Design Practice - 2
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Lecture - 10
Representation of Surfaces

Hello and welcome to this module 10 of Design Practice 2. We were actually trying to plot a surface through various schematics you know either by rule surfaces or tabulated cylinder approach for regular solids or surface patches with an additional degree of freedom and explicit implicit equations therein which were involved to plot such surfaces okay for synthetic surfaces or complex surfaces.

So just as we sort of recall the implicit and explicit way of representing the surface with an additional degree of variable that is addition of a y variable okay so you have different combinations on x and y resulting in some particular value of z, a combination of all these 3 would then be descriptive of an array which would be the surface itself. I am going to first outlay the parametric form of representation.

Obviously because there are 2 variables in this case there are going to be a 2 parameter situation of each variable just as in the earlier case in the 2D curve case the variable x was represented through a parameter t varying between 0 and 1 so that you could go from a global to a local level okay. In this particular case because there are 2 different variables you will have to have 2 different parameters s and t.

Each of them varying between 2 limits so that with a combination of all these different parameters you should be able to again locally estimate a surface given the overall 2-dimensional nature of the surface which is the global nature of the surface which exist.

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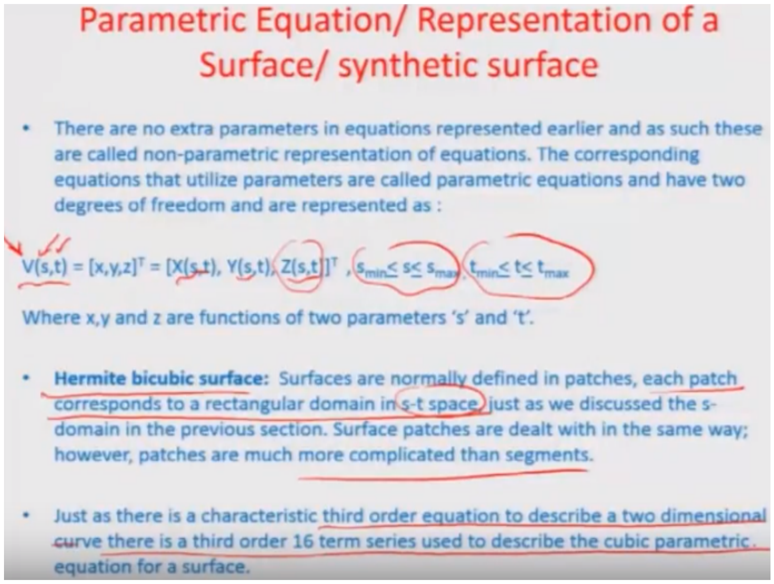

So if we looked at how this parametric form can be represented of a surface we are talking about you know an array of points V represented by 2 parameters s and t where s varies between some maximum minimum value and similarly t varies between again some maximum and minimum value and each of these x, y, and z all the 3 coordinates are made a function of the parameters s and t okay. So x is a function of s and t.

Similarly, y again is a function of s and t and so is z because z obviously depends on x and y. So if x and y are explicitly functions of s and t two variables obviously z is also going to be a function of s and t. So we will try to now plot a bicubic Hermitian surface which is normally defined in patches and each patch would correspond to a rectangular domain in the s-t space just as we discussed the s domain in the previous section and then there is a question of connectivity of this patches so that the whole surface is described by, you know patches getting connected to each other.

It is just little difference you know in terms of operability because in the 2D planar case we are talking about joining curves. In this case we are talking about joining the patches. So obviously the patches would extend both in the s direction as well as the t direction okay to describe the whole surface. So it is slightly more complicated than just joining segments and just as there is a characteristic third order equation to describe 2-dimensional curves there is a third order 16 term series described to describe the cubic parameter equation for a surface which I am going to outlay before you now.

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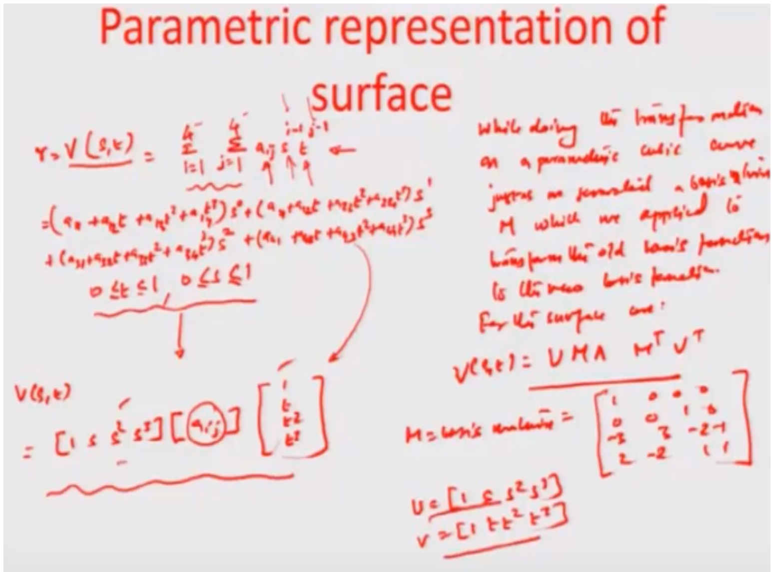

So we are talking about let us say an array of points are described by you know V which is a function of 2 parameters s and t represented in a manner like sigma i varying between 1 to 4. Again j varying between 1 to 4, a ij s to the power of i t to the power of j. So you know the ease of convenience is taken care here because we want to talk about parameters in terms of a for i equal to 1 j equal to 1; 1,2; 1,3 so on so forth on one hand.

And then we also want to use parameters starting from you know s to the power 0 or to the power 0 and you know all the way to the total number of n minus 1. So just as it was a 4 term series with t to the power 0, t to the power 1,2 and t to the power 3. It is a cubic polynomial you remember; in this case also although there are 4 different indices which are there.

We just wanted to make sure that we have a you know 2 different cubic polynomials estimating both the parameters which would eventually result in sort of a 16 term series okay describing the 2-dimensional curve. So we are just trying to represent V in that manner. If I expand this term we would have something which looks like what the Hermitian cubic fit was in case of a planar curve namely a 1 plus a 1 to t plus a 1, 3 t square plus a 1, 4 t cube times of s to the power of 0 okay plus again a 2, 1 plus a 2, 2 t plus a 2, 3 t square plus a 2, 4 t cube times of s to the power of

1 okay plus again a 3, 1 plus a 3, 2 t plus a 3, 3 t square plus a 3, 4 t cube times of s to the power of 2 or s square.

Plus again a 4, 1 a 4 2t we will write this little properly plus a 4, 3 t square plus a 4, 4 t cube times of s to the power of 3 where t varies between 0 and 1 and so does s varying between 0 and 1. So if we wanted to represent this in terms of a more simpler matrix multiplication equation you can conveniently represent it as V s, t equals a multiplication between the s matrix the a matrix and you know the transpose of the t matrix.

So I will just write it down as 1 t t square t cube something like this. So that is what the 16 term series could be represented as where you can find out the parameters independently coming out and you know a i j being the unknown variable which we need to find out through just as we did in the previous case considering both the end points as well as end slopes. In this particular case it is a 3-dimensional surface that we are talking about.

So obviously both the s and t's would vary between 0 and 1 and so there is going to be a slope with respect to the s and the slope with respect to the t at all the 4 corners because obviously as a planar curve would hinge to 2 points, the beginning and the end points a surface would hinge to 4 points which is like a you know sort of a rectangular or square surface corresponding to s 0 t 0, s 0 t 1, s 1 t 1 and then again s 1 t 0.

So you will have four different end conditions this way, 4 slopes okay and slopes at this 4 points along both the directions and you know there are cross slopes. So what I am going to do is to sort of try to write the just as we solved for the Hermitian planar curve example where we independently put the values of V0, V1, V dash 0 and V dash 1 to calculate the variables which were the A0, A1, A2, A3 values.

In this particular case also if we do that computation would be able to transform or produce the transform set of equations through various things which are going to be formulating so it basically consists of the s matrix. It contains the t matrix. It also contains sort of a slope matrix, slope and point matrix which is concerned with all the 4 corners across which this patch is hinged corresponding to the different values of s is equal to 0 t equal to 0 to you know s equal to 1, t equal to 1 all the combinations therein which existed all the four corners.

And then there should be some kind of a basis matrix which gives you a coefficient value or a set of coefficients which would then match the whole equation. This is exactly the same approach as we used for a 2-dimensional case to plot a curve that we are doing on a surface. So in this particular case while doing the transformation on a parametric cubic curve we, just as we generated a basis matrix m which we applied to transform the whole basis function to the new basis function, in the same manner for the surface this transformation may be written down as V s, t equals U M A M transpose V transpose where the M is basically what we know as the basis matrix.

Remember this was matrix full of coefficients okay. So this in particular case the M is given as a 4 by 4 matrix namely 1, 0, 0, 0; 0, 0, 1, 0; -3, 3, -2, -1 and 2, -2, 1, 1 okay. The s matrix and the t matrix are already mentioned in the last step here and we have A matrix further okay which is a 4 by 4 matrix of the vector as in the next page.

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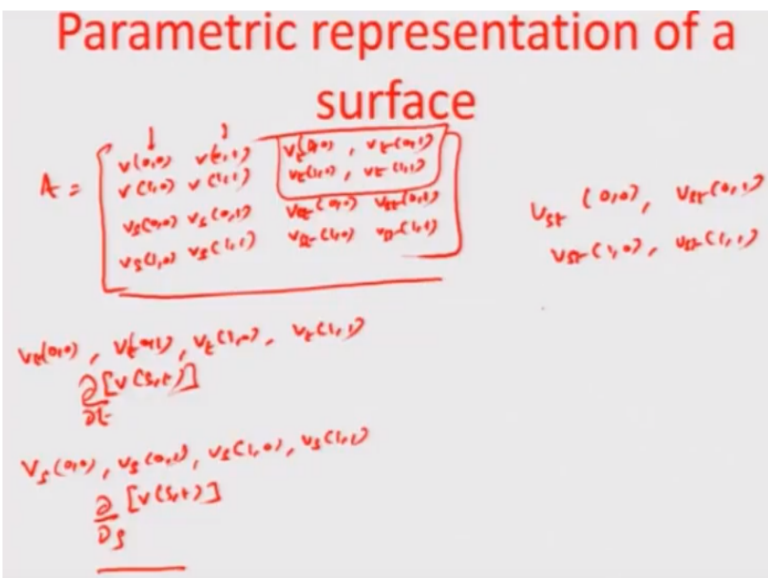

Let us plot that you know by 4 by 4 matrix. A in this case is defined as a set of you know points which is V 0, 0 corresponding to V 0, 1; V 1, 0 and these are all values of s you know at different points or different corners of the rectangle for which we define the patch. So you can call them as hinge points of the particular patch. Then because it is a 4 by 4 matrix and we have 16 terms so we should have a slope in the t direction okay.

So V t 0, 0 V t 0, 1 again V t 1, 0 V t 1, 1 okay. There should be the slopes across the s direction of or at all these hinge points V s (1, 0) V s (1, 1) okay and then there should be the cross slopes which means V s of t 0, 0 V s of t 0, 1 V s of t 1, 0 V s of t 1, 1 so on so forth so that is how the A matrix is defined okay. So when we are talking about these set of terms here that is V t (0, 0) V t (0, 1) V t (1, 0) and V t (1, 1).

We are talking about del by del t of the V function in s and t okay evaluated at the different values of s and t as given in this. when we are talking about the same you know V s (0, 0) V s (0, 1) V s (1, 0) and V s (1, 1) we refer to the slope with in the s direction or with respect to the s parameter and then we are referring to the V st (0, 0) V st (0, 1) V st (1, 0) and V st (1, 1) we are referring to the cross slopes that is del 2 by del t or del t del s whatever it is of V st evaluated again at the different values of the s's and the t's.

So at the 4 corners we are referring to the slope along the physically along the direction of the parameter t of the direction of the s parameter and then we are pulling out a function where we are talking about initially a derivative along the t then along the s direction or vice versa. It is all the same whether it is del 2 by del s del t or del 2 by del t del s is going to be the same as per the conventional you know partial derivatives, scheme of partial derivatives.

So this is how the data structure would then be created for a surface because now if we sending the various values of the s's and t's in terms of fractions, for example remember when we were plotting the Hermitian curve we talked about the various values of t starting between or starting from 0, one-fourth, half, three-fourth, 1 so on so forth. So we do the same for t and same for s and so there is at least 25 different combinations of these values.

Across which you could find out an array of points and if I plotted this array of points on a 3D space that is what we are recording the surface patch as and remember that if there are such patches in series with each other that would help us to describe the surface in totality. So let us actually look at a problem little more you know carefully so we can actually try to plot such a surface.

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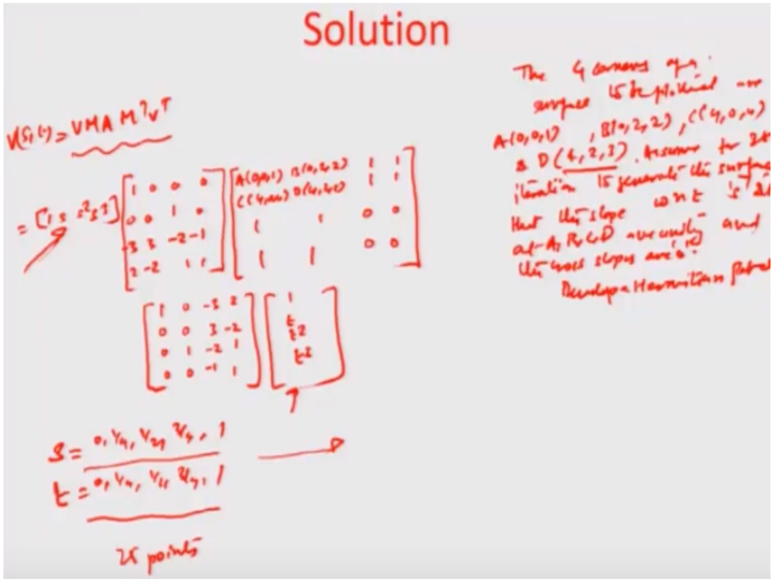

So let us say the problem is about the 4 corners of a surface which are to be plotted and these points are A(0, 0, 1), B(0, 2, 2), C(4, 0, 0) and D(4, 2, 3). So these are the definitions of the points V at the first place. Remember in the Hermitian curve case we had a first point and the last point of the curve so in this case there are 4 points of a surface. So assume for first iteration to generate the surface that the slope with respect to s and t at A, B, C, D are unity and the cross slopes are 0. So we are trying to simplify the computation through this.

So we will try to plot or develop a Hermitian patch. So we want to solve this equation and for this what I would suggest is that rather than manually solving because this problem becomes extensive some of you may have access to package like Matlab which you could use for solving. So this whole transformation equation can be written down as U M A M transpose V transpose.

Which in this particular case we are going to again represent as 1, s, s square s cube times of the basis matrix which is again a 4 by 4 matrix. In this particular case we are talking about 1, 0, 0, 0; 0, 0, 1, 0; -3, 3, -2, -1; 2, -2, 1, 1 times of the A matrix. In this particular case the A matrix has a set of 3 points let us say 3 coordinates x and y and z for the different points the hinge points of the particular surface so let us say A at 0, 0, 1 or B at 0, 2, 2 or C at 4, 0, 4 D at 4, 2, 3.

And further we know that the slopes in the s and t directions for all the 4 points are unity and you know the cross slopes are all 0 okay. So that is how you have formulated the A matrix times the M transpose which in this particular case would be 1, 0, 0, 0. Here let us say we call it 0, -3, 2 and similarly you have 3, -2, 1, 0 here and then finally again -2, -1, +1 and 1 times of 1, t, t square t cube.

So for the various values of s and t let us say supposing we have the values of s equal to 0, one-fourth, half, three-fourth and one and similarly t equal to 0, one-fourth, half, three-fourth and one. So we should have an array of something like the solution for this whole problem close to about 25 points which would then be plotted on a surface grade in the x and the y direction which are exactly 5 points in the you know s parameter side and 5 points in the t parameter side to describe a certain surface. So I am not going to actually you know show you how I did this Matlab oriented computations.

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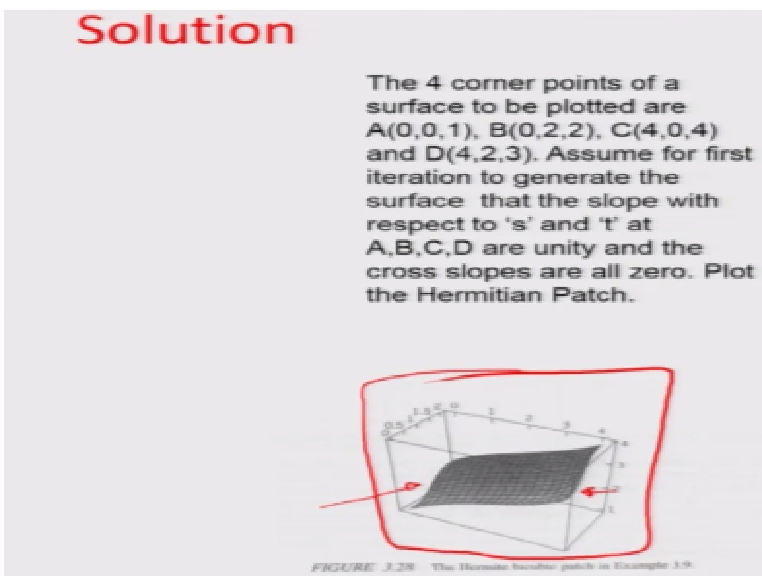

But if I wanted to look at how the plot of the surface would look like it typically looks something like this okay which we are considering. Here probably we took a little more than just you know 5 different points. You could take points in the extent of let us say 0.1 or 0.2 to resolve the surface very well and so you can see a 3-dimensional surface patch getting generated which has variation along 2 independent variables in the s and the t direction okay and a certain value of the z based on whatever are these s and the t values at a certain point.

So in a nutshell what you have observed here is that how extensively your surface can be plotted in terms of you know data which had just not limited to the bounds of the curves which are defining the corners of the edges of the particular surface but you are formulating the data structure for the whole surface for every crevice or every you know hills and valleys which are there on that particular surface.

Further if such data structure exists it is very convenient for us to study the intersection of such data structures and trying to then plot things you know geometrically when such intersections do happen. For example if you had a plane cutting a cone or a plane cutting a cylinder can I get a set of surface points if the plane is at varied angles. So you know this is a very interesting way of finding out the exact coordinate mathematically by storing this vast data structure okay through which you could do computations etc.

So I think that brings us to an end of this module on you know the geometric transformations and computed designing which almost all designers should understand from a perspective of you know what goes in to just you know one of course one aspect is about the how well you are or how well versed you are with the different shapes and forms but the other little more slightly more higher level aspect is about how you generate these in terms of coordinate maps.

So I think the module that has been covered or the few modules that have been covered in this direction make extensively this idea clear that how such data structures could be handled by a designer and this can be very useful for people who are going to design CAD softwares or even you know people who are going to be using curve fits for problems related to reverse engineering of designs, drawings, trying to then you know outlay their own version of the design based on what has been extracted.

So a lot of this can be done if you can go through all this process steps related to how data structures are created from different geometry. So we have not only covered the regular kind of geometries but also have delved quite a bit into the organic forms and also the organic shapes okay which can relate very well to the complex contours etc. associated with surfaces. So I would like to bring an end to this particular module.

Probably in the next module we are going to look into another aspect of this course which is about sensors and actuators MEMS which is micro-electro mechanical systems. So till then goodbye. Thank you very much.