Advanced Composites
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Lecture - 09
Specially Orthotropic Material

Hello welcome to Advanced Composites. Today is the third day on the ongoing week and what we planned to do today is discuss the number of elastic constants for orthotropic materials. So, we will discuss the case of orthotropy and in particular we will discuss Specially Orthotropic Materials.

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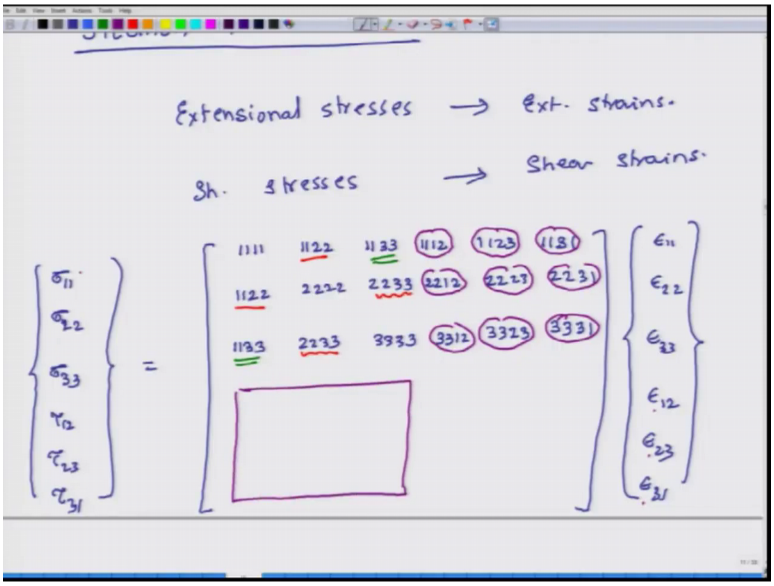

So, we are discussing specially orthotropic materials. So, what we have discussed is that if the such specially orthotropic material is subjected to extensional strains or extensional stresses; then it only generates extensional strains.

And if we are subjecting the material only to shear stresses; then you only get shear strains. So, with this understanding let us look at the stress strain relationship for an orthotropic material. So, on the left side we have sigma 11, 22, 33; tau 12, tau 23 and tau 31. And then we have this large elasticity matrix which is symmetric because we discussed that in the last course that generally anisotropic material has 21 independent constants and this is the symmetric elasticity matrix and then we have the strain tensor and what are the strain tensor.

A strains epsilon 11, epsilon 22, epsilon 33, epsilon 12, epsilon 23, epsilon 31 ok; so, these are strains and they are tensor strains and we have already discussed what these tensor strains are. Let us look at these constants; so this is 1111, 1122, 1133, 1112, 1123 and 1131; this is a symmetric matrix. So, this number is 1122 then the next one is 2222, 2233, 2212, 2223, 2231; the first element in this row is same as the second element in the first row because we have said that this is a symmetric matrix.

You just write one more line and then you can fill up all the other good stuff. So, the third row we will start from what? 1133 and the second element then this will be same as 2233 because this term will be same as this term because of symmetry and this term will be same as this term because of symmetry ok.

And then the third element in this is 33; 3312, 3323, 3331 and so on and so forth. So, you can fill up this entire column this entire matrix in this way. Now what we have said is that if this material is specially orthotropic, then if I subject it to extensional strain only extensional strain; then it will only extensional stress then it will only generate extensional strains.

So, consider one case where I am having a this material and if I am subjecting it to only sigma 11; then it should not generate epsilon 12 which is shear strain, epsilon 23 which is a shear strain, then epsilon 31 which is a shear strain right and vice versa which means that these components should be 0 because they couple shear strains to extensional stresses. So, similar; so this entire block it goes away, this entire block goes away. Similarly all this stuff also goes away right; so all these things go away. So, once they go away when extensional strains will not produce shear strains and shear stresses will not produce extensional strains and vice versa. But there is one more condition which we did not explicitly talk about when we were discussing specially orthotropic materials and we will discuss that now.

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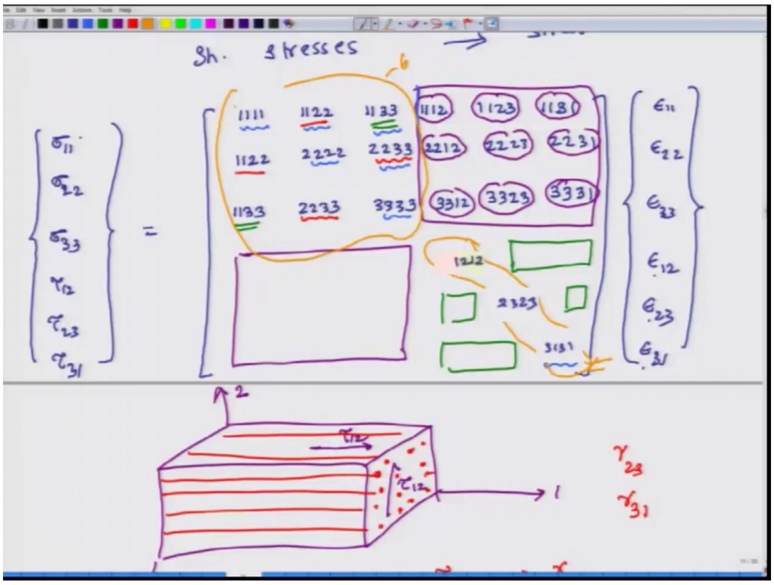

So, consider a case so this is my 1 axis, 3 axis and 2 axis suppose I exert only tau 12 on this right.

So, ideally this top shear should be tau 21, but because tau 12 is equal to tau 21 I have just used the same indices ok. Now if the fibers in this composite are aligned with the material axes and let say that material axes alignment is with respect to axes 1, then on this surface I will see only the ends of this fibers. And this material is been subjected to only the shear stress tau 12; then first thing because this is specially orthotropic it will not have any extensional strains.

But it will also not have gamma 23 and gamma 31 because when you look at the picture; it will only slide in the x direction, it will only shear in the x direction it will not shear in the 3 direction or 2 direction right

So, based on this physics we say that tau 1 2 will generate only gamma 1 2. And then tau 2 3 will generate only gamma 2 3 and tau 3 1 will generate only gamma 3 1 ok. So, what; that means, is that when I populate this matrix further and 1212 I will call it 2323 and the last then I have 3131.

So, these terms these 2 terms and this term; they couple tau 122 epsilon 23 and epsilon 31. And similarly these terms will also; so these terms which are there in the green block they should also be 0; otherwise if I apply shear stress tau 12, it will generate not only shear strain; epsilon 12, but also shear strain epsilon 23 and epsilon 31 right.

But we because it is specially orthotropic tau 1 2 will only has to generate gamma 1 2, tau 2 3 has to only generate gamma 2 3 and so on and so forth; so, all these elements in the blocks are also 0. So, once that is the case then how many independent elastic constants we have? We have 1, 2, 3, 4, 5 everything is symmetric 6, 7, 8 and 9; 9 constants.

So, these guys which are in purple boxes they are all 0; all the elements in green boxes 0 and the remaining elements are because of symmetry we have in this range in this block we have 6 independent constants and in this block we have 3 independent constants.

So, total we have 9 independent elastic constants.

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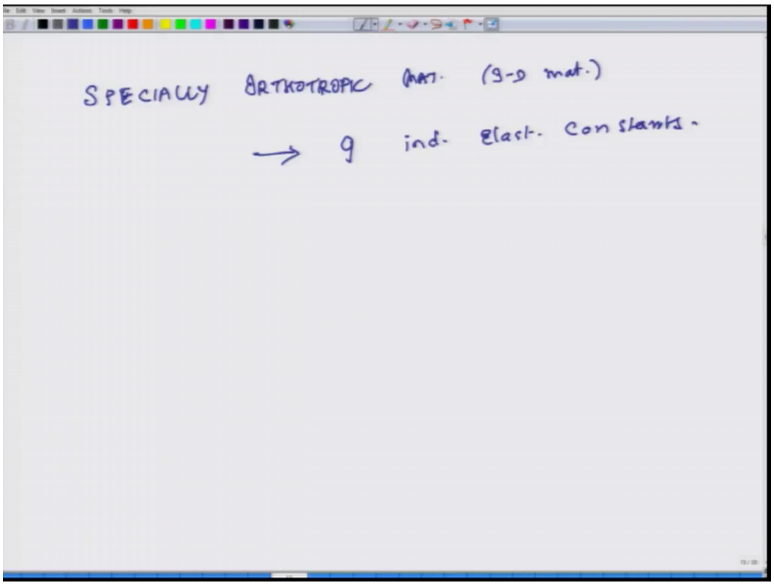

Orthotropic specially 3 D orthotropic material; so, if a material is specially orthotropic in 3 and it is a 3 D material, it is not like a thin plate. This has 9 independent elasticity constants it has 9 independent elasticity constants. So, we will write that because we are now going to make a simplification.

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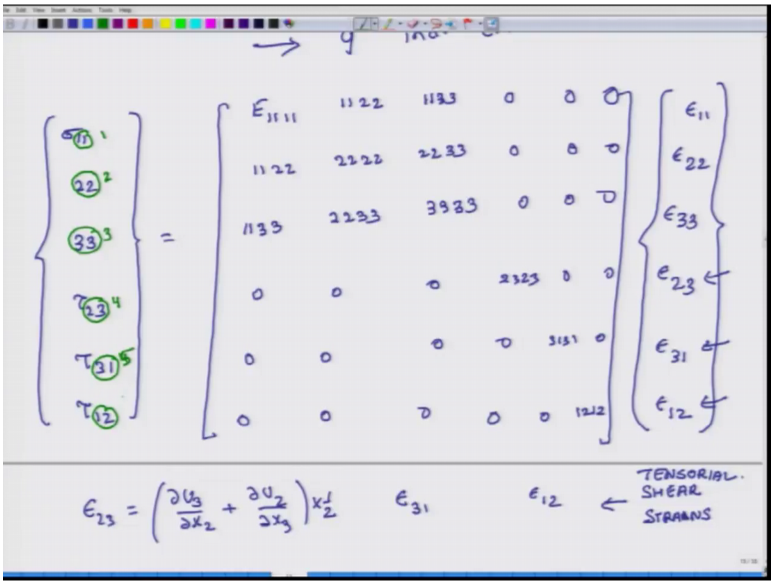

So, what is my stress tensors; sigma 11, 22, 33, 12. So, what I will do is that while I am rewriting these constants; I will just rearrange the order of shear terms. So, instead of writing tau 12 first, I will write tau 23 first just rearranging the equations.

Tau 31 and in the end I will write tau 12; because I want to get the nomenclature correct ok. And here I have my strain tensor which is a fourth order and that is multiplied by; so, this is the fourth order elasticity tensor and this gets multiplied by epsilon 11, 22, 33 and then this is strain 23, elasticity no shear strain tensor 31 and shear strain tensor 12.

So, remember here I am using epsilons even for shear strains these are called strain; you know tensor shear strains. So, epsilon 23 is tensor shear strain; epsilon 31 is tensor shear strain epsilon 12 is tensor shear strain and let us write down the definitions of these guys. So, epsilon 23 is del u 3 over del x 2 plus 2 del u 2 over del x 3 times 1 by 2 there is a half factor.

And similarly we can express relations for epsilon 31 and epsilon 12 ok. So, these are the definitions of shear strains, but these are called tensorial shear strains because they are tensors. There is another thing called engineering shear strain which different, but for the moment this is our definition; this is what we are using in these equations, we are using tensorial shear strains we are not using engineering shear strains.

And what are the fourth order elasticity constants? E 11 11; 11 22 11 33; 0 0 0 so like this, so the second row is 11 22; 2222 2233, 0 0 0 I will write one more line 1133, 2233, 3333, 0 0 0; then here we have all 0s and then we have 4444. So, 44 or you can have actually yeah I will not call it 4, I will call it 2323.

Then I have 3131 and 1212 and all other terms are 0 all other terms are 0. So, this is the stress strain relationship for a 3 dimensional specially orthotropic material. Now we will just rewrite these relations in engineering terms. So, what we will do is we will call sigma 11; so, wherever we see 11 which comes as a group we call it 1, wherever we come get 22; we call it 2 not twice 2, but only once wherever we come get 33, we call it 3 wherever we get 23 we call it 4 and 3 1 is 4 and this is. So, I am sorry this is 5 and this is 6.

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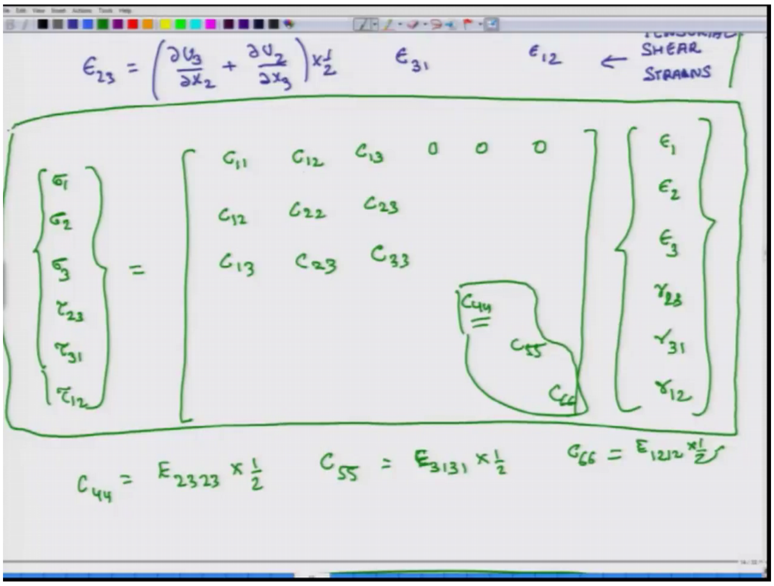

So, my stress tensor becomes sigma 1, sigma 2, sigma 3 actually we will not use these 4 5 6 terms in the stress tensor, but we will use it elsewhere and then we have tau 23 tau 31 and yow 12 because you want to know what directions we are talking about. So, we will use these 1 2 3 terms in the elasticity matrix ok, but before we write the elasticity matrix; here we will write our strain tensor, but instead if engineering strain tensor we will talk about engineering strains.

So, epsilon 11 is epsilon 1, epsilon 22 is epsilon 2, epsilon 33 is epsilon 3 and then instead of writing these tensorial shear strains we write engineering shear strains. So, what are engineering shear strains?

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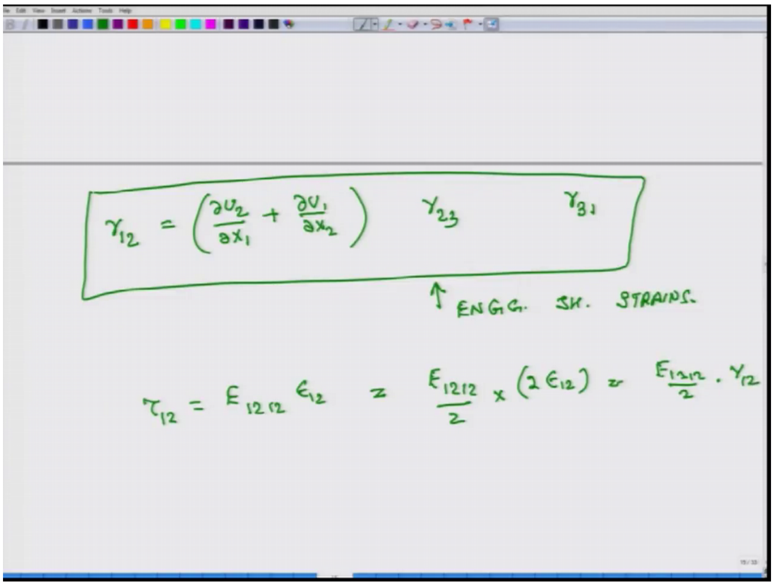

Gamma 1 2 is del u 2 over del x 1 plus del u 1 over del x 2 and there is no factor of half. Similarly gamma 23 we can have a definition and gamma 31 we can have a definition. So, these if we do not use this term half then they are called engineering shear strains ok. So, what is the mathematical relationship between shear stress and engineering shear strains? So, for instance I will give you an example sigma 12 is equal to. So, we have seen this sigma 1 2 is what? E 1 2 1 2 times epsilon 1 2 from this equation right this is tau 1 2 is equal to E 1 2 1 2 times epsilon 12.

So, I am sorry this should not sigma. So, tau 1 2 is equal to E 12 12 times epsilon 1 2. But if I want to express the same relationship in terms of engineering strain; now twice of this tensor strain is the engineering strain. So, this is E 1 2 1 2 divided by 2 into twice of epsilon 1 2 and twice of epsilon 1 2 is gamma 1 2. So, it is e 1 2 1 2 divided by 2 times gamma 1 2.

And similarly we can write expressions for gamma 2 3 and gamma 3 1 also ok. Now remember engineering tensile strengths epsilon 1, epsilon 2, epsilon 3 their definitions with epsilon 11, epsilon 22, epsilon 33 are same. The factor of 2 comes only for the share components epsilon 23, epsilon 31 and epsilon 32; so, it is important to remember that.

So, with that understanding now once we are. So, with that understanding we will rewrite this elasticity tensor and in this elasticity tensor we will use the same convention which I talked about. So, wherever we see 11 we write 1, wherever we see 22; 11 we again write 1, wherever we see 22 we write 2, wherever we see 33; we write 3, wherever we see 23; we write 4 wherever we see 31, we write 5 and wherever we see 12 we write 6 ok.

So, with that; so, here we are going to write gamma 1, gamma 23, gamma 31 and gamma 12. And what are going to be our elasticity constants? And because we are abbreviating it and we are moving from tensor world to engineering world, we will not use this term letter E, but rather we will use a letter C; so, it is C 11.

The first one designates 11, the second one designates again 11 right because here we have 11 11. So, 2 11 means 11 and the second set of 1s means another one. So, it is C 11 C 12, C 13, 0 0 0, C 22, C 23, C 33, C 44, C 55, C 66 ok. And all other terms all other terms in this matrix are 0 all other terms are 0 and what are C 44?

So, C 11 is same as E 1111, C 1 2 is same as E 1122 and so on and so forth, but these C 44, C 55 and C 66; they are different. So, C 44 equals E 2323 times divided by 2; C 55 is equal to E; 3131, this is E 3131 into 1 by 2 and C 66 is equal to E 1212 into 1 by 2. So, now we have a relationship between engineering strains and stresses. And C are elasticity constants which couple engineering strains not shear not tensor strains; engineering strains to stresses.

So, this is the relation for fully orthotropic 3 D material. So, the next thing we will do is we will talk about transverse isotropy.

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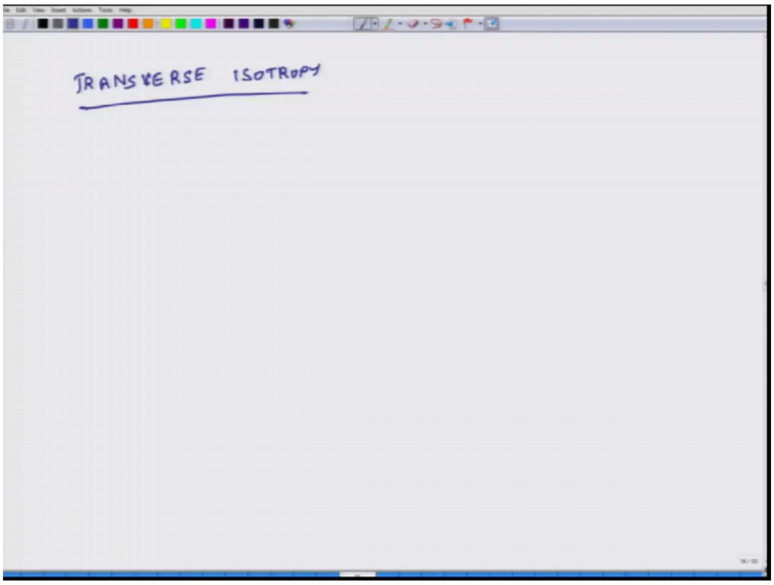

So, this is the case when in a particular plane the material properties are isotropic, but I guess that time for today's lecture is pretty much over. So, we will again continue this discussion tomorrow and we will again consider cases of transverse isotropy, isotropy and the situation when there is plane stress case. So, with this we close our discussion for today and we will meet tomorrow.

Thank you.