

Turbulent Combustion: Theory and Modelling
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology – Kanpur

Lecture - 21
Laminar Non-Premixed Flames (cont...)

Welcome back. So, let us continue the discussion on the laminar diffusion flame. So, we have done our discussion on the jet flows and then the jet diffusion flame which is reacting.

(Refer Slide Time: 00:31)

7: Laminar non-premixed Flames

Historical Theoretical Formulations:

- Burke and Schumann (1928)
 - constant velocity field parallel to flame axis.
 - reasonable predictions of L_f for round burners.
- Roper and Roper et al (1977)
 - relaxed single constant velocity assumption.
 - provides extremely good predictions.
 - matched by experimental results/correlations.
 - round and slot-burners.

Now, we start some basic jet diffusion flame, first we look at some historical aspect of it and when it was initially described. So, I mean, then, that time people were trying to establish the theoretical description of this kind of flames. So, one which is very, very well known in the laminar diffusion flame is the Burke and Schumann kind of flame which is in 1928. So, where the assumption is that constant velocity field parallel to the flame axis, were the reasonable predictions of flame height for round burners so these are the output of that.

Then 1977 Roper and Roper they did a relaxed single constant velocity assumption, this provided an extremely good prediction and their data was also quite matched by the experimental observation or the results and this was valid for round and slot burners.

(Refer Slide Time: 01:50)

7: Laminar non-premixed Flames

Roper's solutions & correlations

Circular port :

$$L_{f,the} = \frac{Q_F (T_\infty/T_F)}{4\pi D_\infty \ln(1 + \frac{1}{S})} \left(\frac{T_\infty}{T_F}\right)^{0.67} \quad \dots (7.24)$$

$$L_{f,exptl} = 1330 \frac{Q_F (T_\infty/T_F)}{\ln(1 + \frac{1}{S})} \quad \dots (7.25)$$

$S =$ stoichiometric molar oxidizer-fuel ratio
 $D_\infty \rightarrow$ mean diff. coef. at T_∞
 T_F, T_F

So, these are the historical aspects. Now, we can look at these are some of the solutions one is this some Roper's solution and correlations. So he has taken circular port, then the flame height is correlated with volumetric flow rate:

$$L_{f,the} = \frac{Q_F (T_\infty/T_F)}{4\pi D_\infty \ln(1 + \frac{1}{S})} (T_\infty/T_F)^{0.67}$$

Now, L_f experimentally observed that:

$$L_{f,exptl} = 1330 \frac{Q_F (T_\infty/T_F)}{\ln(1 + \frac{1}{S})}$$

So, there is a quite nice correlation. Here, S is the stoichiometric molar oxidizer fuel ratio and then we have the infinity which is obviously the main diffusion coefficient of oxidizer.

This is the mean diffusion coefficient of oxidizer at T_∞ and T_F are fuel stream and mean flame temperature so, this is fuel temperature this is flame temperature. So, this is how the correlation provided by these people.

(Refer Slide Time: 04:37)

7: Laminar non-premixed Flames

square port:

$$L_{f,the} = \frac{Q_F \left(\frac{T_\infty}{T_F} \right)}{16D_\infty \left\{ \text{invert} \left[(1+S)^{-0.5} \right] \right\}^2} \left(\frac{T_\infty}{T_F} \right)^{0.67} \quad \dots 7-26$$

$$L_{f,exp} = 1045 \frac{Q_F \left(\frac{T_\infty}{T_F} \right)}{\left\{ \text{invert} \left[(1+S)^{-0.5} \right] \right\}^2} \quad \dots 7-27$$

invert \equiv inverse Erf

$$\text{Erf}(w) = \frac{2}{\sqrt{\pi}} \int_0^w e^{-t^2} dt$$

Now, if instead of a circular port, if you take a square port then we get a different expression from the flame height which is:

$$L_{f,the} = \frac{Q_F \left(\frac{T_\infty}{T_F} \right)}{16D_\infty \left\{ \text{invert} \left(1 + S \right)^{-0.5} \right\}^2} \left(\frac{T_\infty}{T_F} \right)^{0.67}$$

And experimentally observed L_f height was:

$$L_{f,exp} = 1045 \frac{Q_F \left(\frac{T_\infty}{T_F} \right)}{\left\{ \text{invert} \left(1 + S \right)^{-0.5} \right\}^2}$$

Which is now this inverse function is the inverse error function which is defined as:

$$\text{Erf}(w) = \frac{2}{\sqrt{\pi}} \int_0^w e^{-t^2} dt$$

So, this is how the inverse function is defined.

(Refer Slide Time: 06:42)

7: Laminar non-premixed Flames

Slot burner - momentum Controlled

$$L_{f,th} = \frac{b\beta^2 Q_F}{h I D_s Y_{F,stoic}} \left(\frac{T_{oc}}{T_F} \right)^2 \left(\frac{T_F}{T_{oc}} \right)^{0.33} \quad \dots (7.24)$$

$$L_{f,exp} = 8.6 \times 10^{-4} \frac{b\beta^2 Q_F}{h I Y_{F,stoic}} \left(\frac{T_{oc}}{T_F} \right)^2 \quad \dots (7.25)$$

$b = \text{slot width}$
 $h = \text{length}$

$$\beta = \frac{1}{4 \times \text{invert} [1/(1+S)]}$$

Now, if the slot burner is momentum control then the correlation is different and it was given that so, this correlations are also available in any standard book because these are well established correlation proposed long time back. So, one can find that out which is T_{∞} by point. Now, experimentally what is observed is that β which is T_{∞} by this term. Here b is the slot width, h is the length and β is:


$$\beta = \frac{1}{4 \times \text{invert} \left[\frac{1}{(1+S)} \right]}$$

(Refer Slide Time: 08:32)

7: Laminar non-premixed Flames

$$I = \frac{J_{act}}{m_F v_c}$$

uniform flow $I=1$, fully developed $\Rightarrow I=1.05$

 INDIAN INSTITUTE OF TECHNOLOGY KANPUR Ashoke De 46

Now if we define I which is the ratio of the initial momentum flow to that to the slot to the uniform flow that is how you get. Now for uniform flow I would be 1 and fully developed flow if we assume it is parabolic which is fully developed this I would be 1.5. Now, this is where the correlation that we looked at that when the slot burner is momentum control. Now, there could be another situation where this slot burner is buoyancy controlled.

(Refer Slide Time: 09:27)

7: Laminar non-premixed Flames

Slot-burner - buoyancy controlled

$$L_{f,the} = \left[\frac{9\beta^4 Q_F^4 T_\infty^4}{8D_\infty^2 ah^4 T_F^4} \right]^{1/3} \left(\frac{T_F}{T_\infty} \right)^{2/9} \dots (2.20)$$

$$L_{f,exp} = 2 \times 10^3 \left[\frac{\beta^4 Q_F^4 T_\infty^4}{ah^4 T_F^4} \right]^{1/3} \dots (2.21)$$

$a \equiv$ mean buoyant acceleration $= 0.6g \left(\frac{T_F}{T_\infty} - 1 \right)$

$g =$ gravitational acceleration.

In that case, the lift off height of the flame is different and that is given as:

$$L_{f,the} = \left[\frac{9\beta^4 Q_F^4 T_\infty^4}{8D_\infty^2 ah^4 T_F^4} \right]^{1/3} \left(T_\infty / T_F \right)^{2/9}$$

And experimentally observed this thing is:

$$L_{f,exp} = 2 \times 10^3 \left[\frac{\beta^4 Q_F^4 T_\infty^4}{ah^4 T_F^4} \right]^{1/3}$$

Now, here there is one more nomenclature which is your mean buoyant acceleration that is:

$$a \cong 0.6g \left(\frac{T_F}{T_\infty} - 1 \right)$$

And g is as usual gravitational acceleration. So, so, every situation even if you take a slot burner whether it is a momentum controlled or buoyancy controlled the expression of the flame height is different.

(Refer Slide Time: 11:39)

7: Laminar non-premixed Flames

Slot Burner (Transition Regime)

Froude Number: $Fr_f = \frac{(u_e I_{f,static})^2}{\rho L_f} \dots 7.33$

= $\frac{\text{initial jet momentum flow}}{\text{buoyant force experienced by flame}}$

$Fr_f \gg 1$	—	Momentum Controlled
$Fr_f \approx 1$	-	Transition (mixed)
$Fr_f \ll 1$	-	Buoyancy.

Now, we look at the slot burner where it is in transition regime that means, we look at this slot burner in transition regime. Now one parameter which becomes important to define is the Froude number. Now, what it does physically this Froude number represent the ratio of the initial jet momentum flow to the buoyant forces which is explained by the flame.

So, this is initial jet momentum flow to buoyant force experienced by flame so that is the ratio. So, there could be situation where this is very much higher than one. So, this is a special case if this is very higher than one that means buoyant force effect is less so it is more or less the jet momentum, so this becomes a momentum control situation. Now, if Fr_f is 1, this is the transition or mixed that means and the other possibility Fr_f is less than 1, so if it is less than 1 moment of is not that much so it is essentially buoyancy controlled.

(Refer Slide Time: 14:16)

7: Laminar non-premixed Flames

L_f - To be known a priori

Then, $Fr_f \approx 1$

$$L_{f,T} = \frac{4}{9} L_{f,M} \left(\frac{L_{f,B}}{L_{f,M}} \right)^3$$
$$\times \left\{ \left[1 + 3.38 \left(\frac{L_{f,M}}{L_{f,B}} \right)^3 \right]^{2/3} - 1 \right\}$$

... 7.25)

Now, one can note that L_f must be known a priori to establish the appropriate regime. So, it is sincerely in sort of trial and error approach because L_f to be known a priori. So, this is little bit trial and error approach. Now, we can take a situation where Fr_f is of order of 1. Then my $L_{f,T}$ is:

$$L_{f,T} = \frac{4}{9} L_{f,M} \left(\frac{L_{f,B}}{L_{f,M}} \right)^3$$

So that is how you get in that situation. So, depending on the Fraude number and all these things you can get, now, these are about jet diffusion flame what you can have.

(Refer Slide Time: 15:55)

7: Laminar non-premixed Flames

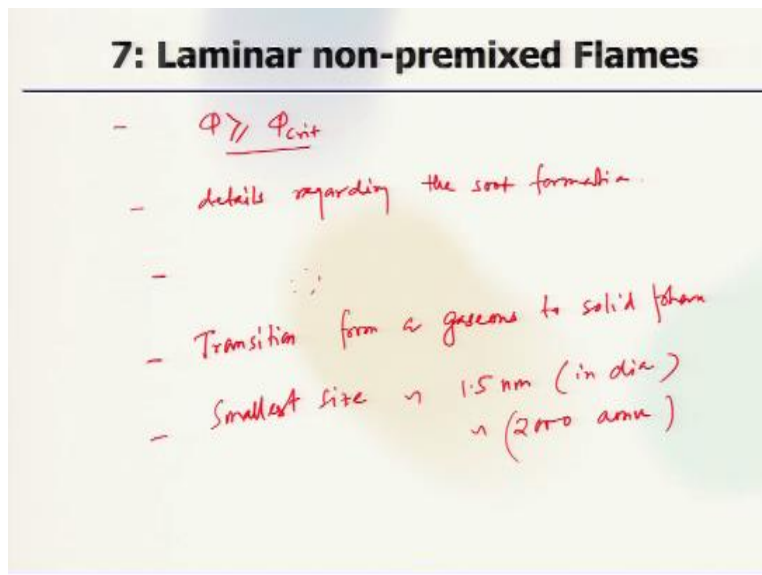
Soot Formed

- Fuel type : Chemical structure & composition
- Dilution : Inert or reactive diluents
- Turbulence :
- Temperature
- Press

Now, what happens in the diffusion flame you get soot which is one of the problems in the jet diffusion flame you get soot formation and in the jet diffusion flame. So, why that happens? There are some factors like fuel type. So, it depends on when you talk about the fuel type it is primarily what happens when you get these hydrocarbon fuels, when you burn the hydrocarbon fuel you get these soot particles these are black particles that you get.

Now, it depends on fuel type. So, what do you mean by the fuel type is that it is essentially the chemical structure and composition then dilution that is another issue. So, it can tell you whether it is inert or reactive diluents. Then the underlying flow field which is another important thing that is your turbulence. So, the time scales and all these things would be important because now once your flow field is turbulent, each has different time scale compared to chemical time scale then obviously, your temperature, pressure these are factors what impact that.

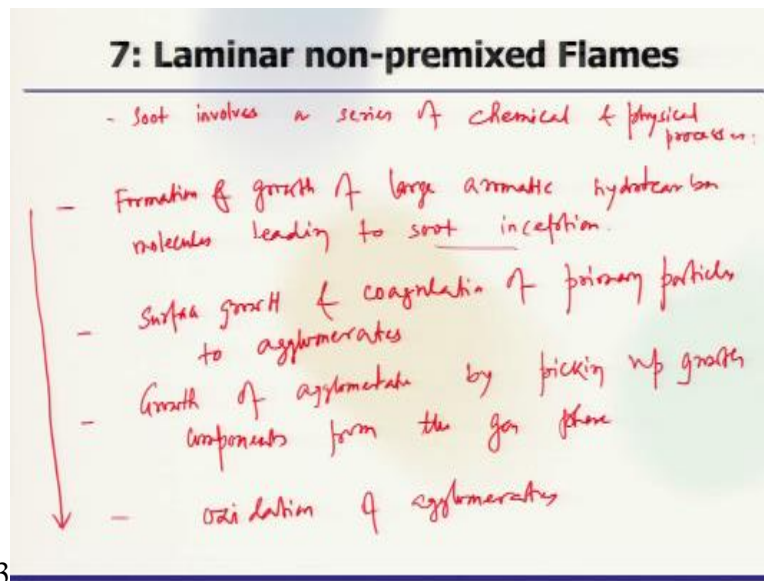
(Refer Slide Time: 17:50)



So, typically this is a problem in your premixed flame, this is not a problem in premixed flame you do not get this unless ϕ greater than $\phi_{critical}$. So, this is a typical problem you see or exist in diffusion flame. And there are details which are available, details regarding the soot formation and it happens because of the conversion of the hydrocarbon fuel with molecules containing a few carbon atoms into carbon SES, agglomerated containing some millions of carbon atoms in few milliseconds.

So, in a nut shell, you get some carbon particles at the end of the combustion process which is by nature and also there is a transition which takes place from a gaseous to solid phase. Now, typically the smallest sizes of these particles are about 1.5 nanometer in diameter which is about 2000 amu. So that means they are such a small particle which are actually formed hydrocarbon, fuel combustion.

(Refer Slide Time: 19:38)



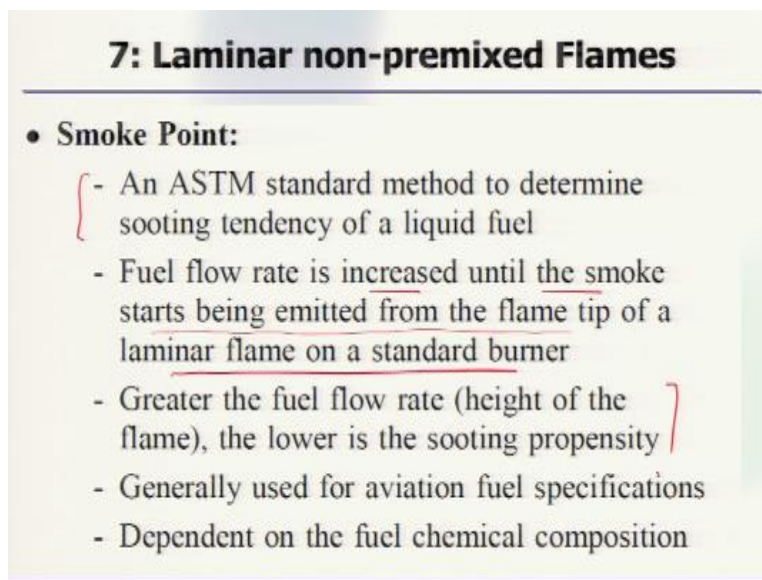
Now, if you look at the soot formation it actually involves A series of chemical and physical processes. So, number one there is a formation and growth of large aromatic hydrocarbon molecules leading to soot inception. So, that is probably the fastest or fast step towards the soot formation one can think about where the formation and growth of large aromatic hydrocarbons molecules so, this is called a soot inception.

So, that means, this is the transition to fast solid particle. So, this is the first transition that is to solid particle then surface growth and coagulation of primary particles to agglomerates. So, that means, this surface growth takes place then they coagulated with each other and there is an agglomeration. Now, thirdly there will be growth so, the growth of agglomerates by picking up growth components from the gas phase.

Then finally, the oxidation process of agglomerates. So, these are the multiple step that soot formation actually occurs and what it happens initially though when these higher order hydrocarbons are bond, they form like a sheet kind of structure lower order hydrocarbon and sheet like in structure. This is the aromatic hydrocarbon molecules leading to soot inception, this is called the inception.

Then they grow and coagulate and they form like a particle like structure where the phase change takes place and then the surface growth process and finally, these agglomerates actually grow and it gets oxidized then you don't get to see this, but it does not oxidize, then end of that process you get these small particles.

(Refer Slide Time: 23:23)



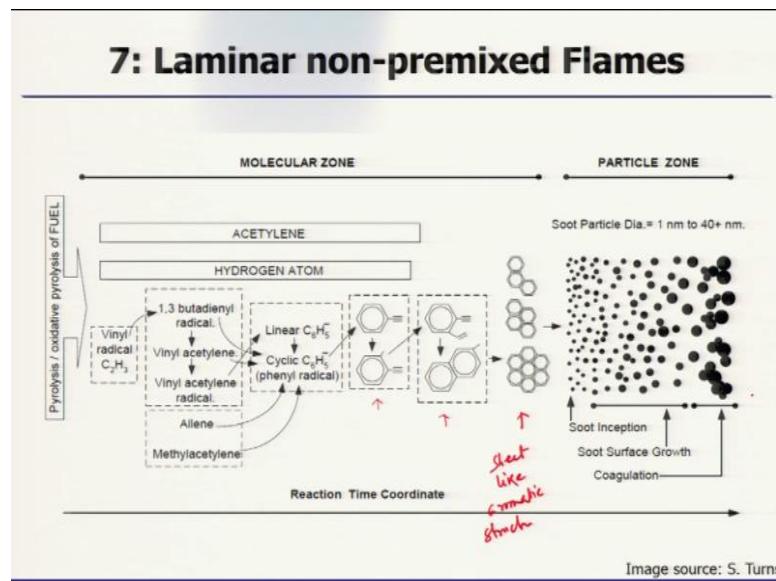
7: Laminar non-premixed Flames

- **Smoke Point:**
 - An ASTM standard method to determine sooting tendency of a liquid fuel
 - Fuel flow rate is increased until the smoke starts being emitted from the flame tip of a laminar flame on a standard burner
 - Greater the fuel flow rate (height of the flame), the lower is the sooting propensity
 - Generally used for aviation fuel specifications
 - Dependent on the fuel chemical composition

Now, the smoke point is that so, there is an ASTM standard which is available to determine the sooting tendency of a liquid fuel, that means every liquid fuel one can actually characterize based on this available ASTM standard. Now, these fuel flow rate increased until the smoke starts being emitted from the flame tip of a laminar flame on a standard burner. So, it is a simple burner where you have a laminar frame unless the smoke starts to emit is the fuel flow rate increased.

So, greater the fuel flow rate, the lower is the sooting propensity. So, generally it is used for aviation fuel specification and dependent on the chemical composition also.

(Refer Slide Time: 24:20)



Now, this is what one can see pictorially that what happens. So, this is the first step, the pyrolysis process of the fuel. So, you get the radical and the lower order hydrocarbon you get then from the lower order hydrocarbon and this happens when you burn the higher order hydrocarbon. So, you get these vinyl radical, then butadienyl radicals, vinyl acetylene all these are formed.


Then there is an exchange of bond or breakage of bond that takes place and then you get to a situation where these kind of structures which you get the aromatic rings. So, essentially this is the stage where you can think about there is a formation of aromatic structure or rather I would say that all these intermediate state finally you come to this formation of this aromatic structure which is a sheet like aromatic structure that forms.

And then from there, first the phase change takes place and this is the step where you call it a soot inception. Now, then, you have a surface growth that means, these small particles grow this is what the surface of these assume ably spherical particles they grow and you get the larger size particles. Now, once you get the larger size particles, now, at this stage these particles may actually coagulate with each other. If they stick to each other, then you can see a chunk of particles they stick to each other and then there will be some agglomeration of this particle this is a coagulation process and then finally, at this convert downstream it gets oxidize presence of the OH and these things. Now, if that oxidize the amount which is formed and oxidization is balancing out then there will be no

soot at the end of this process. But is that does not happen there is an imbalance so you get to see end of this black particles this is the whole thing, the process one can think about.

(Refer Slide Time: 27:14)

7: Laminar non-premixed Flames



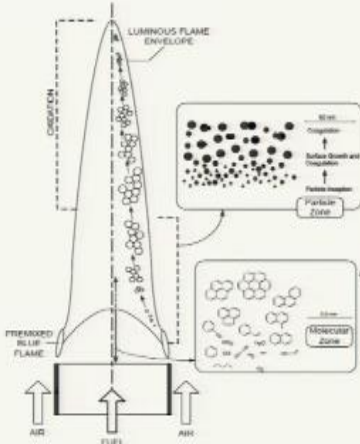
- fuel tube = 11 mm; fuel flow rate = 3.27 cm³/s
- air nozzle = 100 mm; air flow rate = 170 L/min
- visible flame height = 67 mm (Fuel: C₂H₄)

Image source: Internet (google)

It is a simple example, which one can take there is a fuel tube of this millimeter diameter, this is the fuel flow rate, this is the air nozzle of this millimeter, flow rate of this visible flame height is 67 millimeter that means, this is the roughly 67mm fuel is C₂H₄.

(Refer Slide Time: 27:37)

7: Laminar non-premixed Flames



The diagram illustrates the structure of a laminar non-premixed flame. It shows a central fuel jet surrounded by an air jet. The flame is divided into a luminous flame envelope and a particle zone. The particle zone is further divided into a molecular zone and a soot zone. The soot zone is characterized by the presence of soot particles, which are formed through the oxidation of hydrocarbons. The molecular zone is characterized by the presence of molecular species. The diagram also shows the flow of air and fuel into the burner.

Image source: S. Turn

Now, this is what happens when you have C_2H_4 actually this fuel comes here, air comes here and you get this soot particulate formation. So, what it does, if the air comes, so, there will be a diffusion process and this is your luminous flame zone and this portion is your oxidation process and these are the particle structures. So, this example we will discuss in more details in the next lecture, we will stop here today.