

**Engineering Metrology**  
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**Lecture – 30**  
**Strain Measurements (Part 1 of 2)**

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**Strain measurements**

↓  
displacement/deflection

• Force

$E = \frac{\text{stress}}{\text{strain}}$  (with  $\sigma/A$  written next to stress)

• Uniaxial ✓  
• Biaxial ✓  
• Triaxial ✓

Force

- Tension
- Compression
- torsion
- shear

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Today we will see a topic strain measurement, strain is basically nothing but displacement or deflection measurement slash deflection measurement. So, basically what we do is, we try to measure strains, when we are trying to apply force, we will try to see what is the deflection. It is easy to measure strain from the strain, you can try to go back and work out, what is the force you applied for example, you know the Young's modulus of a given material, Young's modulus within the elastic limit is directly proportional to stress by strain. So, what is stress? Stress is nothing, but force per unit area you know area, because that is a design parameter you know a material parameter I measure strain.

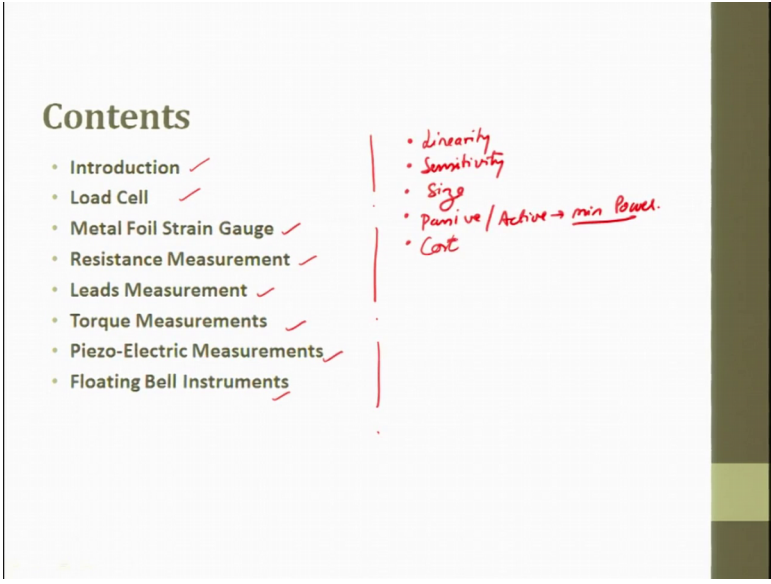
So, naturally I can try to measure the force and force measuring is not very easy because in real time application, whenever there is a force which is acting on a body, it can be it can be in one direction uniaxial. It can be in biaxial, it can also be in triaxial. All the

measurement what we do in a in the universal testing machine is on a uniaxial testing, today slowly the concept of biaxial is also coming up biaxial means, it is basically in 2 directions. The force will be applied and the strains will be measured or the strains will be applied and the force will be measured.

So, whatever it is. So, we try to do biaxial testing today, what we talk about is triaxial testing and when we talk about forces, the basic forces are going to be one is tensile, the other one is compression and the third one is going to be torsion and the fourth one going to be shear. So, these are the typical forces, we generally use to measure and all the measurement which is the measuring strains is very common in solid only, gas and liquid strains are very rarely measured, gases it is very rarely measured and solid we always go focus on it.

So, when we talk about measurement it is not only the part manufacturing and taking measurement. It is also important to measure the temperature temperatures strain pressure also. So, in this course we are also putting little efforts in giving, you the glimpse of strain measurement.

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Contents	
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• Load Cell ✓	
• Metal Foil Strain Gauge ✓	
• Resistance Measurement ✓	
• Leads Measurement ✓	
• Torque Measurements ✓	
• Piezo-Electric Measurements ✓	
• Floating Bell Instruments ✓	

So, we will try to have the introduction for strain measurement, then load cell which is a very primitive equipment, which is used for measuring the strains then metal foil strain gauges, then strain resistance measurement, then leads measurement, then torque measurement, piezoelectric measurement and floating bell measurement.

So, all these things are used for measuring strains ok, as I told you earlier when we talk about any measuring equipment, we try to talk about the linearity, sensitivity and the size. Of course, you would like to have passive, if not passive we will try to have active, but in active also it has to have minimum power requirements ok, these are the basic things which you should always keep in mind while deciding a sensor. So, based on this we will try to choose, what is required for us? And of course, cost is one thing which is very important.

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**Introduction**

- Our standards of force are ultimately based on weight, magnified by levers as necessary.
- Sometimes direct application of weights is convenient, or we use spring balances which are easily checked by attaching weights.
- A minor problem with weights is that gravity varies perceptibly, mainly due to the earth's non-spherical shape.
- Gravity has been measured by freefalling body methods to be about 9.8127 Newtons per kg (or m/s<sup>2</sup>) in London but would be about 9.7725 N/kg in Quito, near the equator and 2800 m above sea level.

Strain =  $\frac{dl}{l} = \frac{\Delta L}{L}$

Our standards of force are ultimately based on weight magnified by a lever as necessary, sometimes direct application of weight is convenient like in spring balance right.

Or we use a spring balance, which are easily checked by attaching weight through it. So, what you measure is basically the spring in a spring, balance you try to put a mass and moment you put a mass the string gets stretched. Now, how much has it stretched? Is nothing, but your displacement, from the displacement you try to find out strain and by the way, what is a definition for strain? Strain is nothing, but  $dl$  by  $l$  or it can be sold as  $dl$  by  $l$  ok,  $dl$  is the delta increase by applying the load the minor problems with weight is the gravity varies perceptively mainly, due to the earth's non spherical shape, gravity has been measured by free falling body method to be about 9.81 Newton per kg or meter per second square in London.

But would be about 9.7725 Newton per kg in Kyoto, Kyoto near the equator and 2800 kilo meters above the sea level, you see there is a difference that is what we are trying to say and this difference we will try to influence in the spring balance.

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## Introduction

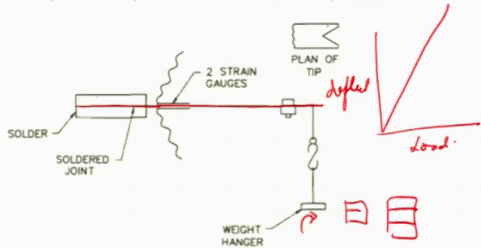
- However, they are cheap, portable, and do not drift unpredictably or go out of calibration.

However they are cheap portable and do not drift and predictably or go out of calibration, this makes the spring balance a very common equipment for measuring strain.

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## Load Cells

- Traditional platform scales have a lever system ensuring parallel motion, so that the potential energy change for a given deflection, and hence the reading, is the same at all load positions.
- Scales using several load cells but no parallel motion mechanism should have matched cells with equal load-deflection properties and should have a highly linear output-load relation, otherwise the outputs will not add up correctly for all loads and load positions.



The diagram illustrates a load cell setup. It shows a horizontal beam with a soldered joint on the left end. Two strain gauges are attached to the beam. A weight hanger is suspended from the right end of the beam. A plan of tip is shown above the beam, with handwritten red annotations: 'deflected' and 'load'. A graph to the right shows a linear relationship between load and deflection. Below the graph are three small diagrams representing different load positions.

Now, let us see the next thing which is called as load cell. Traditionally, platforms scales have a lever system ensuring parallel motions. So, that the potential energy change for a given deflection and hence the reading is the same at all load positions for example, this is traditional one scales have a lever system ensuring parallel motions.

So, that the potential energy change for a given deflection and hence the reading is the same at all load positions. Scales using several load cells, but not parallel motion mechanism should have matched cells with equal load deflection properties and should have a highly linear output load relationship, otherwise the output will not add up correctly to all loads and load possessions. So, what we are trying to say is this is the deflection and this is the load. So, we are trying to say the output should be linear. So, generally what we do is, we try to have a solder which is there, then we try to have a solder joint here. So, here we have a bar or a scale, whatever it is and then we attach 2 strain gauges. We will see what is strain gauge? In due course of time strain gauges and then this is attached to a load.

So, as and when you increase the weight as then when you increase the weight, you can see that the deflection the strains will increase and the deflections will be measured.

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### Load Cells

**Resistance of a load:**  
 Consider a conductor having a uniform c-s of Area  $A_c$  and length  $L$  made of a material having electrical resistivity,  $\rho_e$ . For this electrical conductor, the Resistance  $R$

$$R = \rho_e L / A_c$$

If the conductor is subjected to normal stress along the axis of the wire

$$dR = \frac{A_c(\rho_e dL + L d\rho_e) - \rho_e L dA_c}{A_c^2} \quad \dots \text{---} \textcircled{1}$$

The c-s and Length change resulting in a change in electrical resistance  $R$ , the total change in  $R$  is due to several effects

Eqn ① can be expressed in terms poisson's ratio

$$\frac{dR}{R} = \frac{dL}{L} (1 + 2 \nu_p) + \frac{d\rho_e}{\rho_e}$$

Let us see now, how is the load measured resistance of load. First let us consider a conductor having a uniform cross section of area, this is very important  $A_c$  and length  $L$  made of a material having electrical resistivity for this electrical conductor or this

conductor the resistance  $R$  is given by  $R = \rho \frac{L}{A_c}$ . If the conductor is subjected to normal stress along the axis of the wire then it is given reported as  $dR$ . So, the change in resistance will be  $dR = \rho \frac{dL}{A_c} + L \frac{d\rho}{A_c} - \rho \frac{L dA_c}{A_c^2}$ .

So, here the wire so, what are we trying to say? We are trying to say the cross section and the length change this resulting in a change in electrical resistance  $R$  ok. So, the total change in  $R$  is due to several effects. So, this is what is this  $dR$  represents all about. Ok so, this equation I will say 1, equation 1 can be expressed, can be expressed in terms of poisson ratio, poisons ratio we express because poisons ratio is known to us. It is a material property which is poisons ratio. So, we try to use it in poisons ratio.

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**Numerical Problem**

**Question:** Determine the total resistance of a copper wire having a diameter of 1 mm and a length of 5 cm. The resistivity of copper is  $1.7 \times 10^{-8} \Omega \cdot m$ .

The Resistance may be calculated by

$$R = \rho \frac{L}{A_c}$$

$$A_c = \frac{\pi}{4} D^2 = \frac{\pi}{4} (1 \times 10^{-3})^2 = 7.85 \times 10^{-7} m^2$$

$$Resistance = \frac{(1.7 \times 10^{-8} \Omega \cdot m)(5 \times 10^{-2} m)}{7.85 \times 10^{-7} m^2} = \underline{\underline{1.08 \times 10^{-3} \Omega}}$$

So, that is nothing, but  $dR$  by  $R = \rho \frac{L}{A_c}$  ok. So, now let us try to solve a problem and see determine the total resistance of a copper wire having a diameter of 1 millimeter and the length of 5 centimeter, please see I have given 1 unit in millimeter the other one in centimeter.

It is good to converted the resistivity of a copper is 1.7 into  $10^{-8}$  ohm meter. So, how do we solve it? So, the resistance maybe calculated by  $R = \rho \frac{L}{A_c}$  ok. So, which is nothing, but  $A_c$  can be expressed in terms of  $\pi$  by 4  $d$  square, which is  $\pi$  by 4 into 1 into  $10^{-3}$  the whole square which is nothing, but 7.85 into  $10^{-7}$  meter square ok, resistance is represented as  $R$

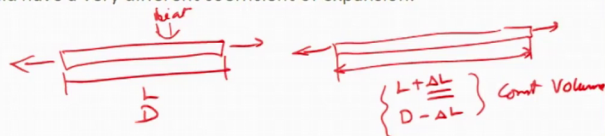
equal to  $10^{-8}$  ohm meter into  $10^{-5}$  to the power minus 2 meter that divided by  $7.85 \times 10^{-7}$  meter square.

So, this is nothing, but  $1.08 \times 10^{-3}$  ohms. So, this is the resistance, which is getting built in this problem the resistivity of a copper is given, because this is unknown see if you see very clearly this is a material property. This is a design or intentionally manufactured thing. So, now, using all these things what are you trying to do is total resistance has to be measured.

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### Metal Foil Strain Gauges

- The resistance of a strain gauge increases when put under tensile stress.
- The main reason is an intrinsic change in the material while under load (not a permanent change) but part of the effect is the increase in length and the reduction of cross-section.
- Thermal expansion of most structurally common metals exceeds 11 parts per million for each degree Celsius, whilst the gauge material could have a very different coefficient of expansion.



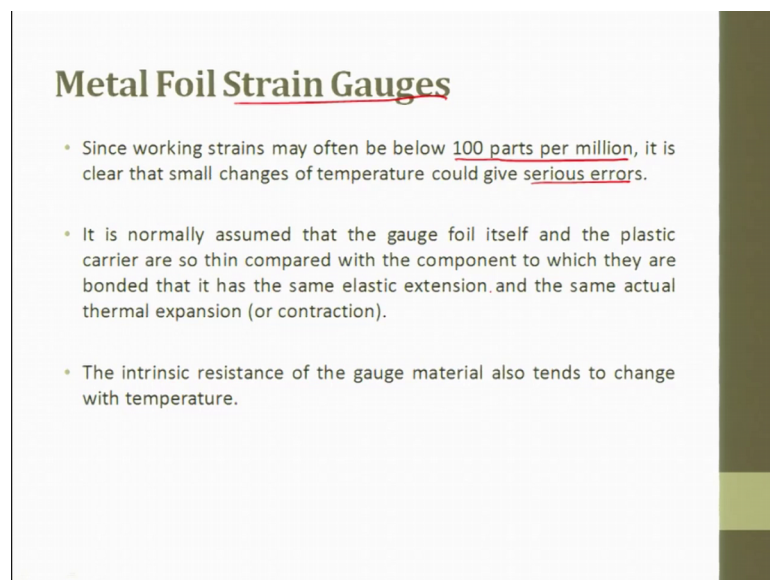
So, load cell is 1, the next one is the metal foil strain gauges the resistance of a strain gauge increases, when put tensile stress tensile compression, but predominantly we used for tensile stress.

The main reason is an intrinsic change in the material while under load, but part of the effect is the increase in length and the reduction in cross section. So, for example, you have a wire. So, when you try to pull this wire this is  $L$ . So, what happens is, when you apply load this can go to  $L + \Delta L$ , when this  $\Delta L$  happens so, naturally the diameter so, that the diameter be  $D$ . So, now, the diameter will be  $D - \Delta D$ , why because it is a constant volume ok. So, the main reason is an intrinsic change in the material while under load, but part of the effect is the increase in length and reduction in cross section. Thermal expansion is most structurally common metal exceeds 11 parts per

million for each degree Celsius, while the gauge material could have a very different coefficient of thermal expansion.

So, we talk about thermal expansion suppose, in this shaft if you start applying heat. If heat is applied then, what happens? What is the material property? You have to take that is what is discussed here.

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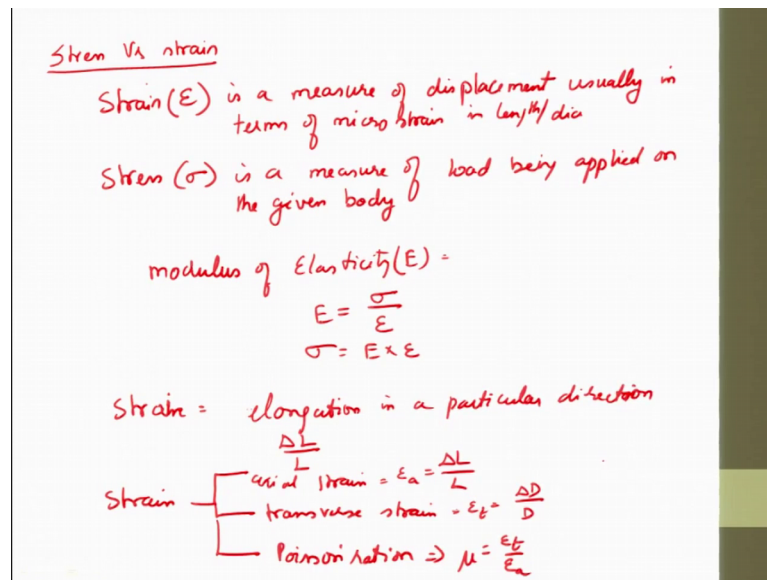
**Metal Foil Strain Gauges**

- Since working strains may often be below 100 parts per million, it is clear that small changes of temperature could give serious errors.
- It is normally assumed that the gauge foil itself and the plastic carrier are so thin compared with the component to which they are bonded that it has the same elastic extension and the same actual thermal expansion (or contraction).
- The intrinsic resistance of the gauge material also tends to change with temperature.

Since working strain may often be below 100 parts per million, it is clear that small changes of temperature could be very serious error. So, please make sure, when we use the strain gauge measurement the temperatures are not dominating. It is normally assumed that the gauge foil itself and the plastic carrier are. So, thin compared with the component to which they are bonded that, it has a small elastic extension and same actual thermal expansion.



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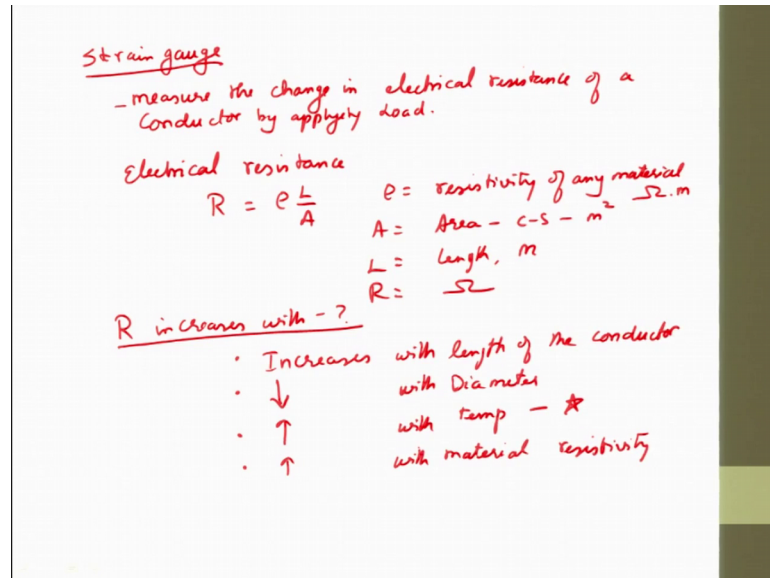
So, let us structurally understand the use of strain gauges. So, far that you will first understand was what is stress versus strain? Though I gave in the introduction very brief, so far, more clarity let us have a detail presentation. Strain is psi is a measure of displacement usually in terms of microstrain; that means, to say very small micro strain in length, length or diameter then, what is stress? Stress is sigma is a measure of load being applied on the given body, where you measure the strain ok. So, the stress and strain are linked by modulus of elasticity E. So, Hook's law says within the elastic limit E is equal to stress by strain or I can say stress equal to E into strain ok.

So, let us understand what is strain? Strain is defined as an elongation, we can say elongation in a particular direction or we can say increase in length by original length ok. So, these strains can be there can be 3 different types of strains. One can be, can be transferred strains or it transfer strain, the other one can be axial strain and of course, you have poisons ratio, what is poison ration? Poisons ratio is generally referred as mu equal to sigma t by sigma a ok. So, then transfers is nothing, but you call it as t which is equal to change in diameter by original diameter. In axial it is going to be a sigma a axial. So, that is nothing, but delta L by L. So, we know that increase in length or increase in diameter or poisons ratio has to be measured.

So, now what you know? You know there is a constant volume material available with you once the constant material available with you expands ok. There is a change in

resistance so, if we can measure the change in resistance, because that is easy for me to process and later use for control, because it is voltage as the output. So, I now try to measure the resistance, which is which is getting generated or a change in resistance.

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So, for that what we use is we use device called as strain gauge. So, a strain gauge is an equipment which is used to measure the change in electrical resistance of a conductor because you cannot use an insulator of a conductor by applying load.

So, if you can measure the change in electrical resistance. So, that device is we use, this is the basic principle for using strain gauge or basic principle behind the strain gauge. So, the electrical resistance R can be measured by rho into L into A, where rho is the resistivity of any material ok, which is generally given in ohm meter, A is the area; that means, to cross section area, which is always in meter square ok, L will be in meter length the unit will be in meter.

So, when I find out R will always be in ohms, if I cut down the units place the units and cut it down I get in ohms. So, there are some interesting facts, which should know about how to increase R or R increases with what? First test it increases with length of the conductor. Conductor is a wire shaft, whatever it is next it decreases with diameter ok, the other thing is it increases with temperature.

Please make a note, many a times we get erratic readings, because of change in temperature either at the conductor or when the conductor is stuck on to a shaft that the change in temperature leads to this and the last thing is increases with material resistivity ok. These are the ways and means you can change, we are or we change in R value will come based on these things.

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Gauge factor (GF)  
 $R = \rho L/A$   
 change R  $dR = d\rho \cdot (L/A) + \rho \cdot d(L/A) \quad \text{--- ①}$   
 expand  $dR = d\rho \cdot (L/A) + \rho/A \cdot dL + \rho \cdot L \cdot (-1/A^2) \cdot dA \quad \text{②}$   
 $\div$  equ ② by R  
 $\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad A = \pi \cdot \left(\frac{D}{2}\right)^2$   
 Now we  $\frac{dA}{A} = \pi \cdot 2 \cdot \left(\frac{D}{2}\right) dD \propto \frac{dA}{A} = 2 \frac{dD}{D} = 2 \epsilon_t$   
 $\frac{dL}{L} = \epsilon_a \quad \text{or} \quad \frac{dR}{R} = \frac{d\rho}{\rho} + \epsilon_a - 2 \epsilon_t$   
 $\frac{dR}{R} = \epsilon_a (1 - 2\mu) + \frac{d\rho}{\rho} \quad \text{or} \quad GF = \frac{dR}{R} = \frac{1 + 2\mu}{\epsilon_a} + \frac{1}{\epsilon_a} \cdot \frac{d\rho}{\rho}$   
 Gauge factor  $GF = 1 + 2\mu$

So, from there what we go is we have to now find out something called as a gauge factor ok. So, we know R equal to rho into L by A ok. So, when there is a change in R. So, what we do? It let us differentiate dR equal to d into p L by A plus d L by A ok, then there is a change in resistance we basically start differentiating d the dR dR is nothing, but this we get.

So, now when we try to expand, what happens is dR is nothing but d P L by A plus rho by A into d L plus rho into L 1 minus A square A square into dA, when we divide the entire thing with R. So, divide equation 2 by R. So, what we get is dR by R equal to dP by P plus d L by L minus dA by A, this is what we get when there is a change in resistance. Now we have to sort out dA we have to find out what is dA? So, dA is nothing, but where A equal to pi into d by 2, the whole square pi ok. So, now, dA will be equal to when you differentiate, this dA can be represented as pi 2 times d by 2 into dB or it can be d A by a because this factor we need da by a which is equal to two times dB by d which is nothing, but 2 times epsilon because that is poisons ratio.

Also we have to find out what is  $dL/L$ ? Which is nothing, but this is axial strain and  $dR/R$ , is nothing, but  $dP/P + E_a - 2 \mu E_t/E$ , which is transverse right. So, now, what we can do is we know what is Poisson's ratio from there we try to because Poisson's ratio is a material property these are designed properties. So, we would like to convert this into Poisson's ratio. So, that is nothing, but  $dR/R$  equal to  $dA/A (1 - 2 \mu) + dP/P$  or this can be defined as gauge factor GF that is nothing, but the  $R/R$  by  $a$  which is nothing, but  $1 + 2 \mu$  Poisson's ratio plus  $1/a$  into  $dP/P$  and this term is almost close to 0. So, we can get gauge factor gauge factor GF is equal to  $1 + 2 \mu$  Poisson's ratio. This is a factor which is very important when we start reading strain gauges.

Thank you.