when the stress gets transferred with the fiber experiences a tensile stress.

So, let us say on the top surface the tensile stress is sigma f and the length of this small element is d z and on the bottom surface, the stress is sigma f plus some change in sigma f ok. So, it may need not be same and the shear stress on the edge is tau shear stress is tau. The other thing, I would like to state is that this diameter of the fiber is 2 r. So, the diameter is twice of radius. So, what we do is we do a force equilibrium. So, if this is my x axis, then sigma f x if it is in a static equilibrium this should be equal to 0.

So, what is the force on the top surface in the positive x direction? So, positive x direction; so this is positive x; so it is pi r square times sigma f right and on the other end is pi r square sigma f plus d sigma f. So, this is the force and then of course there is a shear force and shear force is acting upwards. So, plus 2 pi r which is the circumference times the height of this cylinder which is d z times the shear stress and the sum of all these should be equal to 0.

So, this term and this term goes away. So, essentially what you end up with is pi r square d sigma f is equal to 2 pi r tau dz or d sigma f over d z is equal to 2 tau over r. So, this is the relation between rate of change of tensile stress with respect to the length of the fiber. So, that is the left side and the right side is the shear stress at the interface of the fiber and the radius of the fiber. So, what does that mean that if I have a very large fiber if it is very thick fiber, that is it is diameter is very large, then it will the stress which will develop in the fiber will develop very slowly, because d f over d z is equal to 2 tau over r.

So, if r is large then stress will develop in the fiber very slowly. So, it is good to have as fine fibers as possible then the rate of change of fiber this thing will be fast, this is there.

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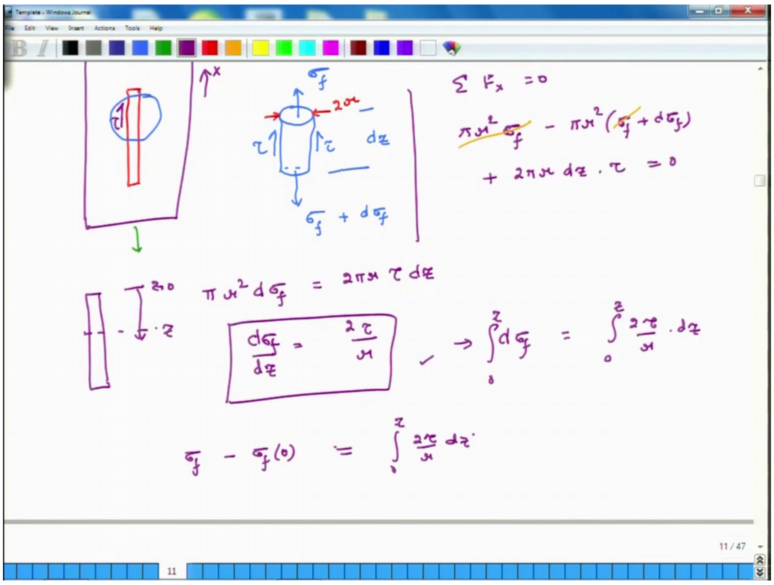

Now, what we have shown here is this relation and if I integrate this, then what I get is d sigma f is equal to 2 tau over r times d z. So, I am trying to integrate it. So, if I integrate both sides and my fiber.

So, what is my domain I am going to integrate it over, the length of the fiber right the length of the fiber or let us say we will. So, this is z is equal to 0 and this is z is equal to z. So, what I am. So, let us say this is the tip of the fiber. So, and the length of the fiber is h. So, this is h over 2. So, I am going to integrate it from 0 to h over 2 and 0 to h over 2. So, what do I get? I get sigma f I am sorry I can.

So, just to be sure; so what I am doing is this is my origin. So, here z is equal to 0 and at some cross section z is equal to z. So, the limits are 0 to z. So, the left side is sigma f evaluated at z minus sigma f evaluated at the tip of the fiber and that is equal to 0 to z 2 tau over r d z.

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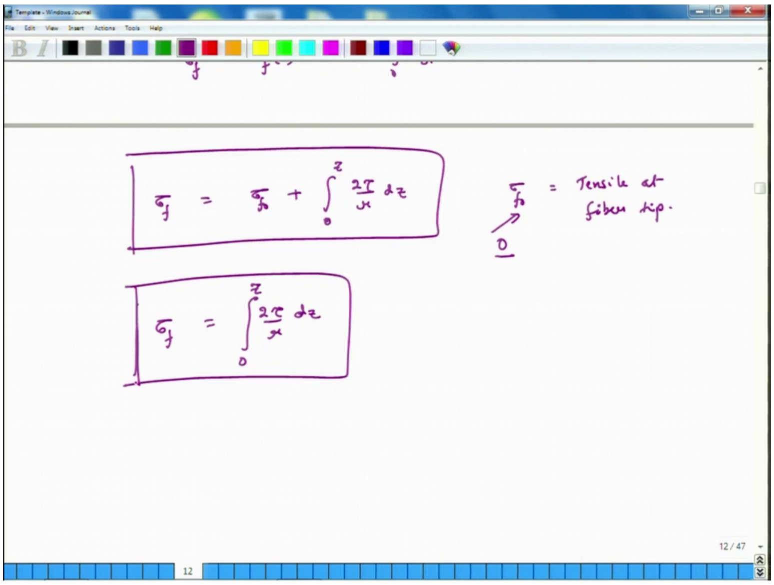

So, sigma f is equal to sigma f naught plus 0 to z 2 tau over r d z.

See, we have not we do not know how tau changes with z. So, we cannot integrate it right away, because we do not know the dependency between tau and z how is it changing. We cannot say that it is constant ok, it could be anything. So, what is sigma f naught sigma f naught is tensile stress at fiber tip fiber tip.

Now, as we had discussed for several reasons typically this value is 0. Why is it 0? First thing is whenever we have a stress concentration. So, at the fiber tip there will be very small cross sectional area and there will be some force and there will be very high stress at the fiber tip and the other thing is the bond of an adhesive joint in tension is normally not that strong. So, in all real cases, this bond if it is there it will break and once it breaks what will be the stress of the fiber at that tip? It will be 0 ok.

So, in all the real cases or most of the real cases this is typically 0. So, we can say that sigma f is equal to 2 tau over r d z integrated from 0 to z. So, this is the relation which helps us understand the transfer of stress that is how stress is transferred through the mechanism of shear stress at the interface of the fiber and the matrix. And once this stress gets transferred, it reflects as the tensile stress in the fiber.

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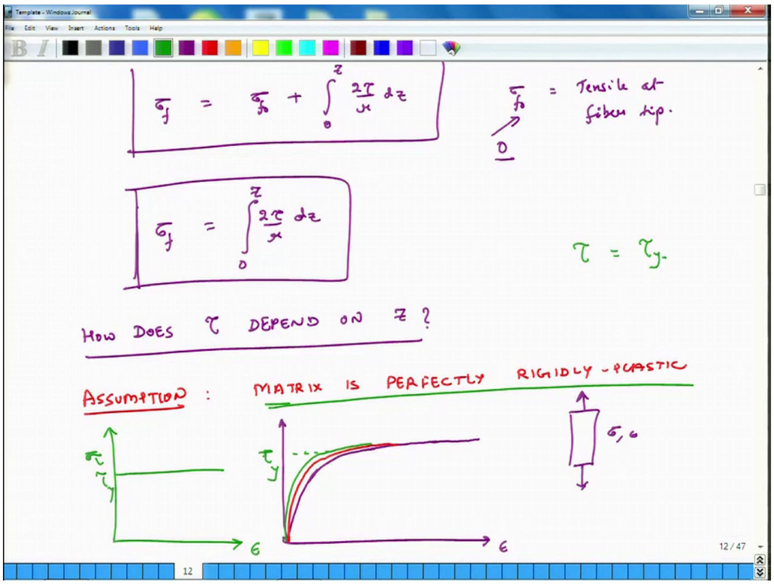

So, the next question is how does tau depend on z because, if we understand how is tau changing with z, then we can actually perform this integral, if we understand this because r is constant, the radius of the system is constant. So, if we know the relationship between tau and z, then we can actually conduct this integration. So now, we will make one very big assumption. So, we will assume for purposes of this course ok.

So, for academic discussion we are going to assume that the matrix is perfectly rigidly plastic. Now this, I will explain that what; that means, rigidly plastic it is perfectly rigidly plastic and a lot of matrix materials are approximately like this. They are not rigidly plastic nothing is rigidly plastic, but this makes mathematics easier. So, we are assuming this. So, what this means is suppose you have a real matrix material real matrix material and.

So, suppose I take a piece of matrix and I pull it, I pull it and I. So, I am applying some stress and strain. So, I am plotting stress versus strain. A lot of these matrix materials they behave like this. So, initially they are everything is linearly elastic. So, they do this, but very soon after they after stress exceeds a certain value they become plastic, and they start just flowing. And when the things become plastic if you release them in that space, it does not come back to it is original configuration.

So, they start flowing and so the shape becomes like this. So, then you do not need any other stress to keep on pulling it further. They just get stretched long and long. So, this is elastic in the beginning and plastic after it has stretched beyond a certain point. Now, if this elasticity is high then it will do this. So, here the elasticity is high in the beginning and if it is higher, then it will do this.

So, what we say is that it is rigidly plastic which means the behavior of the matrix and this is a mathematical idealization is, so this is stress and this is strain it is this ok. So, this is an approximation that the stiffness of the material in the beginning extremely large, still good. So, it is this and what is this limit at which it starts flowing that is the yield point, yield stress. So, this is yield stress.

So, I am sorry this I should call this as a shear this, I should call as shear stress because I am subjecting it to shear. So, this is shear tau y, ok. So, instead of subjecting to this tensile, I am subjecting to some shear thing. So, if that is the case then, what is the relation between tau and z that tau is nothing but that it is equal to constant, and it is equal to tau y and that is true only for a perfectly rigidly plastic material.

Now, such materials do not exist in nature, but this will be the first approximation we can make to do this integration.

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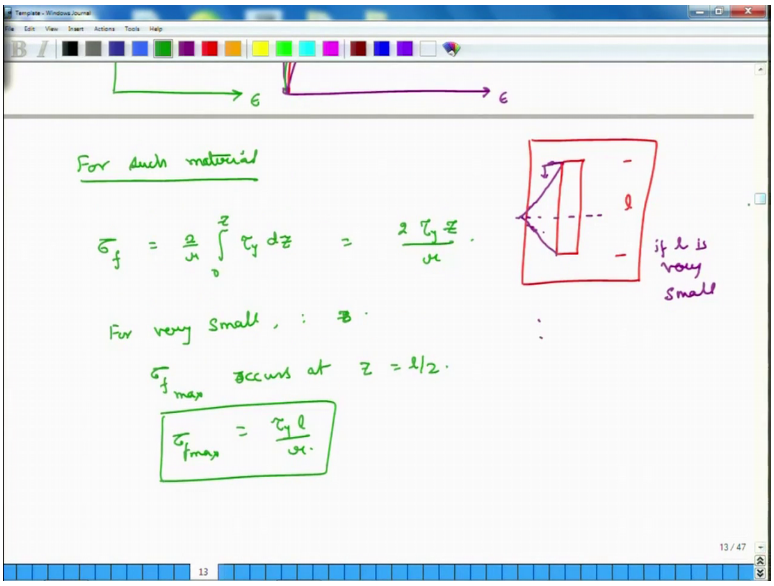

So, if that is the case, so for such material for such material sigma f is equal to 2 by r integration of 0 to z tau y d z and tau y is constant. So, this is tau y z over r 2 I z over r.

Now, suppose this is a fiber and the length is l, suppose the length is l what does that mean and suppose the fiber is in some matrix.

So, this is my origin. What does it mean that fiber stress the tensile stress in the fiber it will just linearly increase. So, this is my z direction. So, the tensile stress in the fiber will linearly increase. It will linear and this is another end. So, it will also linearly increase from this end, because the fiber does not know whether this is the top. So, it will also increase from this end ok. So, it just linearly increases and it will keep on increasing.

And so, it will be something like this. Now, whether it meets at a point or something else happens, we will discuss this later. But if the fibers are extremely small, if the fibers are extremely small, if l is very small then this is how sigma f is going to be in the fiber that from one and it will be linearly increasing and it will also linearly increase from the other end.

And, and in this case, what is the maximum value of this thing it will go to get c at this point, at this point l is equal to z is equal to l by 2. So, if you put in that value, you will get that. So, for very small fibers and we will describe define what is very small later but for very small fibers, sigma f max equals. So, here what is it? This sigma f max occurs at where, it occurs at z is equal to l by 2 hm.

So, sigma f max is equal to; so if I put in z is equal to l by 2 I get tau y l divided by r tau l divided by r. So, for extremely small fibers, the maximum stress which will occur in the fiber will be tau y times the length of the fiber divided by the radius of the fiber ok. So, this is another thing to understand, ok.

So, the now the next question is what happens when fibers become long. So, as I keep on increasing these fibers, theoretically if I just stick to this rule if I make the fiber very long, then theoretically if using this formula, the value of the maximum stress in the fiber can become extremely large it can if I make it one kilometer long fiber theoretically it will be almost infinite. But that never happens in real applications.

So, we will now further understand the value of f max and how it changes from with respect to length in the fiber in a larger context. So, that is the next thing we will try to understand. So, that is the type of discussion we will have tomorrow until then, have a good day. Bye.