**Advanced Composites**
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**Lecture - 08**
**General Anisotropic Material**

Hello welcome to Advanced Composites. Today is the second day of this ongoing week which is the second week of the course. Yesterday we introduced you with all the sign convention for stresses, strains and we also introduced the notion of a tensor.

Further, we explained how stress is the second order tensor, a strain is also second order tensor because in both these cases the quantities stress and strain are associated with 2 mutually independent sets of directions. In case of stress these 2 directions correspond to the direction of the plane on which the force is acting and also the direction of the force and in the case of strain these 2 sets of directions correspond to the direction of displacement and the direction in which its partial difference is been calculated.

We also saw yesterday that the mathematical relationship between stress and strain is through a fourth order elasticity tensor which is E i j k l. So, now, we will further discuss this particular tensor in context of anisotropic materials ok.

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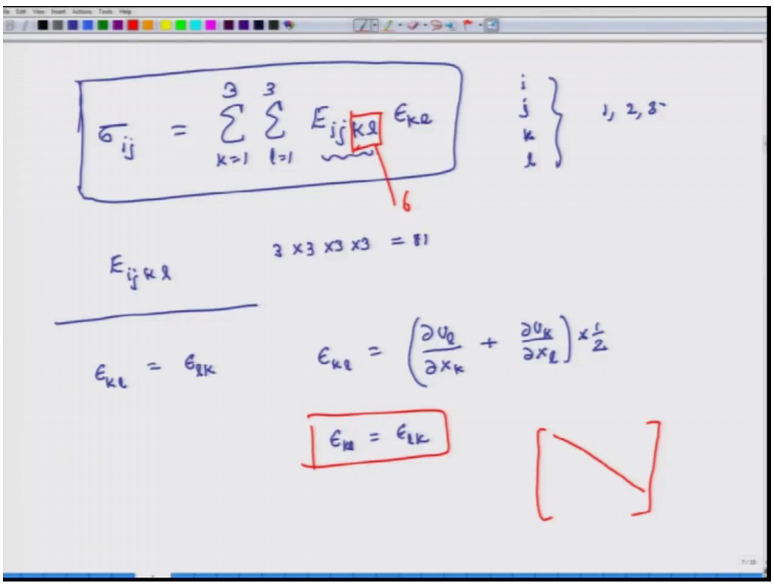

So, what we had developed yesterday was sigma i j equals double summation E i j k l; epsilon k l and here we are summing on these indices. So, l equals 1 to 3 and k equals 1 to 3. So, once again here indices i j k l all of them could assume values of 1 2 and 3 and 1 2 3 are basically axis 1 could be x axis, 2 could be y axis and 3 could be z axis.

Now, if you look at E i j k l tensor; it could have 81 different values why? Because we will have 81 different combinations of E i j k l because i is can assume 3 different values 1 2 3; j can assume 3 different values 1 2 3 and so on and so forth. So, the total number of combinations of E i j k l would be 3 times 3 times 3 times 3 and that corresponds to 81 ok. So, in general this relation between stresses and strains involves 81 different elastic constants. So, for an isotropic material we are starting with the situation where there are 81 different elastic constants ok.

So, the next now we start simplifying this. So, the first simplification we do is based on the fact that epsilon k l equals epsilon l k; why is that? So, the definition of a strain tensor is epsilon k l is equal to del partial derivative of u l differentiated with respect to partial derivative of x k ok. So, x k; x 1 would be x, x 2 will be y, x 3 will be z and so on and so forth.

So, this is 1 plus del u k over del x l and this entire thing into 1 by 2 ok. So, this is the definition of strain and when you; so, this comes from mathematics we will not go into that and what you find is because of this epsilon k l is equal to epsilon l k right.

So, what; that means, is that if epsilon l k is equal to epsilon k l then the number of independent constants in this block k l; will not be 9, if it they were if this equivalence was not true; then this would be the number of independent constants involved with subscripts k and l would be 9; 3 into 3, but now if there is 3 by 3 matrix; then this matrix will be symmetric which means that there will be 6 different possible combinations not 9.

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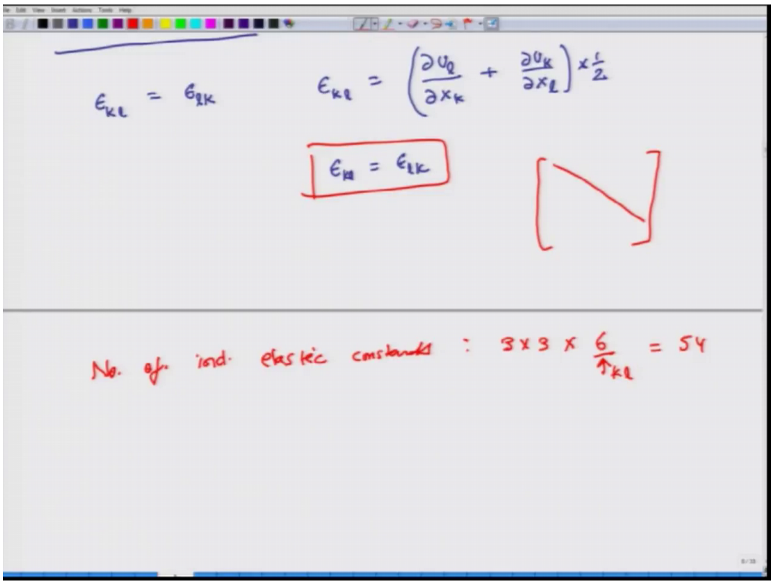


So, if that is the case when the number of elastic constants; number of independent elastic constants it comes down to 3 times 3 times 6. So, this is because k l is symmetric matrix; epsilon k l is symmetric matrix. So that means; it is 54. So, all of a sudden the requirement that we have to find out 81 elastic constant it drops down and it comes to 54.

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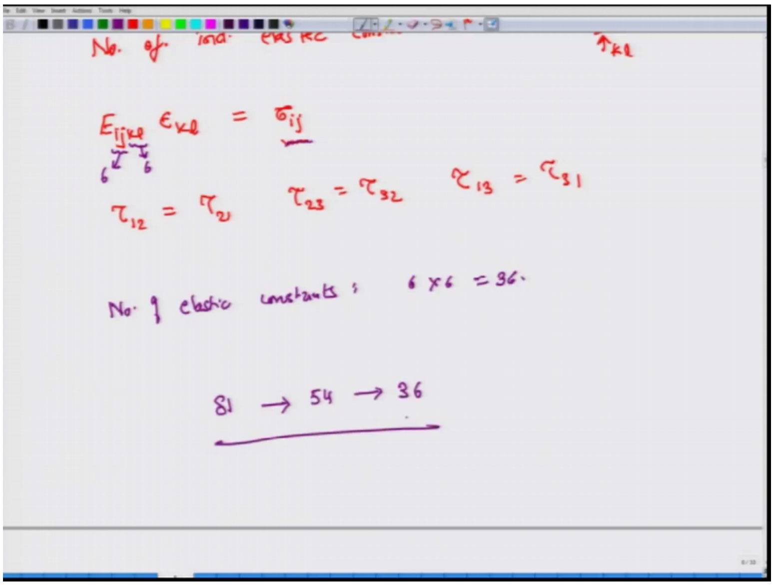


Next look at E i j k l and this is multiplied by epsilon k l. So, that equals sigma i j; now we know from principles of mechanics that tau 1 2 equals tau 2 1 tau 2 3 equals tau 3 2 and tau 1 3 equals tau 3 1.

So, essentially what; that means, is that there are no not 6 independent stress stresses, but rather they are known; we do not have 9 independent stresses associated with sigma i j, but 6 independent stresses. So, we have already simplified the number of combinations associated with subscripts k l n to 6. And similarly we find that this also is associated with not 9 independent situations, but only 6 independent situations because i j relates to sigma i j ok.

So, the number of constants; so, this is i j is related to 6 and k l is related to 6 independent situation. So, number of elastic constants it drops down to 6 times 6 is equal to 36. So, we started with 81 because of symmetry of a strain tensor; it comes down to 54 and because of symmetry of a stress tensor, it now comes down to 36 ok.

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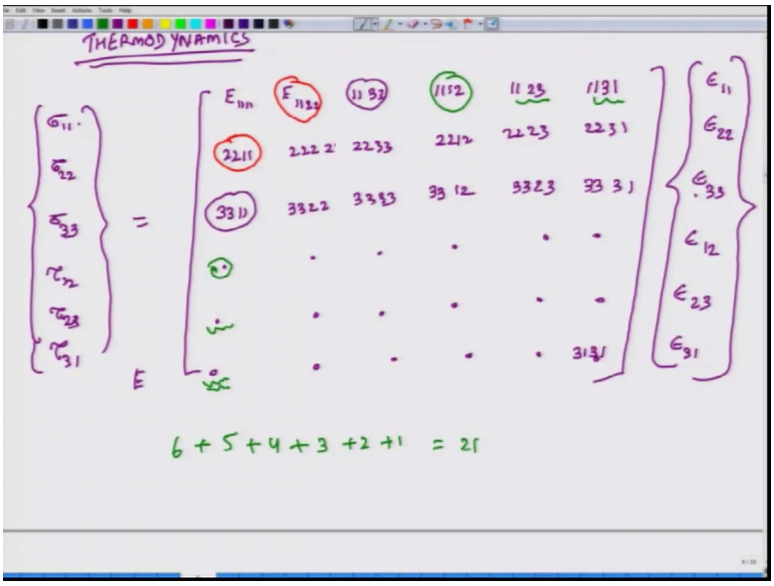


Let us move further, so the next simplification is based from the principles of thermodynamics ok; based on principles of thermodynamics. So, before we talk about this if there are 36 independent elastic constants; then the stress strain relationship would look something like this. So, in this we have a big matrix involving elasticity tensor and this is epsilon 11, epsilon 22, epsilon 33, epsilon 12, epsilon 23, epsilon 31 ok.

And the elasticity matrix would be E 1 111 E 1122 and then I am not going to write E again. So, the third one will be 1133 because this term connects the first row which is sigma 11 with epsilon 33 ok. So, that is why its subscript is 3133 and then we have 1112, 1123 and 1131. Likewise, the second row will have 2211, 2222, 2233, 2212, 2223, 2231 and so on and so forth.

So, you can using this concept you can develop the whole matrix, you can develop the whole matrix. So, will just write one more row; so the next row will be 3311, 3322, 3333, 3312, 3323, 3331 ok. And then we have fourth row, fifth row, sixth row; the sixth row will be 3131 last element.

Now, based on thermodynamic considerations which are related to a strain energy density and the description for this and justification for this has been explained an our introductory course; these are overall 36 independent constants, these are overall 36 independent constants ok. So, these 36 constants come down to 21 because based on thermo dynamical considerations which we are not going to explain, there are further simplifications and those simplifications involve imply that this entire matrix is symmetric.

So, this E matrix is symmetric which means; so, if it has to be symmetric then this number should be same as this, this number should be same as this, this number should be same as this, this number should be same as this number should be same as this and so on and so forth ok.

So, a 6 by 6 matrix initially has 36 independent constants, but now because of symmetry, the total number of constants it drops down to 30 it drops down to 21; why it is 21? Because there are 6 independent constants in the first row, 5 independent constants in the second row, 4 independent constant in the third row plus 3 plus 2 plus 1 and so on and so forth. So, if you add these up together it is 21.

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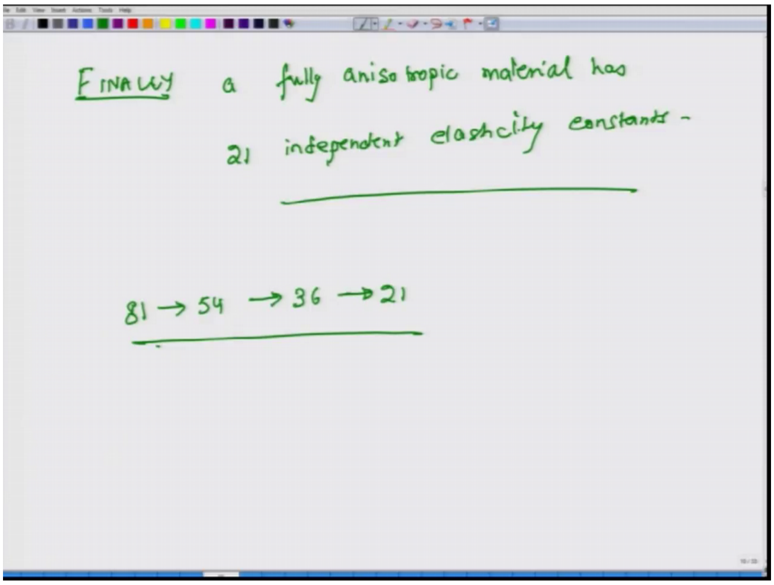


So, this is the final answer; so finally, we can say finally, we can say that a fully anisotropic material has 21 independent elasticity constants it has 21 independent elasticity constants.

So, how did we get here? We started with 81 constants because of symmetry of strain tensor, it came down to 54, because of symmetry of stress tensor it came down to 36. And because of thermo dynamical considerations involving strain and energy density function, it comes to 21. So, this concludes our discussion for today; tomorrow we will continue this discussion and we will show that the number of elastic constants drops down further is the material was specially orthotropic which we have discussed what it means yesterday.

So, for that kind of a situation we will make some more simplifications, but if the material is not specially orthotropic and fully anisotropic; then what it means is that they will be 21 independent elastic constants which will relate stresses and strains. So, that concludes our discussion and I look forward to seeing you tomorrow.

Thank you.