

Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 72
Modulus of Short-Fiber Composites and Closure

Hello welcome to Advanced Composites. Today is the last day of this course and what we plan to do today is conclude our discussion on random fiber composites and also bring this overall course to a closure. What we have learned till so far in context of random fiber composites is that if the fibers are short and they are oriented in a particular direction, then we have a way to calculate how stress gets transferred from the composite from the matrix to the fiber. And based on that understanding we know how to compute the length of the fiber which will give us fairly good performance of the composite and it will it may be comparable to that of a continuous fiber composite.

Now, as I said earlier our another aim of this discussion is to estimate what is the modulus of this randomly oriented fibrous composites. So, for this there have been lots of studies and using principles of theory of elasticity and finite element analysis. And what I will do is I will directly share the results of those studies rather than go through the derivations because that is way beyond the purview of this course.

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MODULUS OF RANDOM ORIENTED FIB. COMPOSITES

$$E_{\text{RANDOM}} = \frac{3}{8} E_L + \frac{5}{8} E_T$$

$$G_{\text{RANDOM}} = \frac{1}{8} E_L + \frac{1}{4} E_T$$

$\left. \begin{matrix} E_L \\ E_T \end{matrix} \right\} \text{Long. and transverse moduli. } \eta \text{ SHORT-FIBER ALIGNED Composites with same } v_f.$

$$\frac{E_L}{E_m} = \frac{1 + 2(\ell/d)\eta_L v_f}{1 - \eta_L v_f} \quad \eta_L = \frac{E_f/E_m - 1}{E_f/E_m + 2(\ell/d)}$$

$$\frac{E_T}{E_m} = \frac{1 + 2\eta_T v_f}{1 - \eta_T v_f} \quad \eta_T = \frac{E_f/E_m - 1}{E_f/E_m + 2} \checkmark$$

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So, modulus, so what we are interested in producing the modulus of randomly oriented random composites randomly oriented fibrous composites ok. So, first I will give you the relations. So, E_{random} is equal to $\frac{3}{8} E_L$ plus $\frac{5}{8} E_T$. And then there may be situations we are also interested in finding out G_{random} . So, G_{random} is equal to $\frac{1}{8} E_L$ plus $\frac{1}{4} E_T$. Now, E_L and E_T are longitudinal and transverse moduli of short fiber aligned or aligned composites, aligned composites with same volume fraction.

So, what does that mean that if I align all the fibers in a particular direction, and the volume fraction is the same, then what is the longitudinal modulus that is E_T , and what is the transverse modulus that is so that is E_T and logical modulus is E_L . So, these relations for E_L and E_T are not the same as the ones which we had developed for continuous fiber composites their different relations.

So, E_L , so I will just give you the relation they are again gotten from (Refer Time: 04:09) equations. So, (Refer Time: 04:11) they have developed these and they worked really well. So, this is equal to $1 + \frac{2l}{d} \eta_L v_f$ divided by $1 - \eta_L v_f$. And here this parameter is $\frac{E_f}{E_m} - 1$ divided by $\frac{E_f}{E_m} + 2 \frac{l}{d}$ ok. So, if you know the volume fraction if you know v_f , E_f , E_m , and you know the length of the fiber and the diameter of the fiber you can calculate η_L and you can use this expression to compute E_L .

And similarly η_T over no E_T over E_m equals $1 + 2 \eta_T v_f$ divided by $1 - \eta_T v_f$. And η_T is different that is equal to $\frac{E_f}{E_m} - 1$ divided by $\frac{E_f}{E_m} + 2$. So, these are the relations.

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The image shows a whiteboard with handwritten mathematical work. At the top left, the word "EXAMPLE" is written and underlined. Below it, the following values are listed: $v_f = 20\%$, $l = 3.2 \text{ mm}$, $d = 10 \mu\text{m}$, $E_m = 2.76 \text{ GPa}$, and $E_f = 72.4 \text{ GPa}$. The goal is to "FIND E_R , G_R , ν_R ". The calculations shown are: $l/d = \frac{3.2 \times 10^{-3}}{10^{-6}} = 320$, $\eta_L = 0.03787$, $\eta_T = 0.89$, and $E_L = 16.26 \text{ GPa}$. The value $E_L = 16.26 \text{ GPa}$ is underlined. The whiteboard interface includes a toolbar at the top and a status bar at the bottom showing "19 / 47".

So, what we will do is we will quickly do a question or an example. So, example, so the data of the problem is v_f is equal to 20 percent, l is equal to 3.2 millimeters, d is equal to 10 microns, so 10 micrometers. E_m is equal to 2.76 Gpa and E_f is equal to 72.4 Gpa ok. So, the question is find E_R , G_R and Poisson's ratio also Poisson's ratio also. So, we do this.

So, first we compute η_L . So, that is equal to. So, what is l by d , l by d is equal to 3.2 into the 10 to the power of minus 3 by 10 to the power of minus 6. So, that comes to 3200, so that is our l by d ratio. So, η_L we do we use all this relation and we compute that it comes to be 0.03787. And η_T , it if we use the relation which is shown here, E_f over E_m minus 1 divided by E_f over E_m plus 2. So, we use this relation and we find that its value is 0.89.

So, the next thing is we compute E_L over E_m and so this is equal to this entire expression $1 + 2 l$ by d beta l over v_f divided by $1 - \eta_L v_f$. So, all I am doing is just using this relation. So, from this I calculate that E_L equals 16.26 Gpa.

Student: (Refer Time: 08:39).

No, E_L is 16.26 Gpa, E_L .

Student: (Refer Time: 08:46).

l by d ratio?

Student: (Refer Time: 08:48)..

Ok, I am sorry. So, l by d ratio should be 320 ok. And if transverse modulus of short fiber oriented composite it comes to be 4.53 Gpa. So, with this E R is equal to 3 by 8 E L plus 5 by 8 E T, this works out as 8.93 Gpa; and G R is comes out to be 3.17 Gpa.

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The image shows a digital whiteboard with handwritten mathematical derivations. At the top left, there is a note $V_f = 20\%$. Below it, the variables E_L, E_T, G_R, ν_R are listed. The derivations are as follows:

$$l/d = \frac{3.2 \times 10^{-3}}{10^{-6}} = 320$$
$$\nu_L = 0.09787$$
$$\nu_T = 0.89$$
$$E_L = 16.26 \text{ GPa}$$
$$E_T = 4.53 \text{ GPa}$$
$$E_R = \frac{3}{8} E_L + \frac{5}{8} E_T = 8.93 \text{ GPa}$$
$$G_R = 3.17 \text{ GPa}$$
$$\nu_R = \frac{E_R}{2G_R} - 1 = 0.41$$

So, the last thing is what is Poisson's ratio? So, Poisson's ratio we know that for isotropic materials the relationship between E and G and Poisson's ratio is this. So, $E R$ over $2 G R$ minus 1, so this the relation used from isotropic materials. So, if I use this, so this comes to 0.41. So, this is what I wanted to discuss in context of short fiber composites. So, we know how to decide the critical fiber length how to decide the length of the fiber, so that we use the composite to the maximum possible extent, and then how to estimate the stiffness of the fiber for shear and tensile in tensile and shear context, and also the Poisson's ratio of the system, so that brings us to the closure of this course.

Over this course we have discussed several advanced topics in area of composite materials. We started our journey by developing by doing a preview of the earlier course and then we in terms of new content we developed equilibrium equations for composite materials laminates specifically plates. And we learned several ways to solve these equations using series methods, Galerkin method, principle of virtual work and closed

form solution. And we used all these methods to solve different types of problems related to infinitely or semi infinitely long beams, problems related to finite plates, subjected to external normal forces as well as in plane forces. We also used these equations to solve problems related to vibrations, their natural frequencies and also buckling.

And finally, in the last week this week particularly we have been discussing how to get some understanding of randomly oriented fibrous composites. So, I hope you have found this course to be very useful and applicable to your professional lives. And if you have any questions about this or if you want to engage with us further in this direction please do let us know.

Thank you very much, bye.