Turbulent Combustion: Theory and Modelling
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology - Kanpur

Lecture-27
Turbulence (contd...)

Welcome back. So, let us continue the discussion on turbulence, we just started the discussion on turbulence and where we have stopped just looking at this thing, the problem of the sphere.

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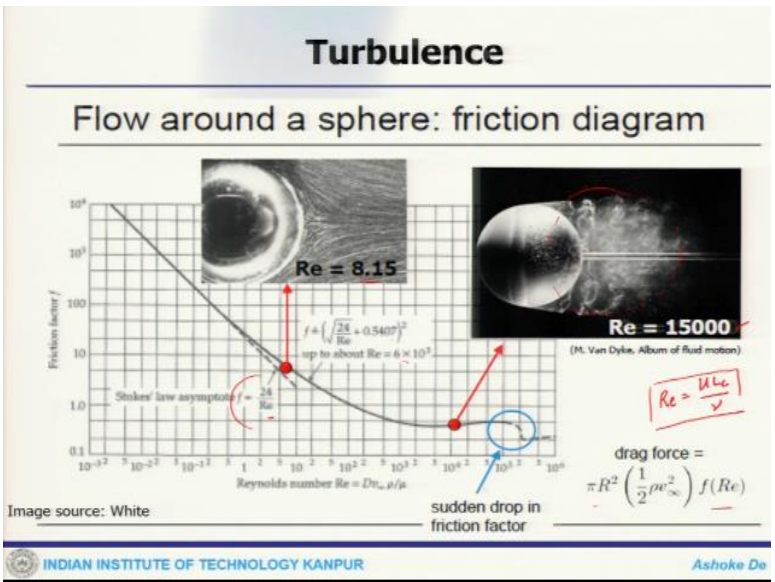

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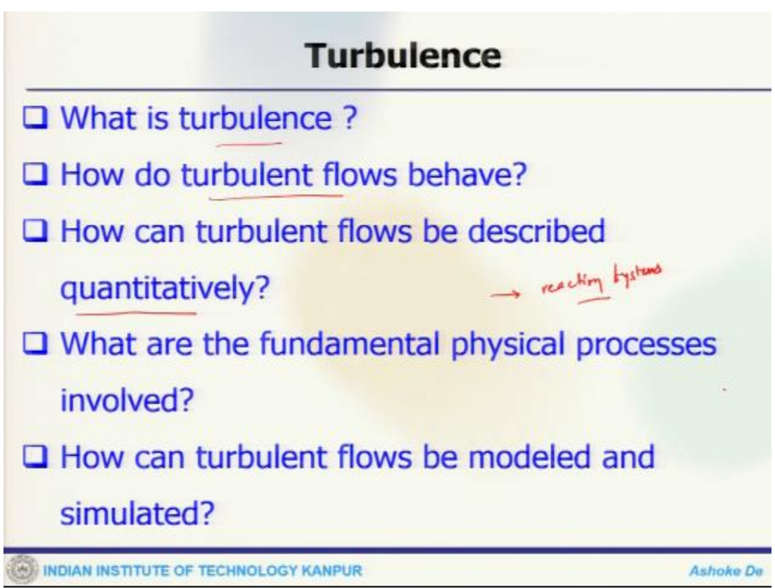

So just quickly review what we are going to do about this discussion on turbulence, we first look at what turbulence their behavior of the turbulent flow, and then we will try to characterize it and then finally, some of these processes which are involved and the modeling aspect. And then we started of this particular example of the sphere. Now, this one can see as we just said that drag force is function of the Reynolds Number and all these things.

So, there is a nice similarity between your cricket balls. If you have looked at it usually the players rub it in one side and keep on rubbing it so, that means what they try to do? They try to keep one side shiny and the other side to be rough. So, they maintain that roughness and all these things to make a difference in the few dynamics sphere features here, everything is turbulent. Now if you make one side smooth and keep it shiny, the flow pattern will be different compared to the rubbed side.

So, that actually generates that kind of swing and also depending on the speed and all these things, there are other issues like how the bowler actually bowl at a quarts speed then the leap is generate when hitting the pitch and then the reverse swing all these are associated with that.

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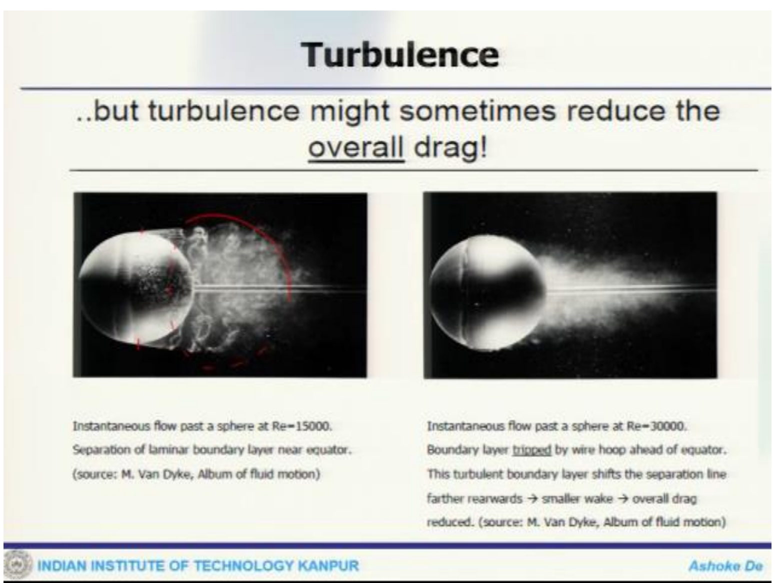

And this is a schematic now more if you look at it. So, one hand if you have things are turbulent and that sometime reduces the overall drag. So, this is the picture that we looked at in the earlier also this is Re equals to 15,000. So, you see the boundary layer which separates near this point. So, this is a separation point, okay? You are that say situation where the laminar boundary layer separates. Because this particular Reynolds number 15,000 that belongs somewhere here, okay?

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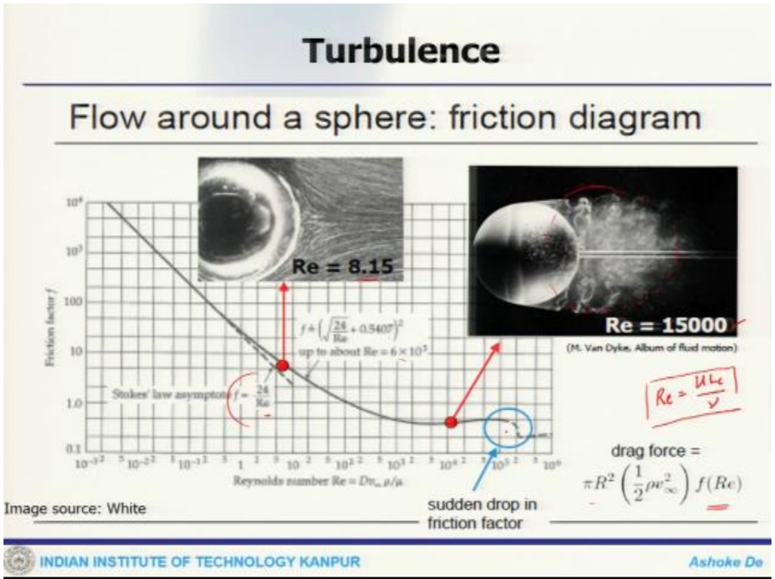

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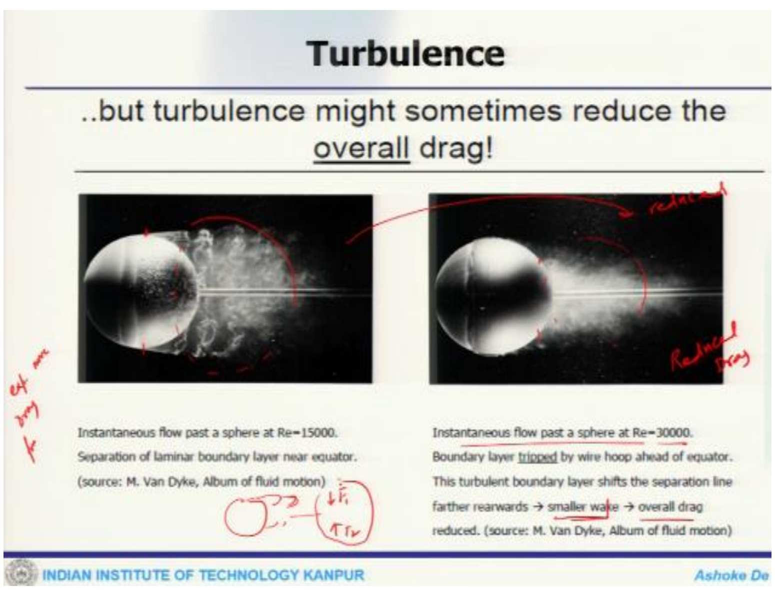

So, that is with the increasing Reynolds Number and the features of so, if you have separation point here, so, there is the zone which will create the adverse pressure gradient and you will get more drag force. So, you experience more drag force. At the same time same sphere, you go to a

Reynolds number of 30,000 which is sort of a higher Reynolds Number situation. So, the tripping of the boundary layer or the separation point of the boundary layer has moved from this position to this position.

So, the zone which is having adverse pressure gradient zone that is also perform here to here that is quite reduced. So, what happens here physically this turbulent boundary layer with the increasing Reynolds Number that turbulence boundary layer see this separator point towards the rear portion of the sphere or towards the rear submission point. So, you have a smaller wake and the smaller wake will reduce the overall drag. So, this case you get reduced drag and this case so, that means, if you have a laminar boundary layer which separates actually creates more drag compared to a situation for if you are and this is why, when it talked about that cricket ball dynamics that is why the player actually rub the ball one side and keep it smooth on the other side. So, they try to make one side to be turbulent, we are actually the separation point shift towards the rear point.

And other side it remain laminar, where it separates earlier. So, if you see a situation of which it separates from here and other cases separates from here. So, obviously, there would be a asymmetric force. So, these 2 forces will be acting in a non-equal fashion and which will create that swing. But this is one aspect of it when the ball is actually traveling in an air but there are other issues like to generate that. I mean to reach to that Reynolds Number of the speed, now, the ball has to be thrown at that speed that it has to hit the pitch and then also that generate some sort of an leap force due to rotation and other thing, but, if you leave those factors for the time being and when you just think about one side, smooth and one side dropped the ball is approaching towards the batsman, then that is where the swing happens because of this asymmetric in the radial forces.

And the primarily the reason behind that is the fluid mechanics one case it says laminar separation, other case it is turbulent separation. So, this case the drag is reduced or the adverse pressure gradient zone is low this case it is high.

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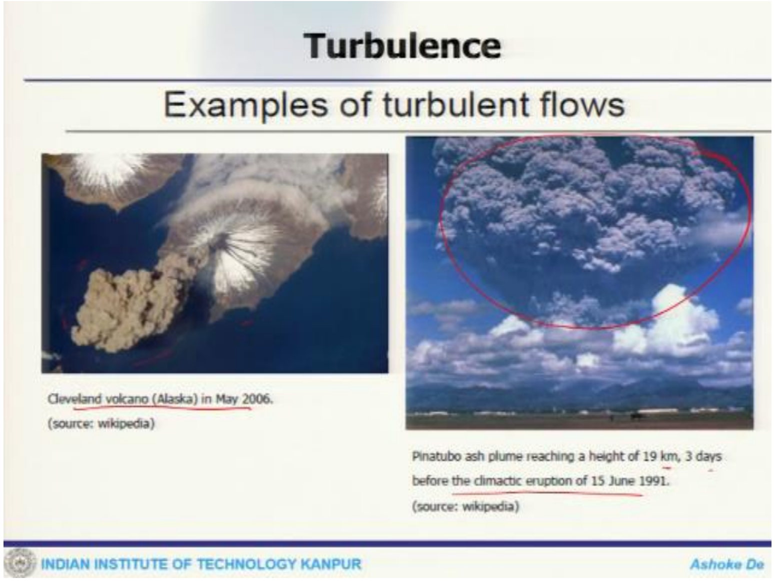

Now going to some more different examples of turbulent flow. These are again what you see in nature very often. So, this is an volcano actually in 2006 which erupted in Alaska and you see this kind of smoke and all these things, this is nothing but the system is turbulent and then another ash plume reacting a height 19 kilometer 3 days before the climactic eruption of 15 June 1991 you see this huge structure these are now think about the scale of these things.

So, this is one hand you are looking at the macroscopic structure of it and this is an apparently it looks quite random in nature, chaotic in nature and that in one single sentence one can characterize this particular flow physics the turbulent qualitatively is that purely random in nature or chaotic in nature, that is what one can define the flow to be turbulent and that is obviously in qualitative sense, when you go quantitatively of a one has to properly characterize it based on scale, they are length scale, time scale and other features.

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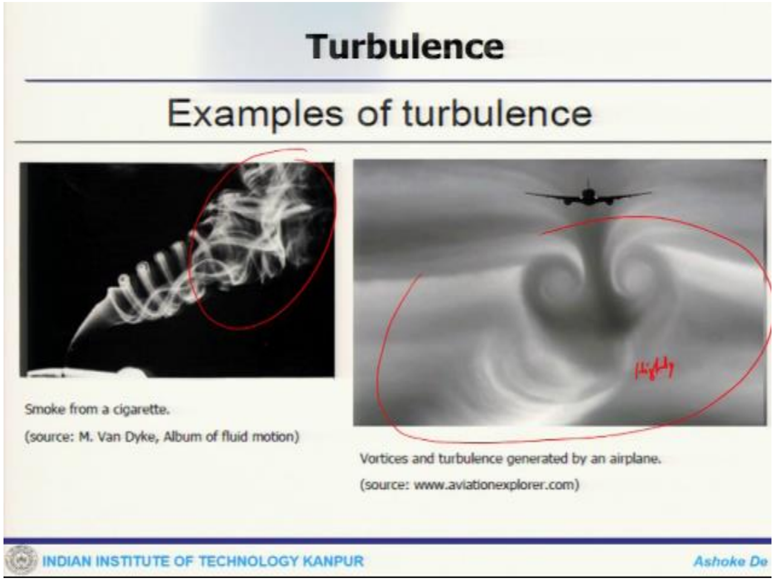

Some more, which you see the smoke coming out of the cigarette, this is also the nice example of characteristics. Another realistic example when there is an aircraft which is flying, you see the nice structure like this so, which is basically what it is and generated by this airplane. That is a big structure. So, these are highly turbulent in nature so, that sort one can look at it different example of turbulence and other application.

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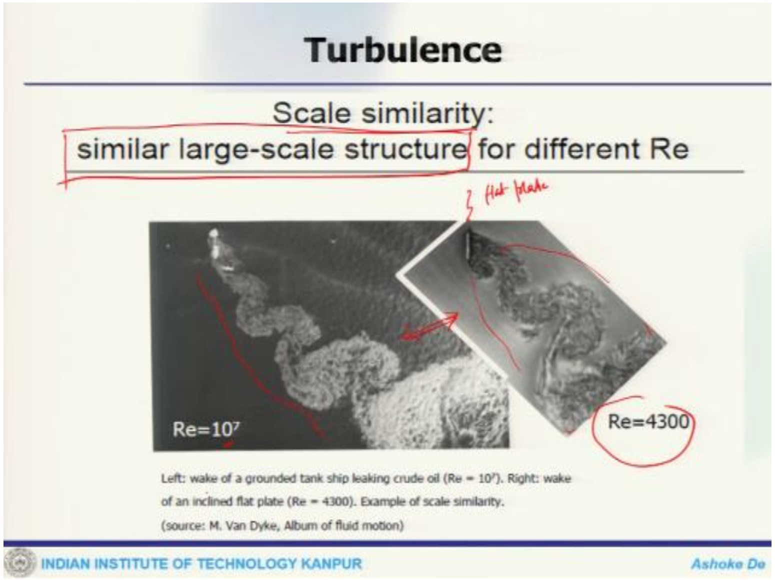

Now, one of the important characteristics is scale similarity, what it says that there is a similarity of large scale structure for different Reynolds number. So, the left hand side picture which is at Re

10^7. This is in wake of grounded tank ship leaking crude oil. So, the ship is leaking the crude oil and you see the kind of structure here behind that, which is obviously turbulence in nature.

There is no doubt about it and the other side this particular one this is a flat and which is inclined at an angle and you see the structure of the large scale structure. So, if you look at qualitatively these 2 structures the large scale structure this whole some sort of a similarity and this is what is meant here this turbulent flow features one of the characteristics of the turbulence flow features and different Reynolds number provided there in all turbulent regime there is a similarity in large scale structure, this is what we call scale similarity. So, that means you take a flow behind grounded tank or you take your flow around flat plate or you take a flow around the sphere, the large scale structure would be similar in nature that is the important point here.

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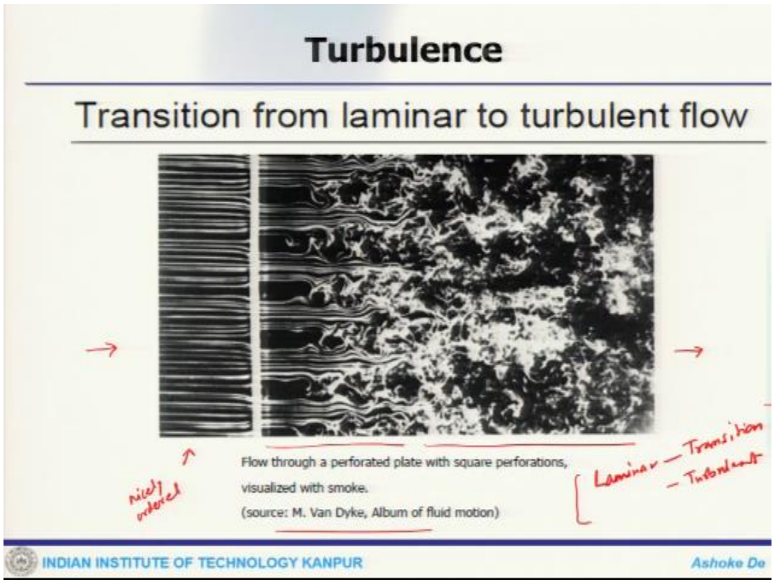

Now, this is a well-known characteristics of turbulent flow. So, this is a flow through a perforated plate with some square perforation and smoke visualization. So, these are the source of these images one can if you are curious you can look at it. So, the flow comes from this side and it goes out from this side when it enters or the plate, this is a nice sim line one can see. I mean rather these are nicely ordered lines, but then it goes through transition to turbulent.

So, there is a transition zone and then finally it becomes completely turbulent. So, the flow initially is laminar, it become transition. So, that means in transition, it will be here with both the characteristics, the laminar characteristics and transition characteristics and then it became finally turbulent. So, you can see the effect of this flow when it becomes turbulent and goes through these different situations.

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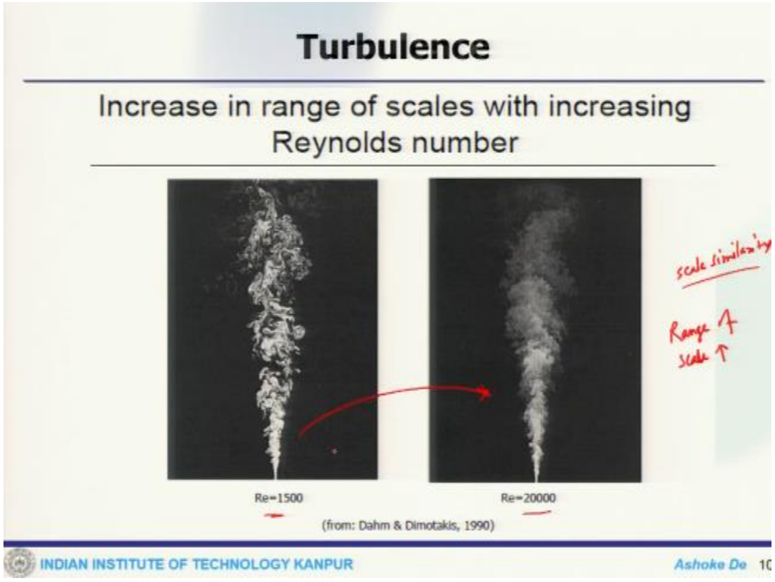

Now, if we increase the range of scale that we will increase Reynolds number. So, this is that Reynolds number 1500 and this is that 20,000. So, the large scale structure remain same. So, there is a scale similarity which talks about the large scale structure remain same, but with increasing the Reynolds Number the range of scale would also go up that means, from these to these scale range has gone up.

So, which essentially says you may have similarity of the large scale structure but that does not guarantee that in the small scale structure would be of the same order. So, there could be huge increment in the other range of scale or the small scale structures with this is what this picture very nicely demonstrate that what happens to this particular thing.

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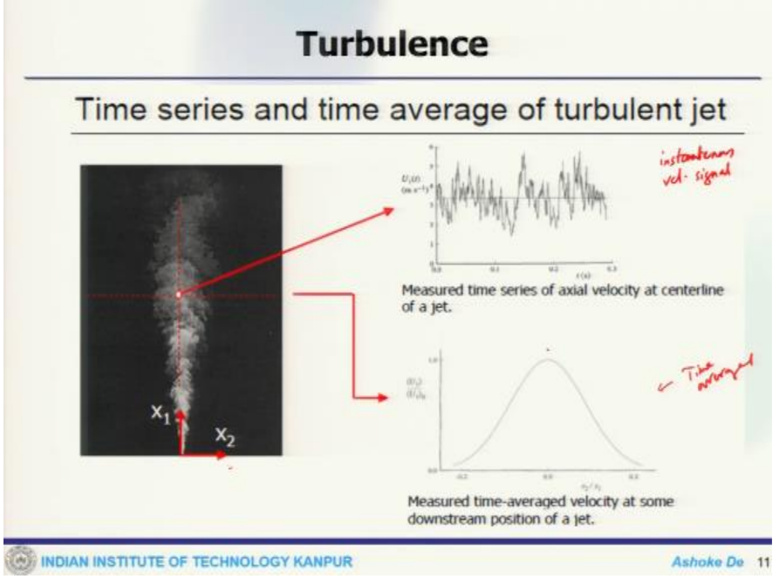

Now, if you go to time series and time averaging of the turbulent jet kind of flame. This is the picture we have taken already mentioned about this source. So, this is the coordinate system x2 and x1 at any particular point. This is an instantaneous velocity signal and you can see how it is actually behaving there is a huge chaotic and this is at the same location when you get the time averaged velocity.

So, the time averaged one will give you some sort of a mean profile and this particular location and from the center line, it will tell you either side of the z axis how it spreads. So, already we have seen for laminar jet, there are 2 important things one needs to know, to characterize this kind of jet flow is the jet spreading and this predict angle which is related to the jet hub width.

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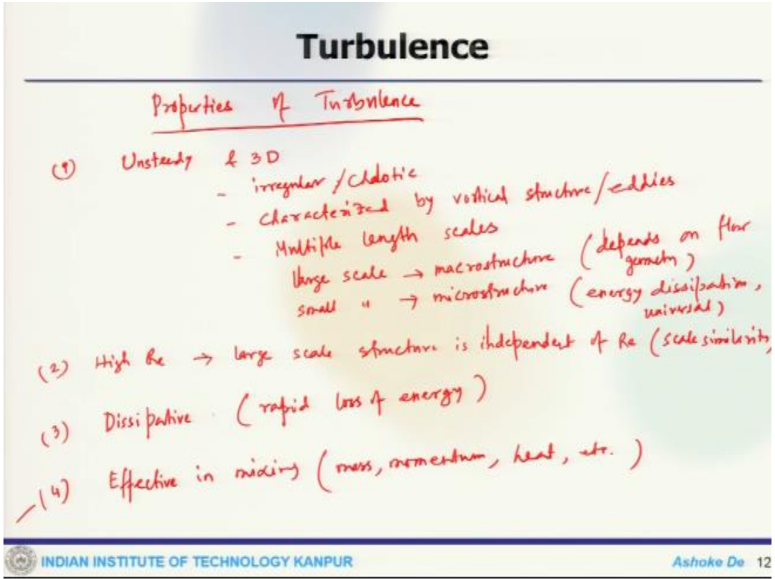

So, whatever we have look so far if we summarize that essentially as the properties of turbulence in that sense if we summarize one point which one has to notice that it is unsteady and 3D. So, under which you can say it is irregular or chaotic in nature. It is characterized by vertical structure or eddies you have multiple length scale one and you have large scale which essentially calls it macro structure and that depends on flow geometry and small scale which one can say microstructure.

So, it depends on energy dissipation and other thing this is universal structure or something. So, again it is a high Reynolds number flow, which is large scale structure is independent of Re which is called scale similarity. Third, this is dissipative in nature that means rapid loss of energy and fourth is it is quite effective in mixing that is mass, momentum, heat etc So, this is one of the very interesting point why it is when you say effective is being safe and that is why in our rating system or practical reacting system one way to look at it we want turbulence to be present there.

Though turbulence itself is going to create a lot of other problem or it creates I mean in case lot of challenges to handle it. But on the one hand, we do want that turbulence to be present, which will make it more effective from the mixing point of view. Better mixing we can always expect a better reaction and complete reaction which is another desired feature of reacting system. So, these are essentially about the broader characteristics that are the properties of turbulence. So, in a simple word one can think about its truly unsteady and 3 dimensional flow field, which is chaotic in nature. Now rest essentially are the secondary level characteristics, that large scale structure, small scale structure, dissipative, either or some scale similarities and all these.

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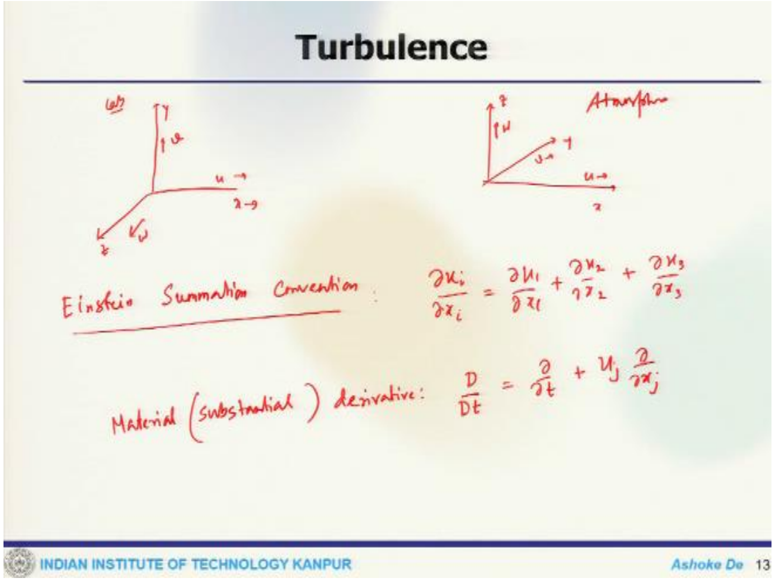

But in now, we can do some talk about some coordinate system and convention. So, we have in the one hand we have this x y which is v z which is w. This is typical we do in the large scale and typically atmospheric combustion atmosphere the convention is this is x this is y and this is z. Now, there is an Einstein summation convention. So, which will essentially make it in terms of initial notation one can write:

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}$$

Now we will write material or substantial derivative which is:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j}$$

So, these are the two things which will be commonly used while we will be discussing more about these things.

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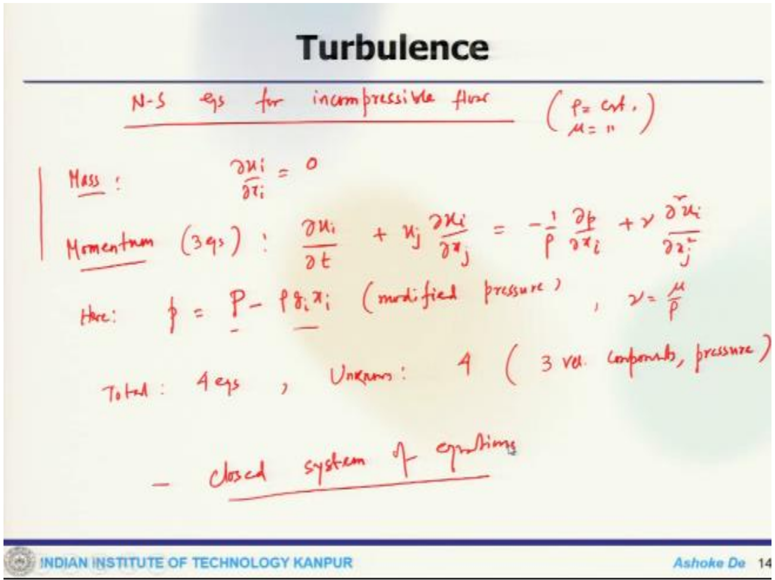

Now we will write our Navier Stokes equation for incompressible flow so, which is ρ constant and viscosity constant. So, that is what we have mass conservation equation. So, that one can write in terms of:

$$\frac{\partial u_i}{\partial x_i} = 0$$

Now we have momentum conservation equation which are essentially 3 equations, but we can write in one single equation:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \nu \frac{\partial^2 u_i}{\partial x_j^2}$$

So, that is my momentum conservation. Here, small p is nothing but:

$$p = P - \rho g_i x_i$$

This is a so called modified pressure. So this is the pressure - hydrostatic contribution. And ν is the kinematic viscosity which is nothing but μ/ρ. Now, when you look at this mass and momentum conservation equation, we have total 4 equation and we got unknown 4 which is 3 velocity components and pressure. So, this is one can think about you have number of equations to number of unknowns. So, this is a sort of closed systems of equations for given initial condition and boundary condition.

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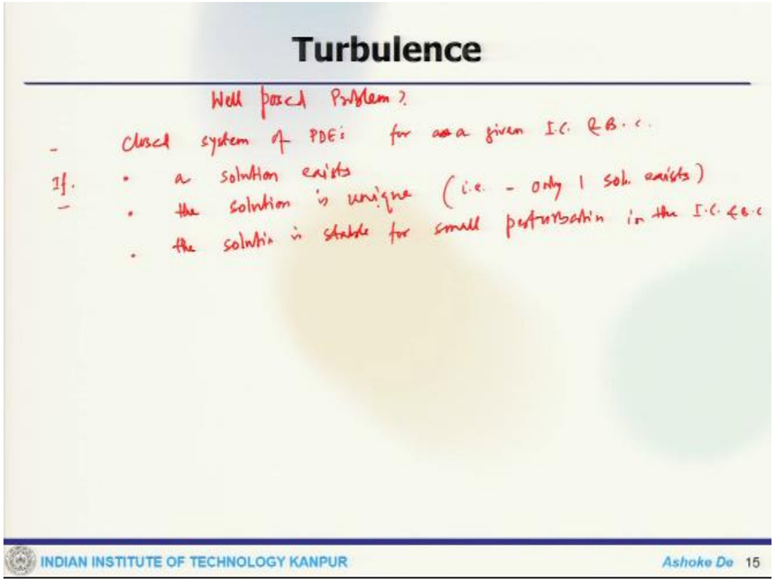

Now we look at it. It is a well posed problem. What is that? Now one can think about a closed system of partial differential equation. Closed system of PDEs can be defined as well posed for a given initial and for a given initial condition and boundary condition and it can be said well posed if a solution exists and the solution is unique which means only one solution exists. Third the solution is stable for small perturbations in the initial condition and boundary condition.

So, the closed system of PDEs can be said well posed for a given IC and BC if the solution is unique, there is only one solution exists and the solution is stable for small perturbation in the initial condition and boundary conditions. So, that is how one defines the well posed problem now, how the Navier Stokes equation would look like so that we can see how Navier Stokes equation whether it is at well posed or ill posed.

Because of the rest of situation, if you talk about the incompressible concerned density system and which is a simple one then you have mass momentum and energy conservation. But if you consider only mass and momentum, and we can see whether that is ill posed or well posed. We will stop here and continue the discussion in the next lecture.