Smart Materials and Intelligent System Design
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Lecture – 10
ABD Matrices

Good morning, everybody welcome to Smart Materials and Intelligent System Design. In the composite materials 4 we had round on the classical laminated plate theory. We are now at the fifth and the final round of composite material.

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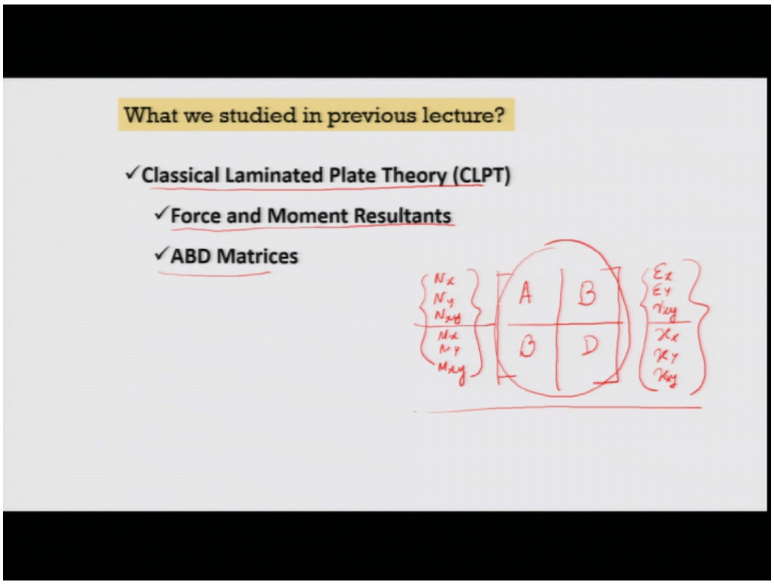

What we had studied in the last lecture if you remember, we had talked about the classical laminated plate theory based on (Refer Time: 00:36) you know assumptions. And, then we also have talked about how to get the force and moment resultants, what was we also referred as stress resultants right. So, there were normal forces, shear force as in the form of stress resultant then we also had if you remember that we had M x M y and M xy that is the two bending and one torsion ok.

So together these things were related in terms of ABD matrix; so, that we have to keep in our mind that the basic relationships always that we had N x N y N x y which is called the n part of it, and M x M y M x y this is the m part the bending part. So, and then we had this ABD matrices. So, I said that we always divide it into four parts right A part B part and D part. We also said: what are the names of each one of them right if you remember that this is related to the stretching stiffness, this is related to the stretching bending coupling this is the bending stiffness. So, here also the corresponding you know the deflections we will be putting, that is epsilon x y or gamma x y if we put it both the ways kappa x, kappa y and the twist that is kappa x y, this was the relationship that we had developed in the last class.

Now, what we are going to do today is that, we are going to see that based on the changes in A B B Ds remember that is the plus point of having composites and this changes if you understand it well you can actually develop very interesting smart composites also. So, we are going to see: what are the changes in A B B D the matrix we can make and how does it reflect in terms of the behavior of the system.

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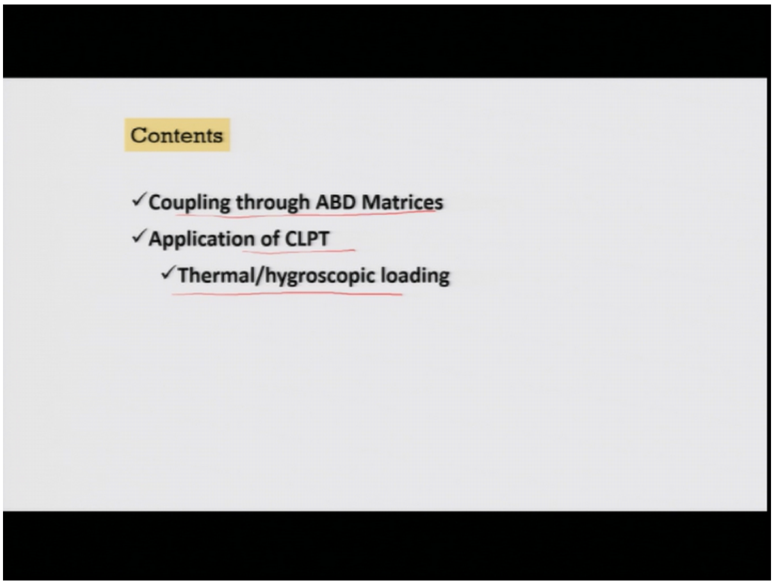

So, we are trying to go through the coupling of this A B D matrices its application in CLPT and also if there is a thermal or hygroscopic loading, how this system is going to behave this is what will be our intention in this particular lecture.

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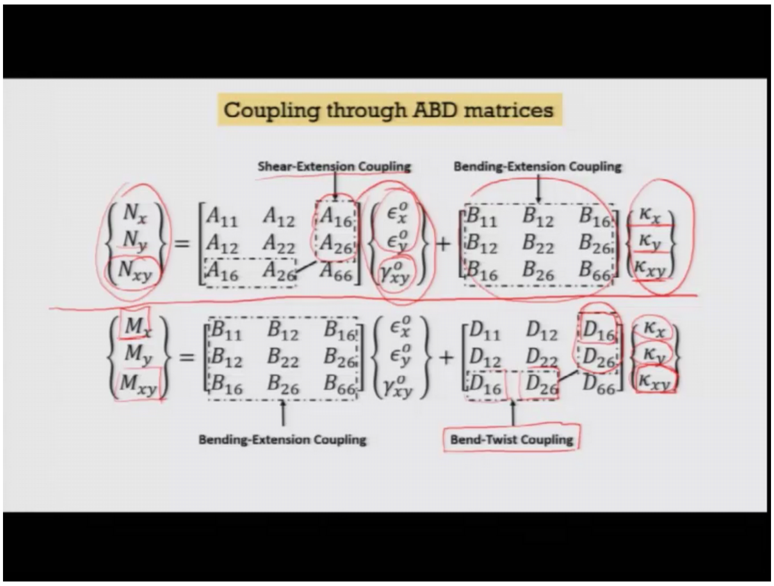

So, if you look at the ABBD matrices, this time we have really split it into two parts so; that means, first of all we made this splitting, that let us look at each one of this you know the normal shear stress part of it the force part to say, and the moment part separately. If you look at force part of it, once again we have subdivided into two part; one is the mid plane part and another is the curvature part. Now if you look at the mid plane part first this part the A 16 A 26 what does it tells us? It tells us that in this case if it is if the matrix suppose because fully populated, then if I apply the normal forces, we will be able to generate shear strain by using the normal force because of this coupling that is happening to the system and vice versa.

If I apply the shear force, we will be able to generate you know strains in the normal direction. So, that is why this A 16 A 26 are commonly referred as shear extension coupling. Now going by the same way here you are going to have the bending extension coupling why it is bending extension? Because in this part as you can see that if you have a bending here, this bending is going to affect the curvature in the x direction. Similarly M y in the y direction or M x y the twist in the kappa x y direction so, that is what is my bending extension coupling.

And interestingly from the forces also we have similar coupling; that means, the normal force can generate you know not only the mid plane strains which is fine, but can also generate curvature ok. N x can generate kappa x, N y can generate kappa y and kappa x

y. In fact, each one of them can generate each one of these things, because there is a fully populated one. And here in this part of the coupling, we also have the D16 D26 terms, which gives us the bending twisting coupling. So, what it means is that, if I give a bending moment here, I am going to get a twisting strain because of the presence of the D16 terms ok.

Similarly, M y is going to give me these twisting and the other side also, if I apply M x y that is the torsion I am going to get you know kappa x because of the torsion and kappa y because of the torsion. So, I am going to get this coupling from both the directions. So, this is: what is my bending twisting coupling. So, in a nutshell, what this is telling us is that if A B D matrix is fully populated, all the stress resultants and the moment resultants and their corresponding you know extensions and the curvatures they get fully coupled with each other. So, I can generate any one of these deformations in a plate by using any one of these terms that is a great advantage that we exploit when we make actually smart laminated composite.

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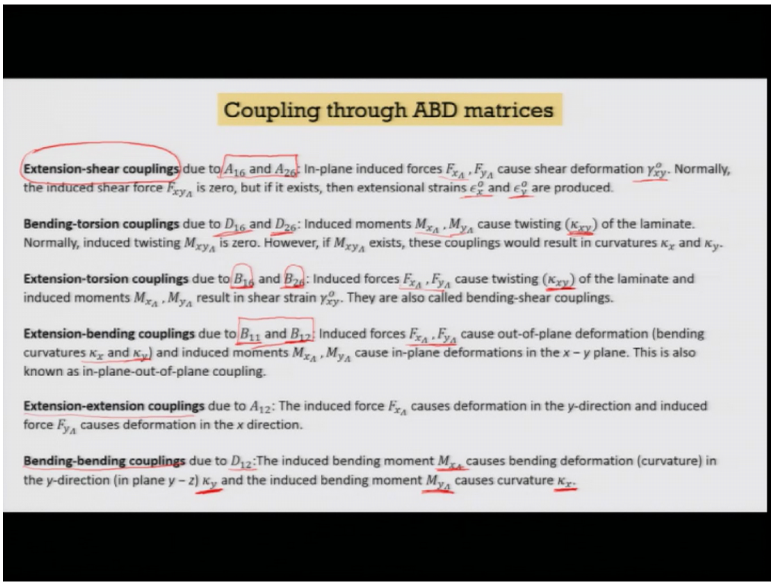

So, this couplings let us just check it once again the extension shear coupling is due to A 16 and A 26 term, which is you know actually developing the in plane induced forces. So, you can see that that is what is F x lambda, F y lambda which cause c r deformation gamma x y 0. Normally F x y lambda is 0, but if it exists then these things will be produced and this can be produced by actually having a smart material layer, you can have the extension shear coupling or by having you know suitable material property you can have this coupling.

Bending torsion coupling that is happening because of D 16 and D 26 terms, and this will be inducing moments in both the M x M y and that will create coup[ling]- you know twisting. So, bending will create torsion and twisting and similarly torsion will create bending deformation and then the extension torsion coupling. So, here it is happening because of the B matrix; B 16 and B 26. So, they will induce forces and that will in turn create you know twisting in the system. Similarly extension bending coupling due to B 11 and B 12 they will induce this forces and that forces which is you know extensional force, that is going to create bending you know kappa x and kappa y.

And what is the role of A 12? Due to A 12, you are going to get the extension coupling in the normal manner. And what is the role of D12? For example, you are going to get bending coupling for example, the induced bending moment M x lambda will cause the bending deformation curvature in the y direction kappa y, and similarly M y lambda will create a curvature kappa x ok. So, A 12 gives us the induced force will give the force from x will create deformation in y or force from y will create the deformation in x and similarly D 12 coupling will give moment from M creating deformation the curvature in the y direction and moment from y creating curvature in the x direction.

So, this is the entire you know coupling thing and each one of the term they gets you know which one is the term, which is responsible for what we have to keep in our mind.

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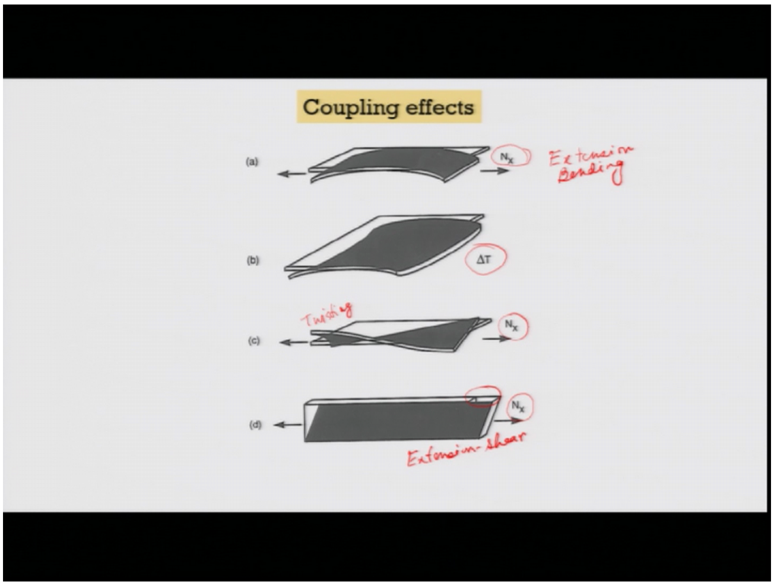

So, there is some of this coupling effect as you can see, try to identify that which one is creating what. For example, can you see that the N x here is the in plane stretching, but it is also creating moment here the bending deformation here. So, there is a stretching bending coupling that is happening. So, which term gives us the stretching bending coupling let us you know go through the terms here and you can see here that the extension bending coupling comes from B 11 and B 12.

So; that means, this particular you know system must be having you know non-zero B 11 B 12 terms. Here we are finding that there is a thermal effect and that is also producing the bending. There is also thermal effects are generally treated as the similar you know just like the in plane stress in the system. So, here also you have a bending stretching coupling. Here it is very interesting you have as you are applying the stretching, but you are getting something which is twisting actually. So, you can see that there is a twisting that is happening in the system. So, this is a twisting and stretching coupling. So, if I go back again. So, which is you know giving me the coupling between extension and torsion when due to B 16 and B 26 we can have it.

So, this could be one of the reasons why we are having it B 16 B 26 (Refer Time: 10:56). If we look at these last example, what is happening here is that we are giving these extensional you know stress here, but we are going you know we are finding that there is a shear that is happening here. So, why is this happening? It is happening because of the extension shear coupling that is present in the system and which is due to the term A 16 and A 26. So, if you have non-zero A 16 and A 26, you are going to get this extension and shear coupling in this system.

So, that is. So, thus you know we are having various systems in which. So, this is for example, extension bending coupling. So, you know you can go through such deformation cases; please practice it that if we if I give you a deformation case you have to find out figure out that what kind of coupling is present in such a system.

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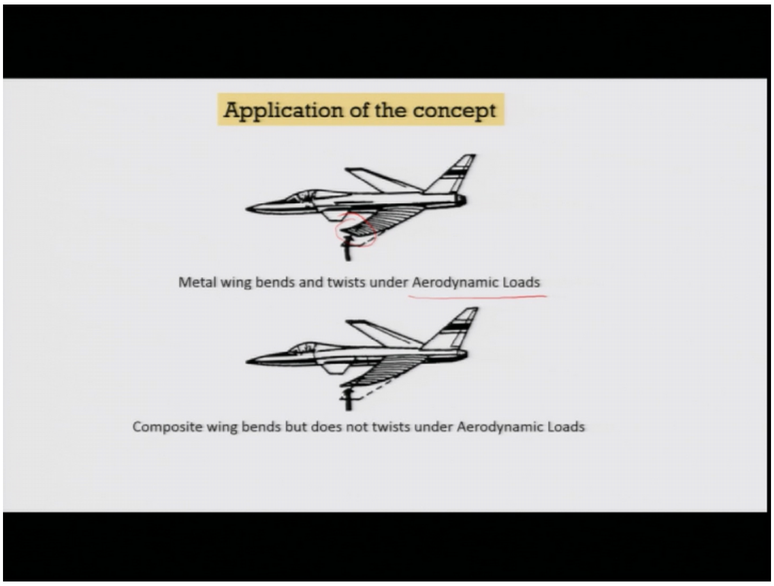

There is a beautiful application of this concept. A the conventional aircraft the metallic wings of this aircraft for example, as you can see the wing here for of course, a fighter aircraft and you can see that the wing actually bends and twists at this point due to the aerodynamic loads. Now how can we you know make a situation that it will bend, but it will not twist? Well if you have a bending twisting coupling that we have earlier told that you can have bending twisting coupling ok. So, you have for example, the bending torsion coupling due to D 16 D 26 terms. So, if you develop a composite wing, having non-zero D 16 D 26 terms you can actually generate a torsional counter force so that it will only bend, but it will not twist.

And that way you know the aircraft will have a better stability. So, these are some of the you know concepts that you can actually apply directly from the composite to modify the behavior of a system.

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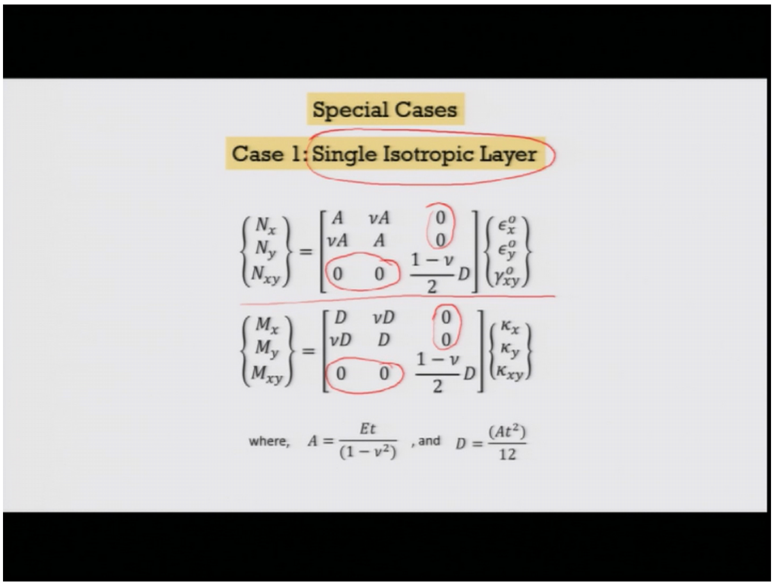

Now, there are some special case we will discuss, the first special case is that what if I have a single isotropic layer isotropic; which means the properties will be same in all the directions. Well in that case you will get you know first of all a decoupling, because as you know that the stretching will not affect the bending anymore and not only that the normal forces as you can see has no affect on the shear strain shear deformation and vice versa.

And similarly here you would see that the moments will not have any effect on the twisting ok. So, these terms are 0. So, for single isotropic layer all this couplings you will be missing, that is why we would not go for a single isotropic layer because we want to create most of the times all these things ok.

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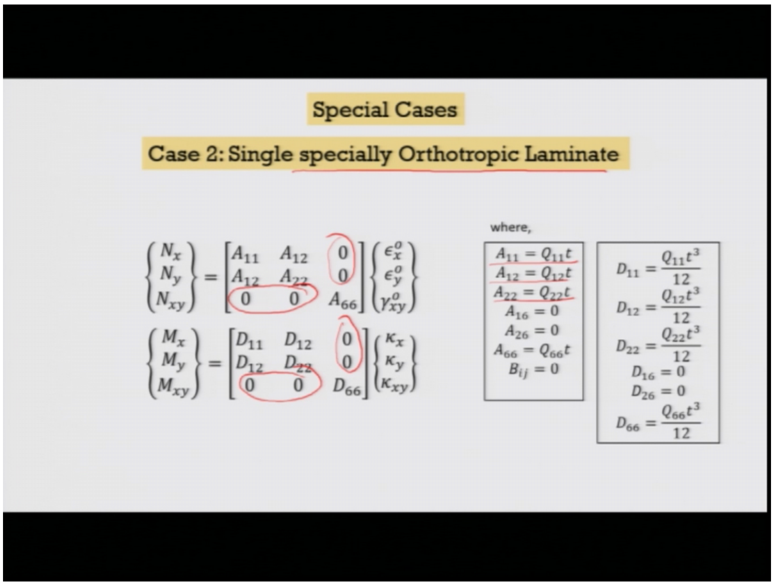

How about if I go for a single specially orthotropic laminate? In that case also as you can see that this parts will be 0 and also these particular parts will be 0 in this particular case. You can in fact, actually calculate it that, what will be A 11 A 12 etcetera. It is a good exercise because it also verifies that how you are developing the A B D matrix and quickly whether you know by giving such an conditions whether the A B D matrix is perfectly done or not. So, these are two very simple cases.

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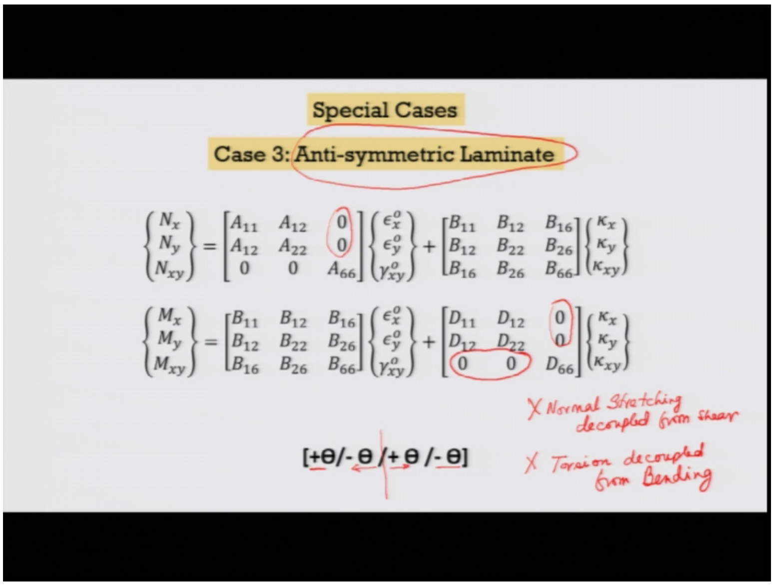

Now, from there if I go to slightly different case in which ply angles are such that, with respect to the meet plane. If the next one is minus here, it is plus here then the next one is plus here and minus here, this is what is called antisymmetric laminate. For the antisymmetric laminate if you carry out the same exercise, what you are going to see is that there is no connectivity between the in plane stretching and shear. So, this stretch shear connectivity which is absent there. But B matrix is present there so that means, you are going to have stretching bending coupling and once again this will be 0.

So, which means there is no connectivity between the bending and the twist or torsion. So, antisymmetric laminate we are having you know two things which is not possible. One is that normal stretching decoupled from shear. So, this is absent and what is absent? That is the torsion decoupled from bending. So, antisymmetric sequence will give us the special case, everything else will be present in the system.

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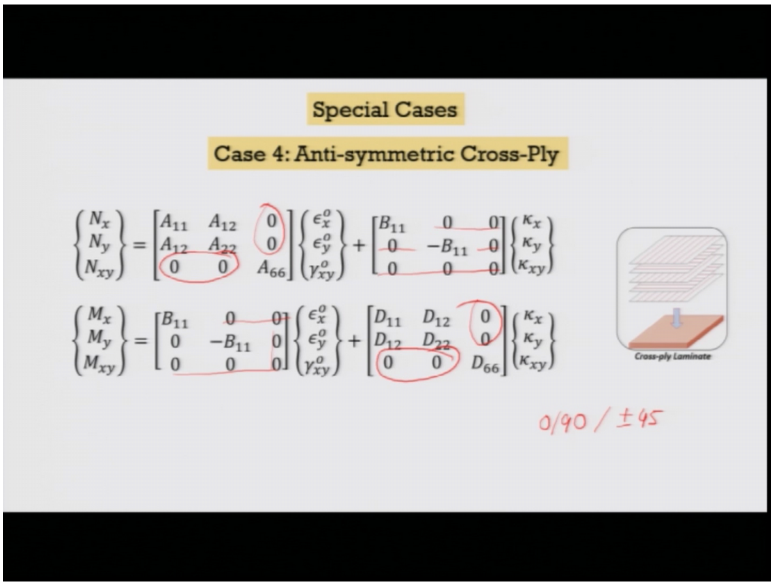

If we go to antisymmetric cross ply so; that means, you have these plus minus theta, but also you have these cross ply sequences, which is generally 0 90 and consists of these things of plus minus 45 etcetera. So, if you go to this kind of you know cross ply sequences, you will see that not only your these you know this part 0 just like it was in the earlier case, but also B matrix most of the terms will be 0 as you can see here ok. So that means, there are many B matrix terms which gives us the stretching bending coupling kind of things, there are many such terms which will be 0 in it ok. So, that is a special case of antisymmetric laminate itself.

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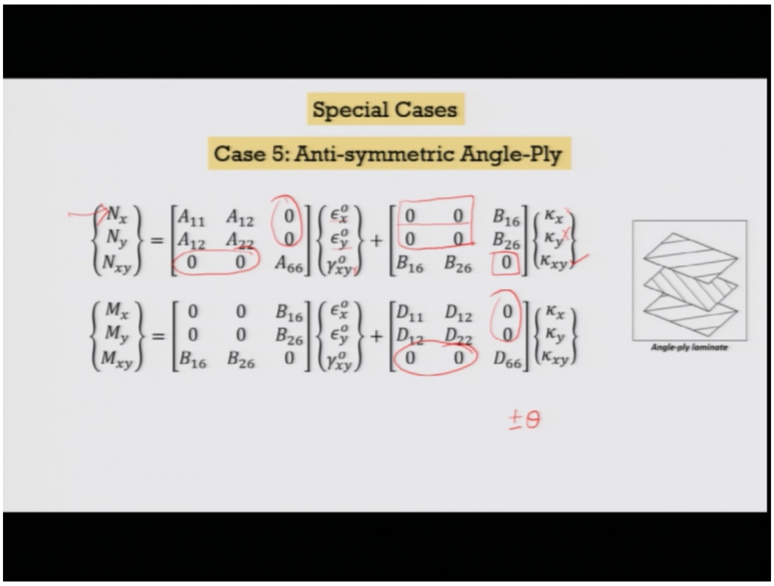

Next is if I go to antisymmetric angle plies. For antisymmetric angle plies which is once again of plus minus theta consists of that, you would see that this parts are 0 as usual ok. So, these couplings are not there the stretching and shear coupling and the bending and twisting coupling, but also a good part of B matrix are 0 actually ok.

So, for example, this part is 0. So, what it would mean is that, even if I you know apply say for example, the normal force along the x direction, what it is going to affect this that I am going to get affected in epsilon x 0 epsilon y 0, but not in the shear suppose corresponding to this condition, and also I am not going to generate kappa x, I am not going to generate kappa y, but only kappa x y because of this. So, this kind of things you know peculiar it is you can find in the special case of antisymmetry angle ply system.

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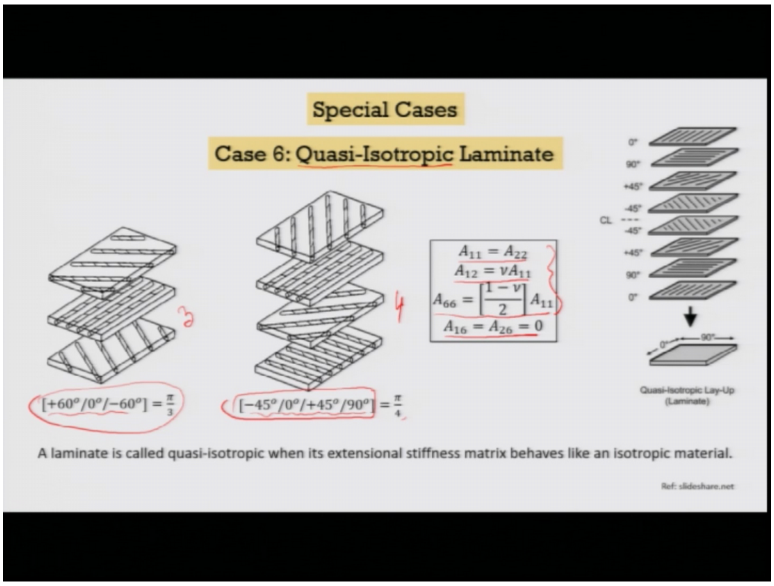

Now, we have another very special case system, which is known as quasi isotropic laminate ok. Why is it called quasi isotropic? It is because it behaves almost like a isotropic laminate in the planar direction, but not out of plane direction. That is why it is not fully isotropic, but pseudo or quasi isotropic laminate. Now you need very special angle conditions for it. In the ply angles are of these type plus 60 0 minus 60 that is of pi by 3 or of this type minus 45 0 plus 45 90 that is of pi by 4 ok. So, for 3 layers this is the sequence and these are 3 layer system.

For four layer system this should be the sequence ok. So, for some very specific such layer conditions, what you are going to see is that first of all A 11 equals to A 22 well that is what isotropy is telling us, A 12 is nu A 11 and A 66 is 1 minus nu by 2 over A 11. So, this is: what is the quasi isotropy that is coming into the system. Also A 16 and A 26 equals to 0. So, this you can develop only if you develop this type of a particular angle conditions in terms of you design of the ply, then only it will behave like this that in plane it will behave like an isotropic material.

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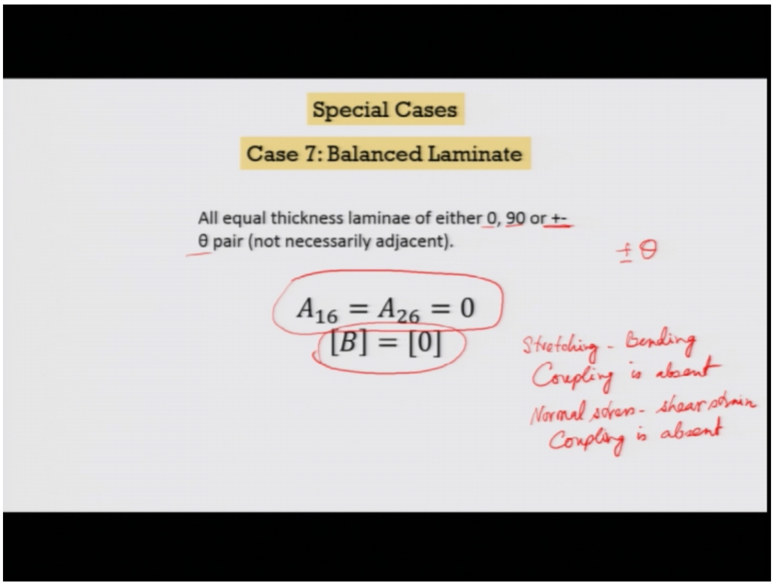

There is another very special case which is known as the balanced laminate. In the balanced laminate you know it should be all equal thickness lamini of either 0 90 or plus minus theta pair has to be there. So, in the ply angles you should be having either 0 degree plies or 90 degree plies or you should be having plus and minus theta.

So, in this case what you will see is that, B matrix is 0. So, B matrix 0 means stretching bending coupling stretching bending coupling is absent coupling is absent coupling is absent and A 16 and A 26 equals to 0. So, what it means is that the normal stress and the shear strain coupling that is also 0 coupling is absent. This is a very special case where, if the plies consist of all of equal angles of equal thickness and angles of either from 0 or from 90 or from plus minus theta, then we get this kind of a balanced condition of a laminate ok.

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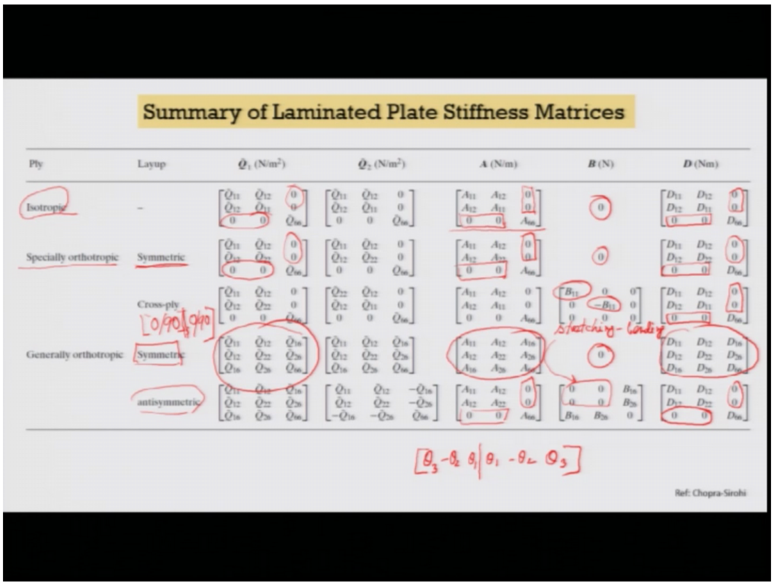

So, now we come to the summary of all these laminated plate stiffness matrices. So, if we consider first of all what I have said that the isotropic system, then what will be the Q matrix that would look like? The Q matrix will have all these 0 terms there as you can see the (Refer Time: 21:47) for isotropic system Q 1 bar and Q 2 bars and what the A matrix if I look at it? You can see that the a matrix has this coupling terms, which are 0 A 16 A 26 are 0 the B matrix is totally 0 and the D matrix also has this 0 terms in it ok. So, that is the isotropic case.

Now, we would not be you know much using an isotropic case, we will come to specially orthotropic case which is like along the layer you know we have the conditions of Q matrix of this type that this particular things are 0 ok. So, the elastic constants you know they are not affected along the normal and the shear directions. So, with that kind of conditions if I consider only a symmetric laminate what is a symmetric laminate you remember? That for a symmetric laminate you should have an angle conditions such that, with respect to the meet plane, suppose I am having 3 layers. So, this is plus theta 1 then this also should be plus theta 1, if this is minus 2 then this also should be minus theta 2 and if this is plus theta 3 then the last layer here also is plus theta 3.

So, this kind of conditions if you have, then you are first of all going to get this part as 0 just like the isotropic case. So, very much similar to the isotopic case, B is 0 and you are going to have these parts also as 0. So, in many sense in a specially orthotropic symmetric laminate it behaves very closely to an isotropic laminated system. But if I go to a cross ply laminated system; cross ply laminated systems I already told you could be something like 0 90 ok. So, symmetric or 0 90 you know in this case it will not be symmetric actually, it will be something like 0 90 0 90 type of ply system.

So, if it is a cross ply system like this, because symmetric any where it will be 0 (Refer Time: 24:03) B matrix. If it is a cross ply system like this, then you would see that the B matrix is not fully 0 there are some terms in the B matrix which are present and however, in the D matrix it will behave just like it was in that symmetric case. But because of that non-zero B matrix what you are going to have in this case is that, you are going to have the stretching and bending coupling.

So, this will give us the first time in considering all the other cases so far the stretching beding coupling we will get in this particular case. Now we come to generally orthotropic case. So, there is symmetric and antisymmetric of them. So, if I consider symmetric generally orthotropic case then you can see that the Q matrix is fully populated and you can see that A matrix is populated there are no zero terms there. D matrix is fully populated, but just because it is symmetric then this part will be 0. So, there is no stretching bending coupling here just like the other case.

If however, I go for generally orthotropic antisymmetric, then we are going to see that once again just stretching bending coupling is back here although some terms are 0, but it is back there. And because of antisymmetry once again we are losing this coupling between normal and shear you know strains coupling normal stress shear strain or shear stress normal strain coupling we will be losing that. Also the moment we lose that we will be also using bending and torsion couplings those terms will be 0. So, this keeps us a summary of the various laminated plate stiffness matrix.

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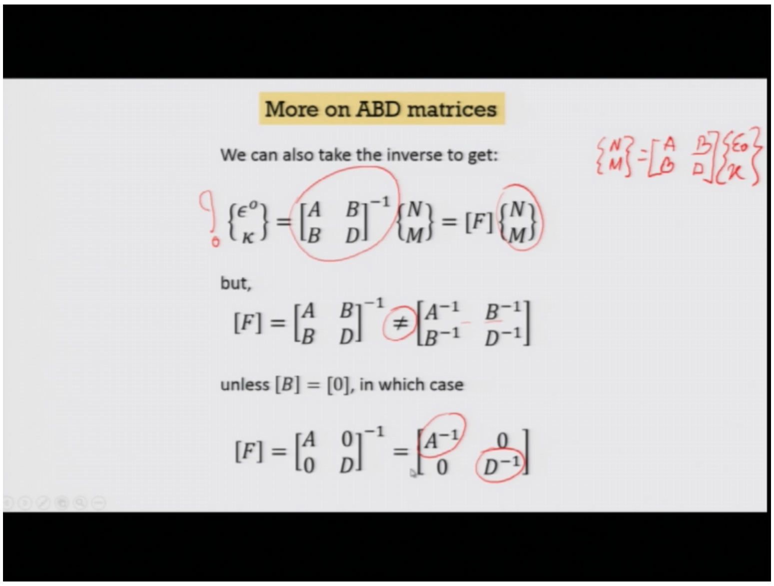

Now, we can also look at the A B D matrix from another point view that is you know earlier I told you that N M what was earlier relationship if you remember, was A B B D epsilon 0 kappa. Now what if I am interested to know that the forces are known to me, many cases it will be like that. That is why the stress resultants will be known to me and I am interested to find out what is the deformation or what is the curvature etcetera, then it will be an inverse of the ABD matrix and remember that A B D matrix there is no guarantee that it will be a symmetric one.

So, you know it is inverse can we actually you know it is not equals to inverse of each one of them it is not equals to that unless B equals to 0. So, so we need to actually find out that what it will be. If B is equals to 0 then of course, this becomes A inverse and D inverse.

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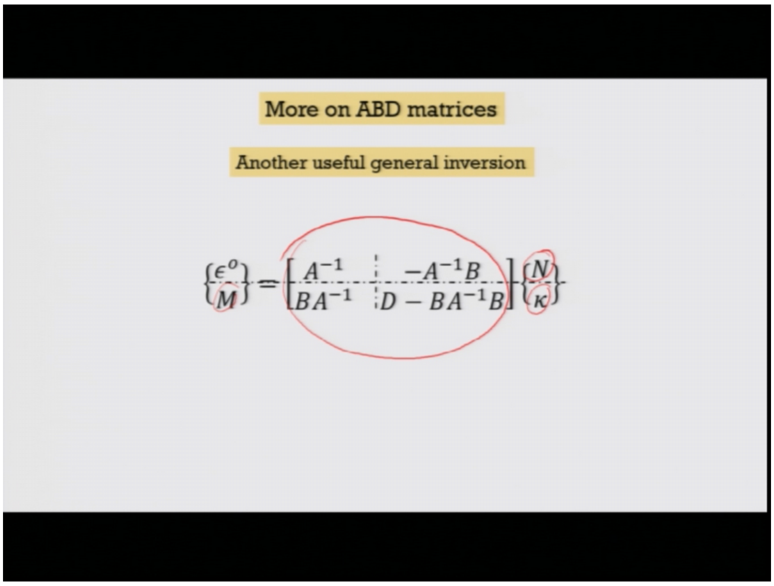

We need to find out otherwise the inverse and we need to find out the new A bar B bar C bar you know D bar matrix etcetera. Sometimes we will have a mixed condition; that means, we will be given you know the stresses in the that is applying normal stress shear stress etcetera and the curvature, and we have to find out what is the normal strain shear strain etcetera and the moment that is coming to the system. So, in that case the A matrix that we know will be modified to a new A B D matrix as has been shown here. These are actually generally useful inversions, which you can you know keep with you for ready requirements.

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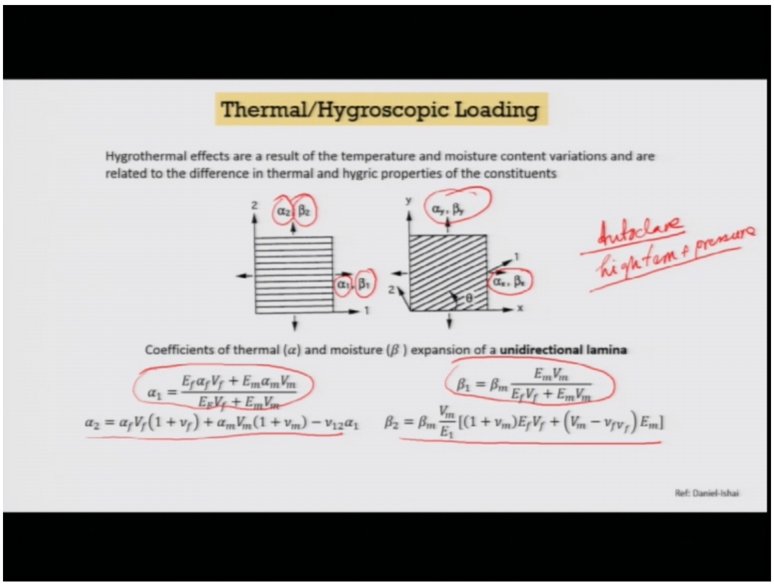

Now, one last case is that what if there is a thermal or a hygroscopic loading how can these happen? Well as you know that composite laminate ones we make it, we also place it in. So, what we call you know autoclave machine right that is a first source where this can happen because in the autoclave we apply high temperature and pressure for curing. So, during this process we are applying the temperature. So, there will be a thermal expansion in the system. Now, everywhere itself is anisotropic first of all and secondly, it you know the layer to layer also because of the change in the ply layer angle, there will be you know the this stress that will be developing may vary.

And similarly once you have this you know composite coming off, then if it is subjected to moisture, composites are more prone in terms of absorption of moisture ok. So, in that case the absorption will happen and once again it is directionally dependent and it is layer angle dependent. So, that is why we have one thermal coefficient along one, another along two direction and similarly a moisture coefficient absorption along one direction another along two direction so; that means, for x y also have alpha x beta x and alpha y beta y.

Now, alpha 1 beta ones etcetera they can be found out by this you know simple relationships, why did I know what is the fiber and the matrix volume fraction and I know the thermal expansion coefficient of each one of them, I can find out alpha 1. And similarly, I can find out beta 1 through experiments people can determine these things. Similarly, alpha 2 can be found out by this expression and beta 2 can be find out by this expression. Now, once I know suppose alpha 1 beta 1 alpha 2 beta 2’s then we go to how do we integrate it in our system.

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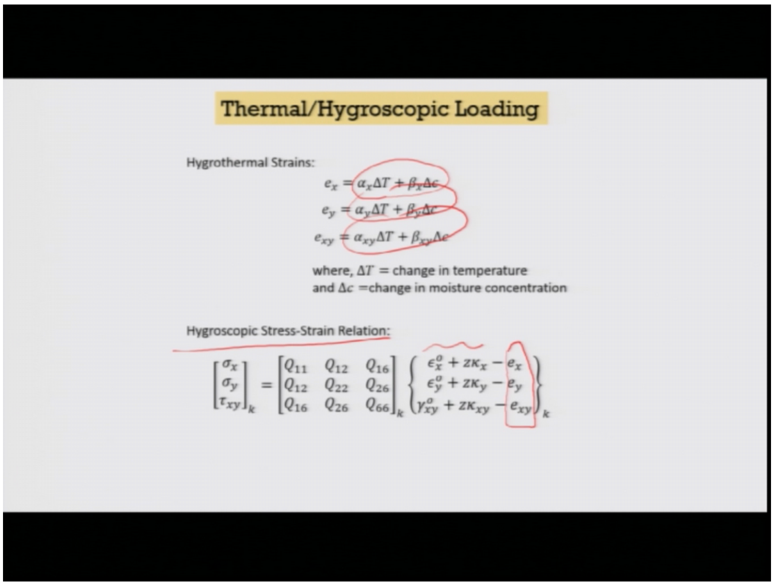

Now, we have this thermal and hygroscopic loading, in this case we have you know the effect it in terms of the strain we can very easily write it in terms once we know these coefficients. And then once I know the strains in e x e y and e x y we can very easily integrate that as additional terms. We have earlier talked about these two terms as additional terms in the strain and then the stress strain relationship is developed.

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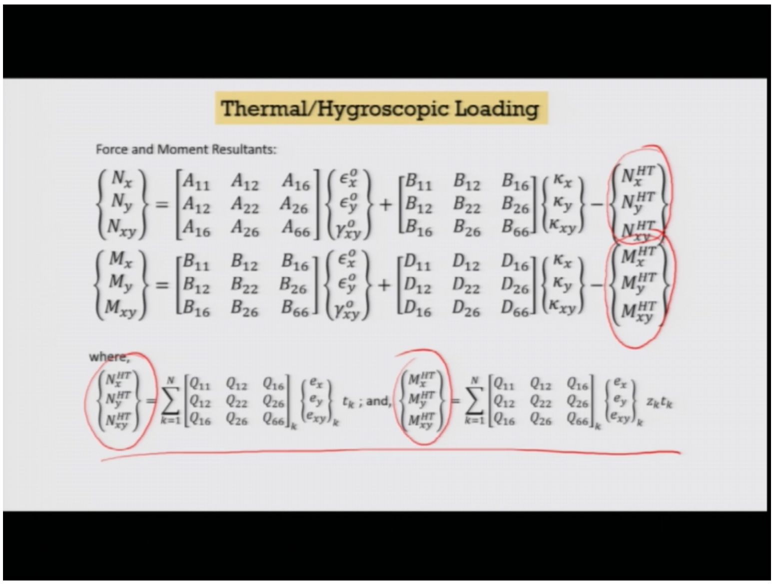

So, this can be done for thermal as well as for the hygroscopic. So, if I do that we can actually represent this additional terms in the form of additional loading. As you have seen that h or t each one of them is giving, additional loading in shear or additional loading in moment where the additional loading in shear or normal stress or additional loading in moment they are defined by this relationship. So, this is an simple extension which will be useful in shape memory composites and also in other you know simple composite systems, but for a smart composite like shape memory it will be also useful.

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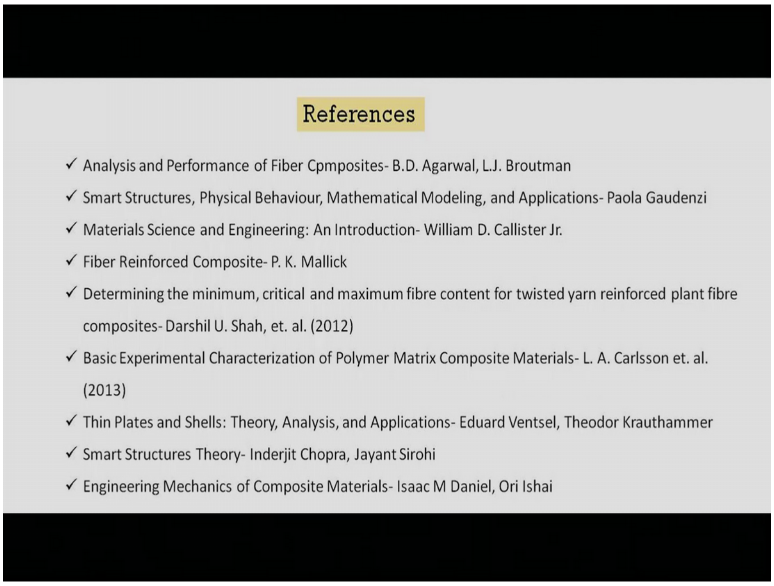

These are the references that you can use for your further you know study in this course. So, this is where I will put an end.

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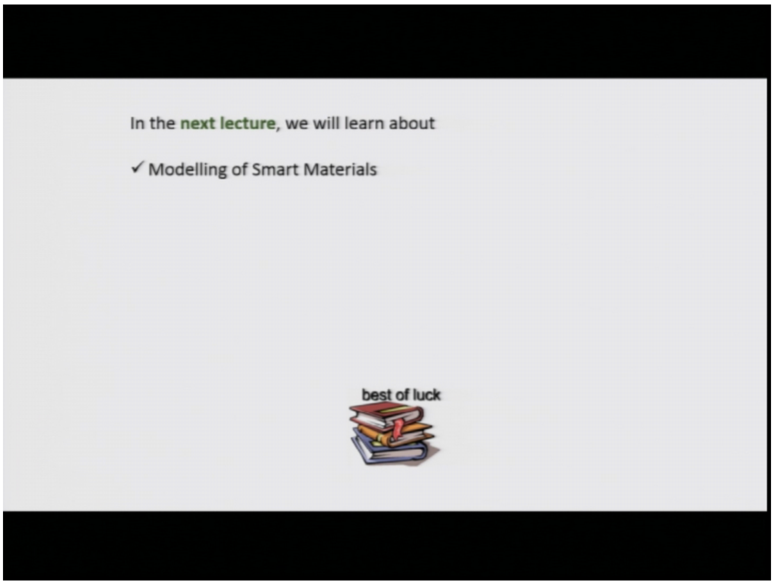

In the next lecture we learn about modeling of smart materials.

Thank you.