Design Practice - 2
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Lecture - 08
Curve Fitting Problem (Hermite Case)

Hello and welcome to this Design Practice 2 module 8. We will be solving the curve fit problem for the Hermitian case given the end points and the slopes and magnitude of the slope. So let us start with looking at a particular equation for this in the x and y direction.

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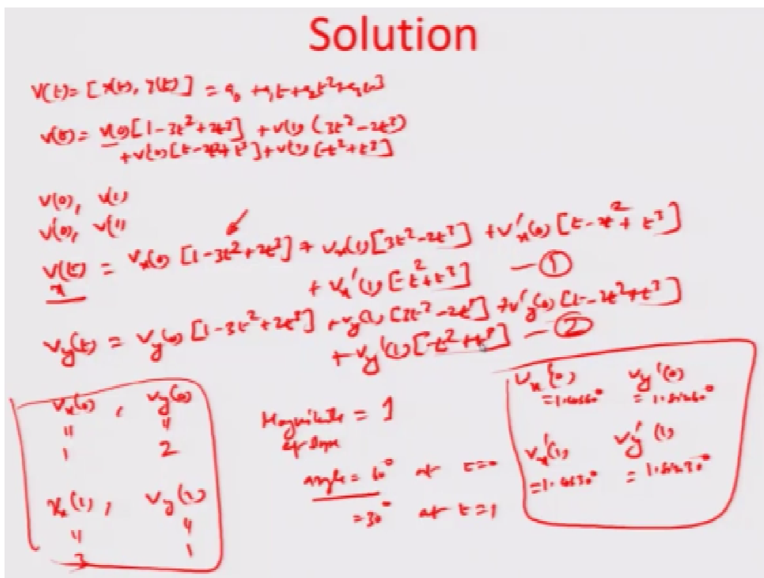

You know that V t is really defined by an x t and y t in case you are talking about the parametric representation and in this particular case, the V t is actually defined using again a term a 0 plus a 1 t plus a 2 t square plus a 3 t cube and further what we know of is that we can write this V t again in terms of the end conditions that is V of 0, V of 1 and similarly V dash 0 and V dash 1 in terms of V t equals V of 0 times of 1 minus 3t square plus twice t cube plus V of 1 times of 3t square minus twice t cube plus V dash 0 times of t minus twice t square plus t cube plus V dash 1 times of I am sorry this is t cube V dash 1 times of minus t square plus t cube.

And if we looked at Vt, Vt could be represented in terms of the x coordinate at V 0 that is I call this V x 0 times of 1 minus 3t square plus twice t cube plus again V x1 that means at the other extremity what is the x coordinate times of 3t square minus twice t cube plus again V dash x0 that means the x slope at the initial boundary that is corresponding to t equal to 0 boundary plus the again the x slope at 1 that means the other boundary that is the n boundary of the particular curve.

And this is actually the V x t that means this is the coordinate corresponding to different variations in t which will plot up with the V y coordinate corresponding to different variations of y. The all you need to change is to put the y coordinates here and the equation for the y can be plotted as the equation being written here. We already know that V x is 0 and the V y 0 are the initial coordinates which are given in this particular problem example is equal to 1 and 3.

And similarly I am sorry 1 and 2 and similarly the V x 1 and the V y 1 are given in this particular coordinate as 3 and 1 okay and similarly as the magnitude of the slope is unity and the angle is 60 degrees at t equal to 0 and 30 degrees at t equal to 1 corresponding to both the extremities and both the end points I should have the x slope and the y slope okay respectively at both the points that means V dash x 0 and V dash y 0.

And similarly V dash x 1 and V dash y 1 okay as the values 1 cos of 60 degrees and 1 cos of 30 degrees and similarly 1 sin of 60 degrees and 1 sin of 30 degrees.

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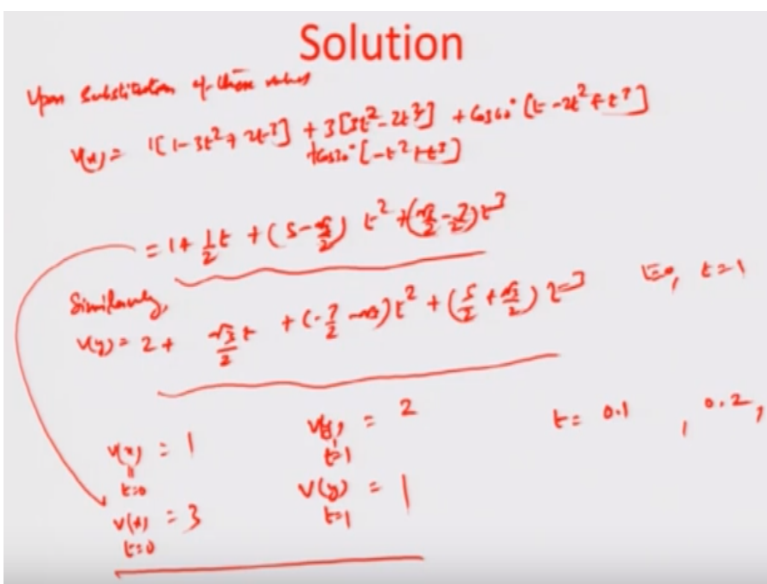

So having said all these values if I put it back into the equations 1 and 2 which I formulated right here the formulation that would eventually happen upon substitution of these values will be the coordinate V x in terms of t which is 1 times of 1 minus 3 t square plus twice t cube plus 3 times of 3t square minus twice t cube plus cosine 60 times of t minus twice t square plus t cube plus cos of 30 times of minus t square plus t cube and similarly so this comes out to be equal to the equation 1 plus half t plus 5 minus root 3 by 2 t square plus root 3 by 2 minus 7 by 2 t cube.

And similarly in case of Vy t we can write down as 2 plus root 3 by 2 t plus minus 7 by 2 minus root 3 t square plus 5 by 2 plus root 3 by 2 t cube. There is a way we can verify further by putting the values of V equal to 0 or sorry t equal to 0 and t equal to 1 to find out if the end conditions match exactly and here what we find out is that if t equal to 0 then Vx becomes equal to 1 okay and at t equal to 0 okay and Vx becomes equal to exactly at t equal to 1 it becomes equal to exactly 3 from substituting the value of t in this equation and finding out.

And similarly if I wanted to see the Vx and Vy at t equal to 1 in both the cases these values would turn out to be exactly I am sorry this is Vy 2 and 1 okay. So that is what cubes are test of whether the solution to describe the x point and the y point in terms of the parameter t provided the t varies between 0 and 1 is going to be appropriately making up for the boundaries where t equal to 0 and t equal to 1.

So this gives you a very good you know flexibility now on plotting the curve because now you can take any value of t for example t could be as small a value as 0.1 or 0.2 and then at every value you could do a local plotting of the curve so that you could have resolution of the curve okay and now the question that remains with us is that this parametric form can we do something so that we can generate a family of such curves one of which may be able to fit the contour of the topology that we are mapping very closely.

That is the whole purpose behind this particular exercise. And so in order to do that let us now try to see first of all whether the particular you know plot would change or curve would change or how it actually plots up when we talk about let us say different values of t.

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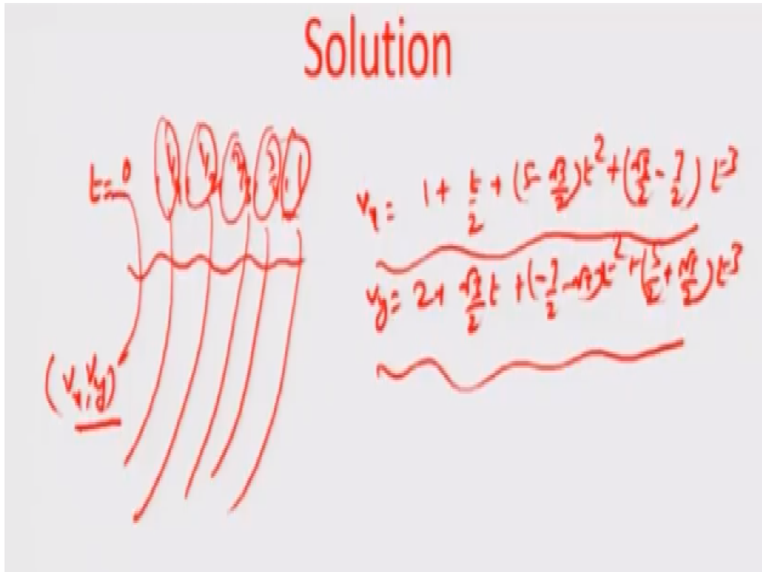

So in this particular instance for example if you were to take the values of t equal to at the magnitude of the slope 1 if you were to you know use values of t equal to 0 so corresponding to the value t equal to 0 let us say one-fourth, one-half, two-third, three-fourth, and one we could actually independently find out both the solutions how the Vx and Vy would behave. So as you know the Vx in this particular case is written down, again I am just going to repeat this equation.

T by 2 plus 5 minus root 3 by 2 square of t plus root 3 by 2 minus 7 by 2 t cube and similarly Vy is or written down as 2 plus root 3 by 2 t plus minus 7 by 2 minus root 3 square of t plus 5 by 2 plus root 3 by 2 cube of t okay. So in these two instances if I put all these values of different values of t we could find out a bunch of different coordinates okay of Vx Vy we should be able to satisfy the you know the plots or the plot by making an array of points.

Which should fall corresponding to this values of t on the curve, the whole curve between the two extremities corresponding to equal to 0 and equal to 1 which are the two points A and B.

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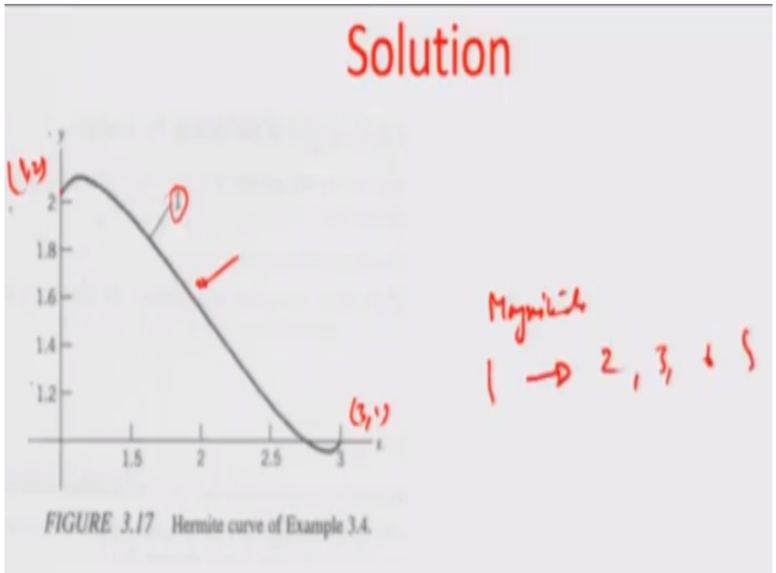

So in such a plot something like this would be generated where you can have the initial value here corresponding to the value let us say 1, 2 and starting from there to a value 3, 1 okay and the plot that is generated is actually represented here with a magnitude 1. What is also very interesting here is that if we wanted to vary the magnitude of the tangent vector from one to multiple values including 2, 3, 6, and 12 then generation would happen of a family of curves.

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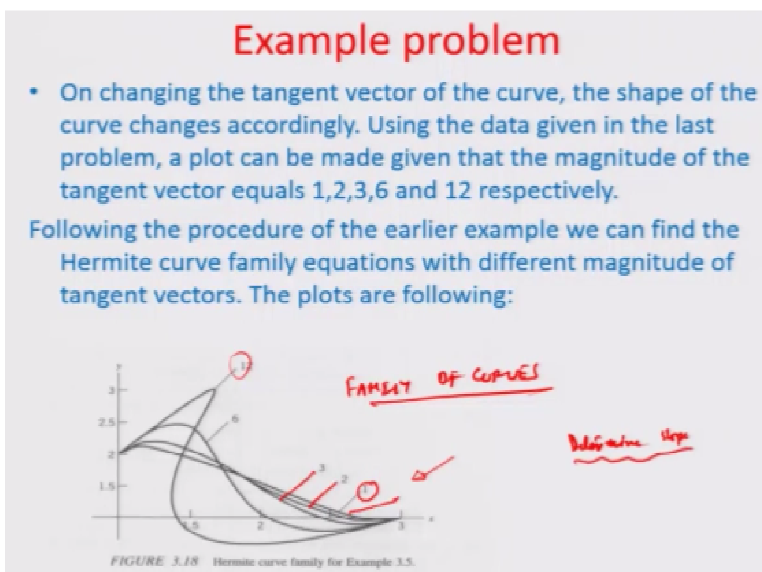

Something like this where this is corresponding to the initial value 1 which you saw earlier. This is the curve corresponding to the value of 2 then again 3 and 6 and 12. So you can see that you know there is a family of curves generated for one to be flexible enough now to actually compare this with the surface a complex topology that you are mapping and generate exactly the segment you know the shape of the segment which you are trying to map.

And so if supposing such curves are connected to each other in terms of C0 continuity or C1 continuity I should be able to generate the whole surface typically as pieces you know and these pieces can be contributed by each one segment of the curve that you saw here corresponding to the magnitude where it fits the particular part of the surface.

So this is the beauty about this process that you can divide a surface into small curves where each of these curves are being indicated and complex organic forms which need to be now mapped into a computer in terms of some data and some data you know management can be very easily stored okay in on the basis of such synthetic curves. So this is one very key example of how you can do a contour mapping through curve fitting exercise using the simplest Hermitian cubic fit polynomial.

But however, the problem in this particular application as you can see is about is almost a single fold problem that how you determine the slopes. And for a designer it is very very hard to determine slope at a certain point because there are not many measurement systems which would say that instantaneously that particular point what is going to be the exact nature of the slope.

And so an average slope has to be concerned and then you know there is a question of again how much will that induce errors in terms of the plot that you are trying to make and so the curve family that eventually would get generated may not be able to fit properly to the contour that we are trying to fit in and there may other problems associated with it.

So we need to method where we get rid of the slope and are only concerned with maybe one or two or maybe a bunch of different points through which we can actually generate okay a curve and for that there is a very nice you know way of estimating through rather than points and slopes only points, a particular curve behaviour and such curves are also known as the Bezier curves.

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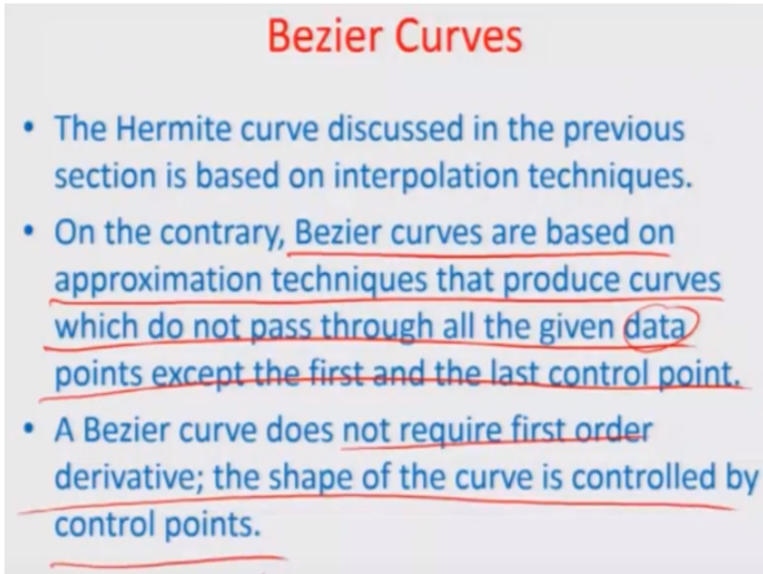

So Bezier curves are based on approximation techniques that produce curves which do not pass through all the given data points except the first and the last control point but the implication of the data is felt upon if the data is varied for example if it goes up or down the overall nature of the curve, the geometry of the curve may change accordingly okay based on which the Bezier is plotted okay with the set of points in mind.

So Bezier curve does not require first order derivative. The shape of the curve is controlled by control points.

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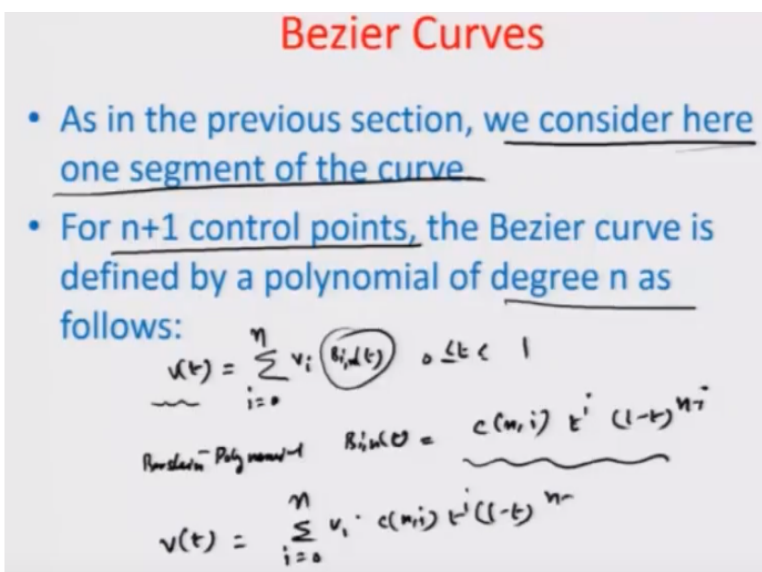

So we consider here also one segment of the curve just as we did for the Hermitian case and here we will like to plot the curve keeping into mind n + 1 control points, including both the end points where the Bezier curve should necessarily pass from and the remaining n - 1 points it may not be needed for the Bezier curve to pass through all the points. However, the nature of the curve is such that the points moving across or the points going to higher place or lower place will influence definitely the overall shape of the curve even if the curve does not pass.

So it has, so the points in the neighborhood which are considered for the Bezier fit would have an influence on the on the Bezier curve even if they are not intersecting with the curve itself. So it is defined by a polynomial of degree n and the polynomial and the Bezier polynomial is expressed by the Bezier function is expressed by a polynomial which is otherwise known as the Bernstein polynomial.

The array of points Vt in this particular case would be represented by you know discrete points for indices varying between 0 and n. So exactly there are about n + 1 points and this would be represented as V i B int where t varies between 0 and 1 is the parameter and this Bin is the Bernstein polynomial okay. So the Bernstein polynomial is defined as commination N i t to the power of i 1 minus t to the power of n – i.

So in fact the whole Bezier equation can be written as V i times of c and i t to the power i 1 minus t to the power of n - i and there are exactly n + 1 points, control points through which such a curve would move.

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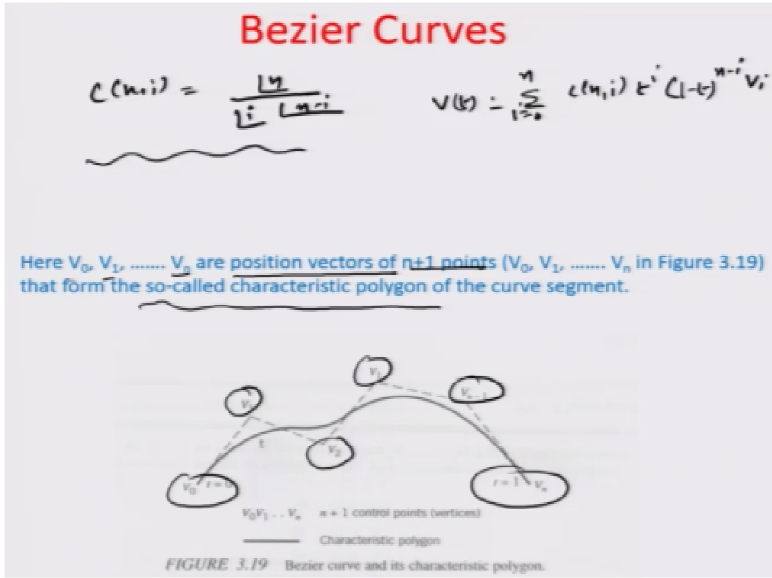

So we all are aware that the combination of ni is further defined as factorial n by factorial i times of n – i factorial. So that is how we will define the Bernstein polynomial okay and the Bezier equation Vt equals sigma 0 to n c and i e to the power i 1 – t to the power of n – i V i. Here the points V0, V1, V2 so on up to Vn are the position vectors of the respective n + 1 points.

I think I had mentioned earlier that the curves necessarily pass through t equal to 0 and t equal to 1 but whatever points are in between the first and the last point it may not necessarily pass through these points and so if you look at the lines which are joining these points they so called represent the characteristic polygon through which a curve segment has to necessarily move for the curve to be qualifying a Bezier fit.

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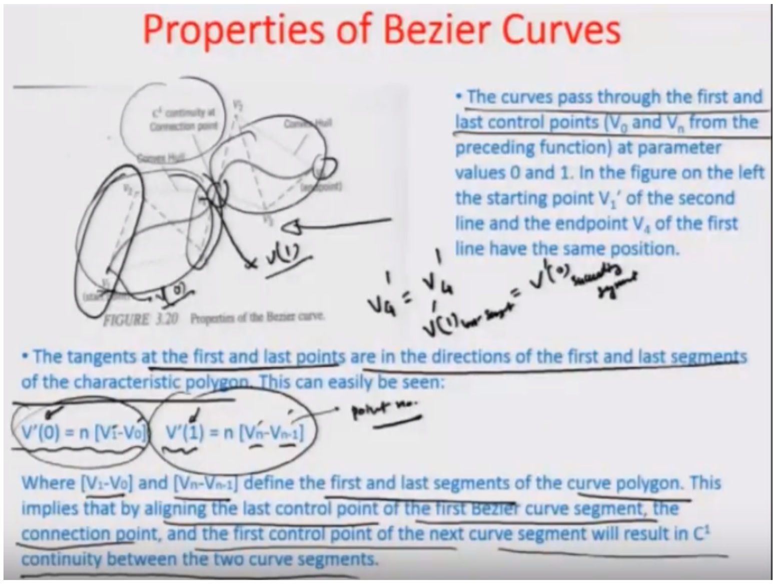

So for example in this particular figure represented on the left here, it shows a C1 continuity with radii of curvatures on both sides and a common tangent. One of the beginning of the next segment, this is the next segment, this is the earlier segment and one at the end of the last segment or the preceding segment the curves pass through the first and the last control points V0 and Vn.

In this particular case, V0 should be point which you know as V1 here corresponding to t = 0 okay and the point here V4 which is somewhere here I am sorry in the first segment which is actually corresponding to t = 1 okay and so V0 and V1 are the parametric forms of representation of these point vectors you know representing V1 and V4 in space for the first segment and V1 dash and V4 dash are therefore the second line segment or second curve segment and naturally the slopes at V4 and V4 dash of this particular geometry.

In other words what I will say is that the V1 of the last, the slope at the V1 of the last segment is equal to the slope at the V0 of the succeeding segment. So the tangents at the first and the last points are in the directions of the first and last segments of the characteristic polygon. You can see here for example the tangent at V1 is in the direction of V2 – V1 okay and similarly the tangent here is in the direction of V4 – V3 okay is the last segment.

And we can represent this better in terms of V dash 0 is n(V 1 – V 0) and V dash 1 is n(V n – V n - 1) where this 1 and 0 and this n and n – 1 describe the point number okay, not the t values. The t values are in brackets here that is being considered. So from the equation that we had described earlier we need to prove this point whether we will have an equation which will satisfy these two conditions for a Bezier fit to necessarily exist where the V 1 – V 0 and V n – V n-1 define the first and the last segments of the curve polygon through which the tangent should necessarily pass for a curve to be called a Bezier fit or a Bezier curve.

This implies that by aligning the last control point and the first Bezier curve segment the connection point and the first control point of the next curve segment, there would be a resulting C1 continuity between the two polygon segments 1 and 2 which are being connected as you can see here in the figure.

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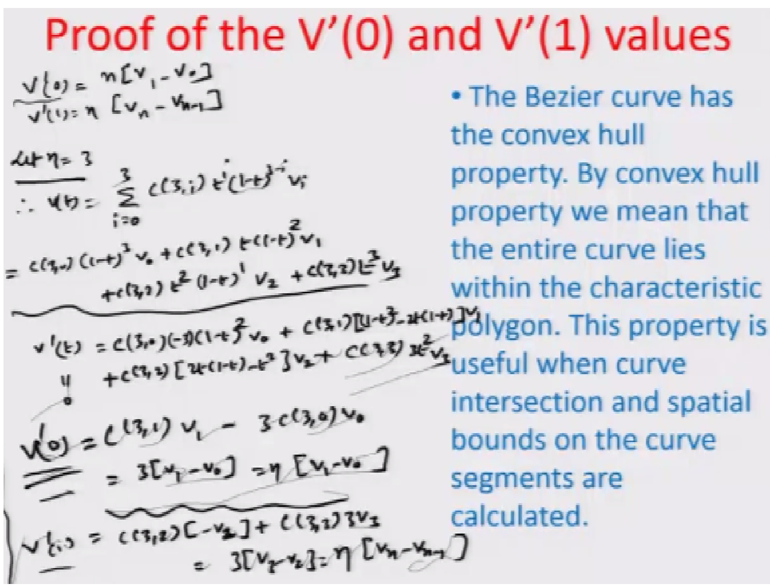

So let us look at how we obtain this expression. So let us say when we talk about the first condition necessary for a fit to be Bezier it says that V dash 0 should be equal to n times of point V 1 – V 0 okay and the other conditions says, condition number two says V dash 1 equals n times of V n – V n-1. So if I supposing if I suppose the number of points to be 4 and the index n to be 3 so the Bezier function V t in that even can be written down as i varying between 0 and 3 C 3i t to the power of i I am sorry, 1 - t to the power of 3 - 1 V i.

Which essentially means that if you substitute now the values of i's, the various values of i's we can represent this in terms of C 3 0, 1 - t to the power of 3 times of V 0 + C 3, 1 times of t 1 - t to the power of 2 V 1 + C 3, 2 times of t square 1 - t to the power of 1 V 2 + C 3, 3 t to the power of 3 V 3 so on so forth. And if I wanted to now solve this corresponding to t equal to 0 just to find out what is going to happen to this V dash 0, so the V dash 0 here could be represented as first of all let us calculate what is V dash t.

So this is actually now if I just differentiate C 3, 0 which is 3 actually times of minus 3 times of 1 - t square V 0 + C 3, 1 times of 1 - t whole square plus or minus twice t 1 - t times of V 1 + C 3, 2 times of twice t 1 - t minus let us say t square V 2 + C 3, 3 times of 3 t square V 3 and so on so forth. So if you put a value of t equal to 0 here for example I would be left with V dash 0 equal to C 3, 1 V 1 – 3 C (3, 0) V 0 and that will further be boiling down to 3 times of V 1 - V0 which is actually equal to n times of V 1 – V 0. So that is how you can actually look at V dash 0.

Similarly, if I were to look at what is V dash 1 in this particular case it can be represented as C(3, 2) times of minus V 2 + C (3, 3) times of 3 V 3 which actually comes out to be 3 V 3 – V 2 nothing but n times of V n – V n-1. So we can see that both these conditions V dash 0 and V dash 1 are satisfying for something to be called a Bezier curve. So we will actually like to end the particular module here.

But in the next module I am going to let you fit in a realistic situation a Bezier fit and try to estimate what are the in between points based on different values of t's so that we can locally plot a very complex surface and then generate a family of those surfaces or family of those curves based on the initial Bezier fit. So as of now thank you very much and see you in the next module.