Design Practice - 2
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Lecture - 04
Composition of Geometrical Transformation

Hello and welcome to this Design Practice 2 module 4. We were talking and discussing about 2-dimensional and 3-dimensional transformations and how computationally you can try to predict the mapping you know after any operation in space related to objects and when we are talking about multiple operations done to a single object obviously there has to be a combined form of equation to compute the final coordinates given the initial coordinate values.

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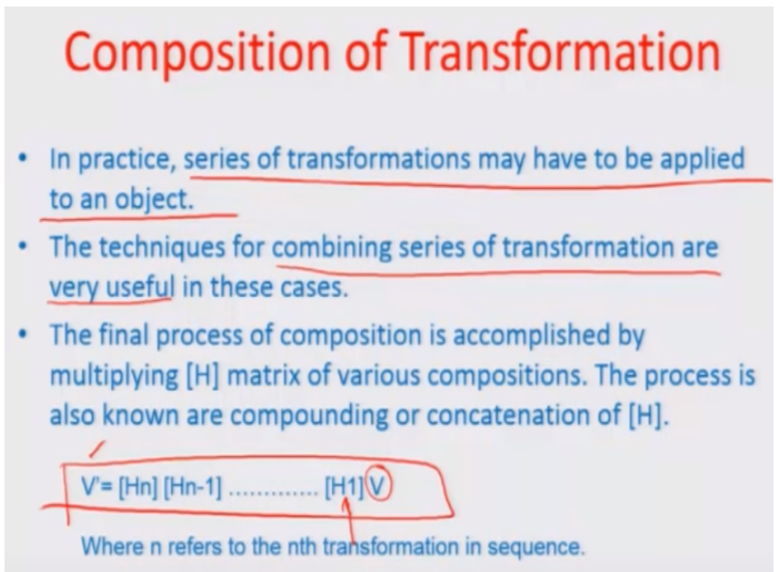

And so in practice a series of transformations may have to be applied in such cases where you need to modify the object through you know sequential geometric operations and the technique of combining series of transformations are sort of very useful because it makes computation very fast and something like this would emerge which highlights the different transformations that are to take place to a vector V in order to give the final value V dash.

And so for a sequence of coordinates which represent a certain geometry in space if these series of H matrices which are nothing but the homogenous form of the matrices executed in different operations like rotation, scaling, translation so on so forth are applied then quickly you can find out a set of final coordinates given all those transformations in between. This will be more clear when we talk about an example problem where we will actually apply this theory and show you how rotation can be carried out of a 3-dimensional object, a complex 3-dimensional object space. Let us look at this problem.

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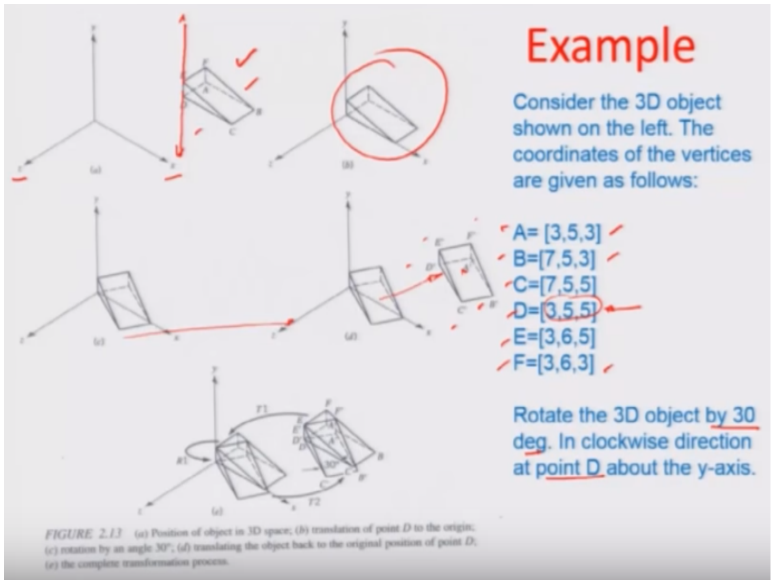

So we consider in this particular example problem an object here with respect to xy Cartesian axis and the various coordinates of the different points of this 3D objects from A to F are mentioned here; 3, 5, 3; 7, 5, 3 so on so forth up to the value of F being 3, 6, 3. So want to execute a rotation here and rotation has to be by 30 degrees and further in the clockwise direction on the y axis.

So the y coordinate does not change and if we supposing have the rotation around the point D about the y axis we are talking about the side which is actually parallel okay to the y axis so we want to rotate this in space by 30 degrees okay and we want to find out what is the final transformed coordinates between A to F which describes this object so that we can re-represent after rotation the whole object in space.

So this is a very simple operation of a dragon rotation kind of an operation which you normally do on a CAD platform but what I want to show you here through a series of these geometric transformations is how complex is the computation, the backend computation which keeps on happening and obviously because of the process of speed you can get immediately the re-representation probably within you know less than a microsecond probably would be taken for doing all this so that you can immediately almost view the object being reconstructed okay.

So in this particular operation because we have mentioned about rotation carried out along the you know along a certain plane this happens to be xz plane along which the rotation would happen. Also we must remember that the rotation cannot just happen at this point in space because the vectors that we talked about while generating the rotation equations were typically with respect to the origin.

So therefore, this set of points being several vectors have to be mapped with respect to the origin in order to see what is the rotation being carried out about the origin. So a best strategy here which will simplify the process is to translate the object from its existing location back to the origin. This is shown in figure B here that object comes to the origin and then you rotate the object around the origin by you know about the y axis about the D point.

Actually about the DE line because E and D are almost parallel to the, is parallel to the y axis. So rotate the object by 30 degrees in the clockwise direction, in the xy plane around the y axis after translating it to the origin. So D becomes the origin. So therefore we need to apply a translation equation to these set of coordinates where we exactly deduct the coordinates by -3, -5 and -5 so that D becomes 0, 0, 0.

And the remaining become in accordance you know the new coordinates and then after performing rotation and using the rotational matrix we again retranslate it back for example from D to from C to D you can see that object is now rotated the final coordinates are formulated and D you are retranslating it back by 3, 5, 5, this particular value so that the position of D does not change or probably in this case even E does not change because D and E are parallel to the y axis.

And the remaining others change because of such a translation and it gives you an idea of what is going to be the final coordinates A dash, B dash, C dash, D dash, E dash, F dash of the object. So this is quite complex. So you are going back to the origin, rotating at the origin, taking it back into space into the same point and these three set of transformation equations can be applied by again this concatenation theory where H1, H2, H3 three matrices are multiplied with the initial vector position which actually represents all the set of all points you know which are in this particular figure. So there are about 6 points which are there.

So we can try to have a matrix with all those 6 points and then try to have a homogenous transformation on that with different transformants. One of them is translation into origin, so that is the first transformant. The second one is rotation about the origin that is the second transformant. Further this rotation would be around the y axis, so it represent Hy and then again translating it back so that, that is the third transformant.

And so it is actually a series of multiplications being performed to the initial vector containing initial point. So we want to actually solve this problem now and try doing the, calculating physically the coordinates given the values A to E which we have already, A to F that we have already as initial values of the figure.

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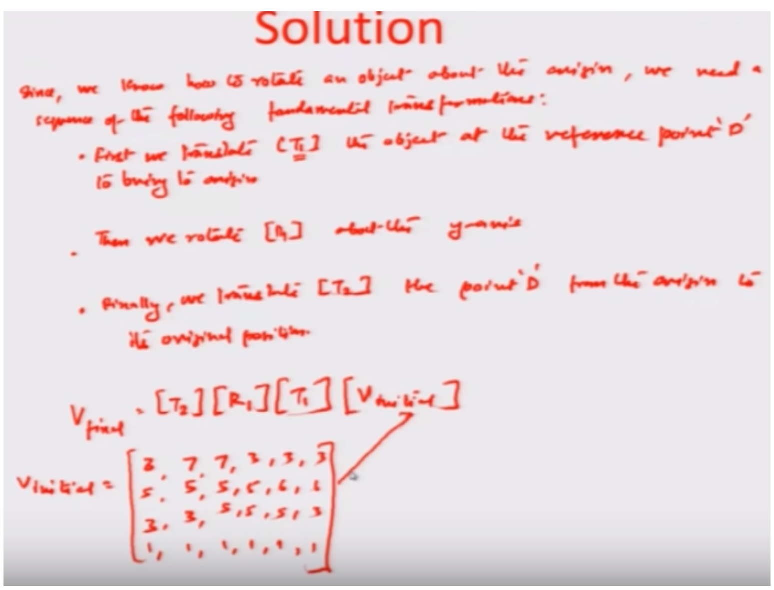

So since we know how to rotate an object about the origin we need a sequence of the following fundamental transformations. First we translate this calls operation the T1 operation the object at the reference point D to bring it to the origin. Then we wrote it, we call this R1 about the x axis oh sorry about the y axis and finally we translate, we call this operation T2, the point D from the origin to its original position.

So the whole transformation concatenated equation can now be written as the V final that is the final coordinates of V point V equals the multiplication of three different matrices T2, R1, T1 to the initial coordinate matrix of the geometry that is at hand. So we want to now gauge what are those different coordinate geometries which are in hand. So I can say that the V initial in this case can be represented as a matrix comprising of all the different points, the 6 points.

So it is a 6 by 4. So we call this 3 5 3 1, 7 5 3 1, 7 5 5 1. Let us keep separating these entities through commas so that there is no confusion and then we have 3 5 5 1. We have 3 6 5 1, and we have 3 6 3 1. So let us just shorten it a little bit just to change the layout little bit. So that is how the initial matrix can be written. This can be plugged back into this zone right here and let us look at the different other transformant matrix which could be used here.

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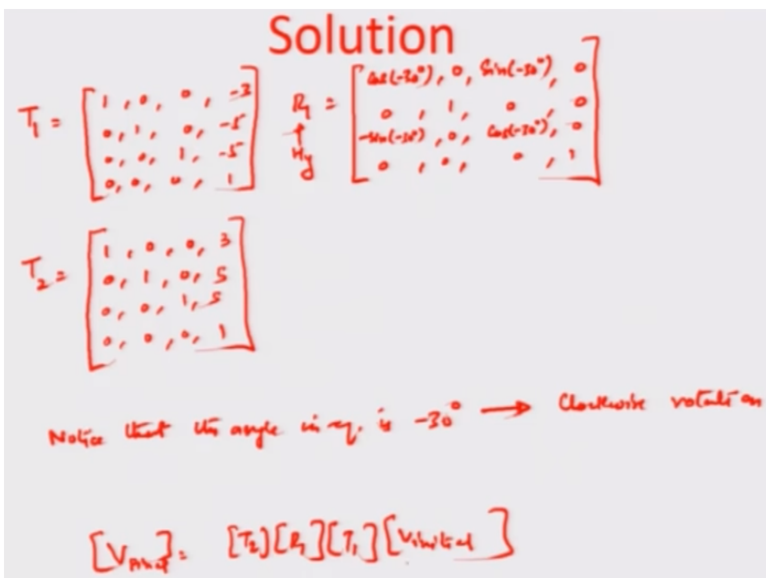

So I will write T1 and so as you know that T1 transformation is about translating the figure in a manner so that D becomes origin. So in this even we will have a transformant written as 1, 0, 0, -3; 0, 1, 0, -5; 0, 0, 1, -5; 0, 0, 0, 1. So this is the first transformant T1. Similarly, we will have the rotational matrix R1 and this can be because it is a clockwise rotation. So angles in the clockwise directions are treated generally as negative angles and of course the rotation is about y.

So we are talking about Hy here. So this is cos of -30 degrees. Please refer to your notes from earlier modules where I have already mentioned this for an angle theta. 0 sin of -30 degrees, 0 and similarly 0, 1, 0, 0 and minus of sin of -30 degrees, 0, cos of -30 degrees, 0, 0, 0; 0, 0, 0, 1. So that is how R1 is and then we have transformation matrix T2 represented as again 1, 0, 0 and now it is +3 because obviously you have to translate the D back from the origin to its initial position, so +5. 0, 0, 1, +5 again and 0, 0, 0, 1.

So these are the three different operations okay or concatenations that serially have to be applied to V initial so that you could obtain V final. So just to notice here that the angle in equation is -30 degrees indicating clockwise rotation. This is the normal sign convention that we are following from trigonometry. So that is how angles are symbolized. And so if I wanted to actually now apply the transformation equation V final equals T2, R1, T1 times of the initial.

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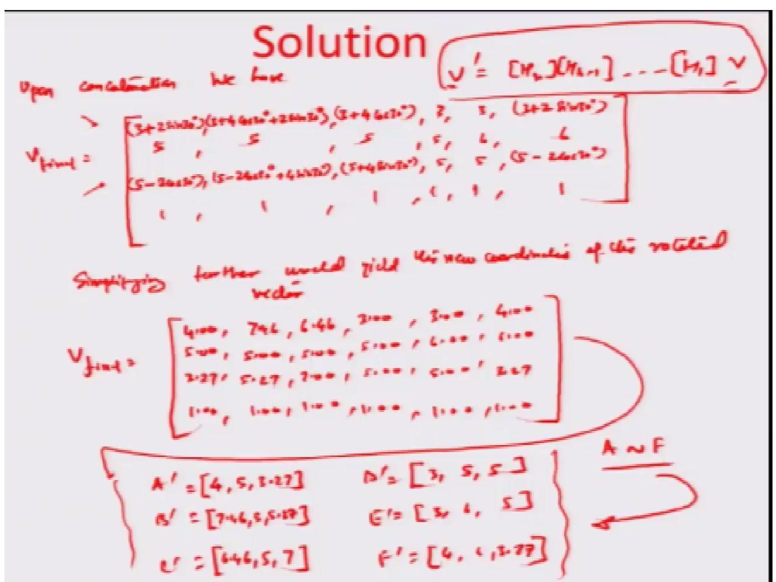

And I computationally calculate what I am going to get in this particular case is upon concatenation you know you remember that V dash was a series of such transformant matrices which would lead to the calculation of the final coordinates given the initial coordinates. So we will have in this case V final as so I am going to now write down the different terms here. So in one case we will have 3 + 2 sin of 30.

We will have 3 + 4 cos of 30 plus twice sin of 30. In a similar manner we will have 3 + 4 cosine 30, 3, 3, 3 + 2 sin 30. Similarly, the second y coordinate would be 5, 5, 5, 5, 6 and 6. You can see the y coordinate does not change really from the initial to the final because rotation is around y. So x has changed quite a bit and now we will also calculate. So this is the concatenated form. I am not for the simplicity sake, I am not doing the whole computation.

I understand that I expect that you will do it you will put all the rigor for the different multiplications steps between T1 and V final, add 2 and T1 V final and then again T2 and the whole product which is there. This is actually the whole product that I am representing here. So then we have 5 minus twice cos of 30 degrees. We have 5 minus twice cos of 30 degree plus 4 sin of 30 degrees. We have 5 + 4 sin of 30 degrees.

We have 5, 5, and 5 minus twice cos of 30 degrees and similarly, we have 1, 1, 1, 1 as a dummy variable, everything coming of. So if I wanted to compute this and simplify it further, so simplifying further would yield the new coordinates of the rotated form vector the V final becomes equal to 4.00, 5.00, 3.27. I am just trying to simplify through computations okay whatever has been represented in the earlier step.

So you have 7.46, 5.00, 5.27 again the dummy variable. You have 6.46, 5, 7 and the dummy variable. You have again 3, 5, 5, 1. Note that the position D does not really change and so does not E. So the E also is similar 3, 6, 5, the dummy variable 1 and then finally the point F changes to 4, 6, 3.27, 1.00. So if from this I extract what are my new positions A dash, B dash, C dash, E dash and F dash so I would be like able to you know write A dash as 4, 5, 3.27.

B dash as 7.46, 5, 5.27. C dash as 6.46, 5, 7. D dash as the same value 3, 5, 5. E dash to be the same value 3, 6, 5 and F dash to be 4, 6, 3.27. So that is how the coordinates change. These are the final forms of the coordinate given the values A to F as in the question earlier. So after this rotation this is how the new coordinates have to be mapped and based on that it will give you an idea of how the object is going to be like.

So I am going to close this module now and then in the next module we will do some more modifications or some studies related to solid modeling and then how you can actually get into plotting of 3D surfaces and from there we will go from just regular objects to irregular objects where we will use some theory and some concepts of fitting into different situations so that you have different geometries or a family of curves which can roll out.

So we will try to cover up in the next one or two modules more or less the regular geometries part and the solid modeling part. As of now I like to end this module. Thank you very much.