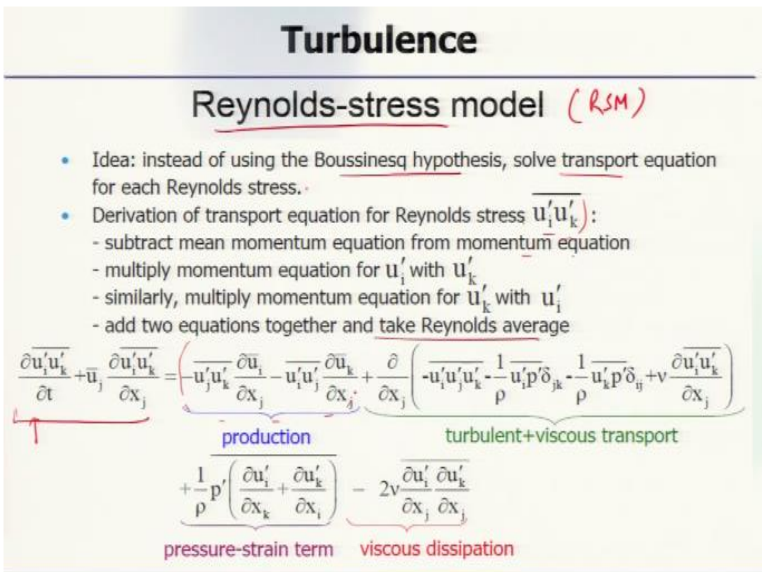
Turbulent Combustion: Theory and Modelling
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology – Kanpur

Lecture - 38
Turbulences (cont…)

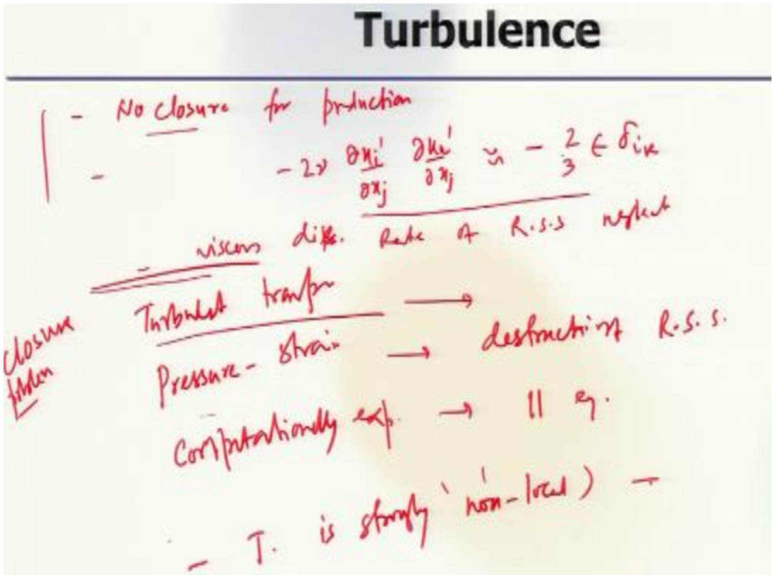
Welcome back, let us continue the discussion on the Reynolds stress model and we looked at the complete equation, and there are different terms. And we will make some comments on those different terms and then look at the other analysis.

(Refer Slide Time: 00:32)



So, this is what we looked at it, this is the total component of the Reynolds stress term this is the material derivative, then it has production, dissipation and all these things.

(Refer Slide Time: 00:42)

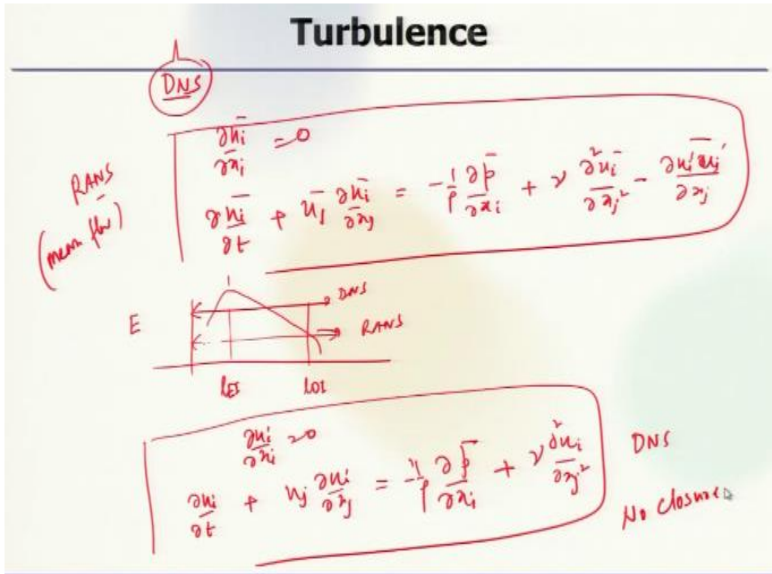


Some comments one of the important is that for production term there is no closer required then microstructure they are except close to the wall they are isotopic, so, that will approximate like this then also microstructure level negligible contribution to the reynolds stress. So, viscous, dissipation rate of reynolds stress neglected. Now, these are some of these things, but there are certain other problems, problems like turbulent transport.

So, this is one of the terms which can create problem for the closer problems. So, these are closer problem, turbulent transport is one term then pressure strain term which is essentially responsible for return to isotropy or other destruction of in all reynolds stress. Now, in this particular set of equation this is computationally, because you have it is total 11 equations, 3 power momentum. 6 equations from the Reynolds stress and one equation from the viscous dissipation then you have the.

Now, important thing is that one can use this Reynolds stress model when turbulences is strongly non local. For example, if you have a strong swirling flow, recirculation flows in pages of the main flow. So, in those cases this RANS model is superior to all these 2 equations model and all these things, so that is a RANS base situation.

(Refer Slide Time: 02:52)



Now, one can have to have DNS that means, you solve all these problems where your average equation looks like this

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_i}\frac{\partial \overline{u_i}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu\frac{\partial^2 \overline{u_i}}{\partial {x_j}^2} - \frac{\partial u_i'\overline{u_j'}}{\partial x_j}$$

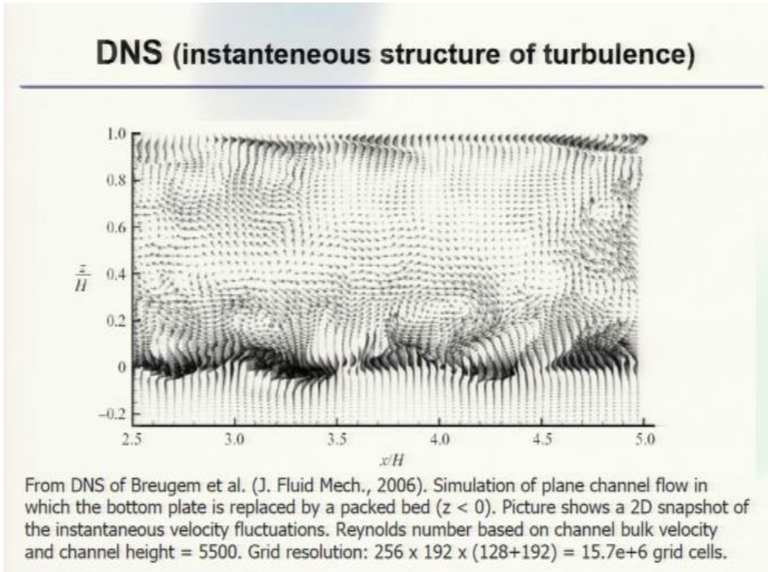
So, this closure was used this is in the RANS context. So, closure used for the Reynolds stress. Now in direct numerical simulation so, here actually use all for the mean flow.

So, that means, if you look at the energy spectrum, there is a huge energy spectrum where if you look at kinetic energy, energy spectrum so this is my l_{EI} energy containing range, inertial sub range, this will be the dissipation range l_{DI}. So RANS actually for all the spectrum, this is what RANS thus means the mean flow so, it models the complete spectrum. Now, actual turbulence you have large scale structure the energy transfers at the intermediate scale then the small scale.

This is where did it numerical simulation becomes important which actually solves for a skill that means, DNS is instead of modeling it solves. So, here the equation system will look like now you have the govern equation and that is what it solves for without any closures. So, that is $\nu\frac{\partial^2 \overline{u_i}}{\partial {x_j}^2} -$

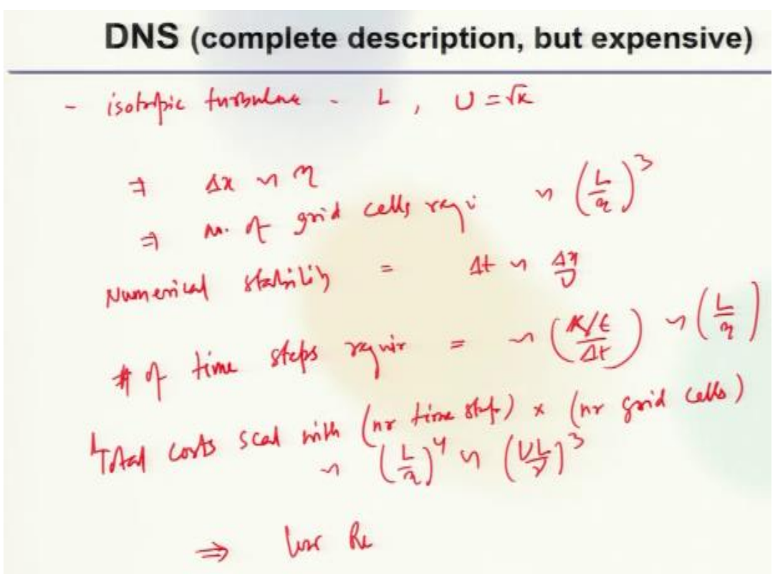
$\frac{\partial \overline{u_i' u_j'}}{\partial x_j}$. So, these are the equation which are solved for DNS that means the governing equations are completely so no closures essentially no closures.

(Refer Slide Time: 05:02)



Now, that is in picture of how DNS looks like. So, this is taken from a channel flow simulation, where you can see the small scale structures are also captured.

(Refer Slide Time: 05:13)



So, what you can note here DNS can give you the complete description, but it is quite expensive. Now, if you consider an isotropic turbulence that an again a specific case with a size L and velocity scale is like this then all scale of turbulence is to be solved. Now, energy continuing large scale to carnival scale in the dissipation range so, which will get you that Δx is order of η and your number of grids cells required would be $\left(\frac{L}{\eta}\right)^3$.

And for the stability on can look at the numerical stability point of view which will be time step is required Δt to you order of Δx/u so that is your required and the number of time steps required that would be like $\frac{k/\varepsilon}{\Delta t}$ which is $\frac{L}{\eta}$. So, the total scale cost scale width essentially if you look at the total cost the cells with nr time steps into nr grid cells, which is

$$\left(\frac{L}{\eta}\right)^4 \sim \left(\frac{UL}{\nu}\right)^3$$

So, that is why there is a limitation serious, limitation for low Reynolds number application, but, these days even today the, I mean architecture or competition architecture is available and people can do large scale DNS.

(Refer Slide time: 07:26)

Computational costs of DNS of isotropic turbulence (I)

Table 9.1: *Estimates, for DNS of isotropic turbulence at various Reynolds numbers, of modes required in each direction, N (Eq. (9.7)); total number of modes, N^3; number of time steps, M (Eq. (9.11)); number of mode-steps, N^3M; and the time to perform a simulation at 1 gigaflop (assuming 1,000 operations per mode per step)*

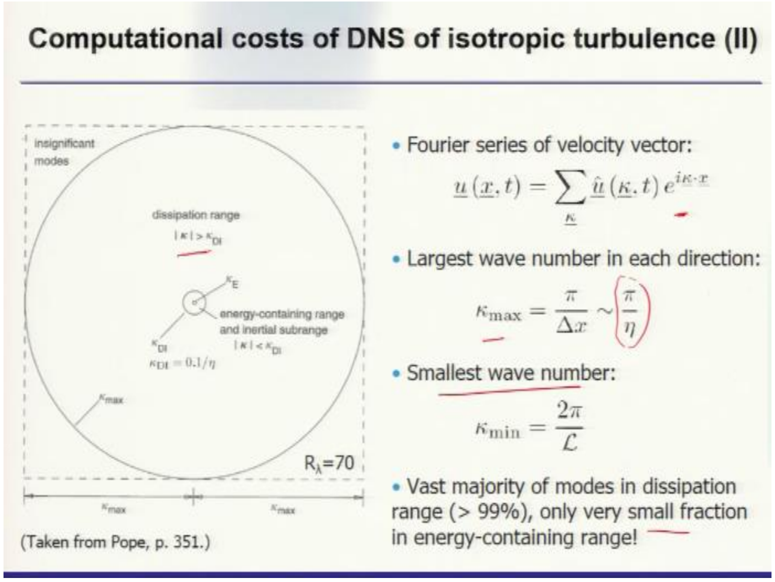
R_λ	Re_L	N	N^3	M	N^3M	CPU	Time
25	94	104	1.1×10^6	1.2×10^3	1.3×10^9	20	min
50	375	214	1.0×10^7	3.3×10^3	3.2×10^{10}	9	h
100	1,500	498	1.2×10^8	9.2×10^3	1.1×10^{12}	13	days
200	6,000	1,260	2.0×10^9	2.6×10^4	5.2×10^{13}	20	months
400	24,000	3,360	3.8×10^{10}	7.4×10^4	2.8×10^{15}	90	years
800	96,000	9,218	7.8×10^{11}	2.1×10^5	1.6×10^{17}	5,000	years

$$\frac{k^{1/2}\mathcal{L}}{\nu} = 3.44 Re_L$$

(Taken from Pope, p. 349.)

So, that is this is taken from the Pope, so you can see for different in lambda. So, this is an example for isotopic turbulence, different R_λ and Re_L, what is the computational time required and like this, but, as I said with the advancement of the computer architecture and permanent of the available processor now, people can actually look at the larger reynolds number DNS kind of calculation.

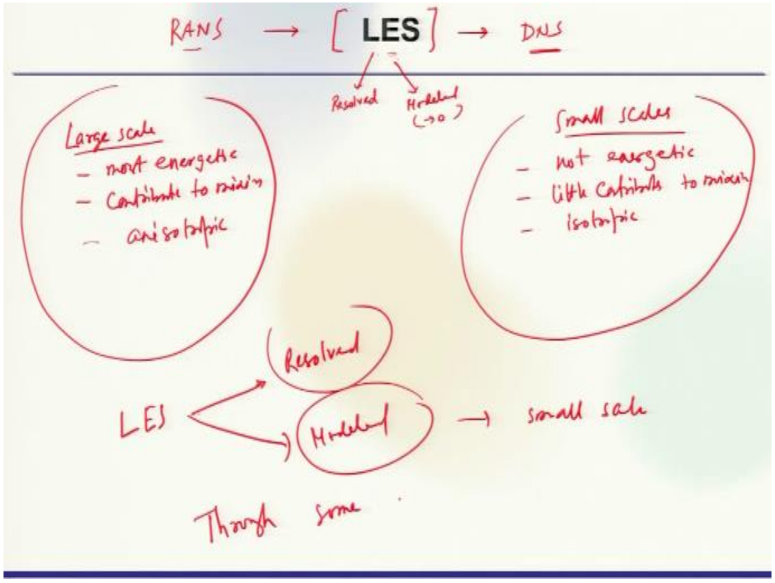
(Refer Slide Time: 08:02)



Now, we can estimate the competition cost for an isotopic turbulence like you have this situation where dissipation range would be at this. So, then free a series of the velocity vector can be expressed like that. So, we can find out the large wave number in each direction that is phi by delta x which is order of phi by eta. Now, smallest to wave number that would be 2 phi by L and he said majority of the most dissipation range.

So small fraction of that is in energy containing range, so that is what give you an idea what DNS is all about DNS requirement of the DNS for large scale large Reynolds number application.

(Refer Slide Time: 09:04)

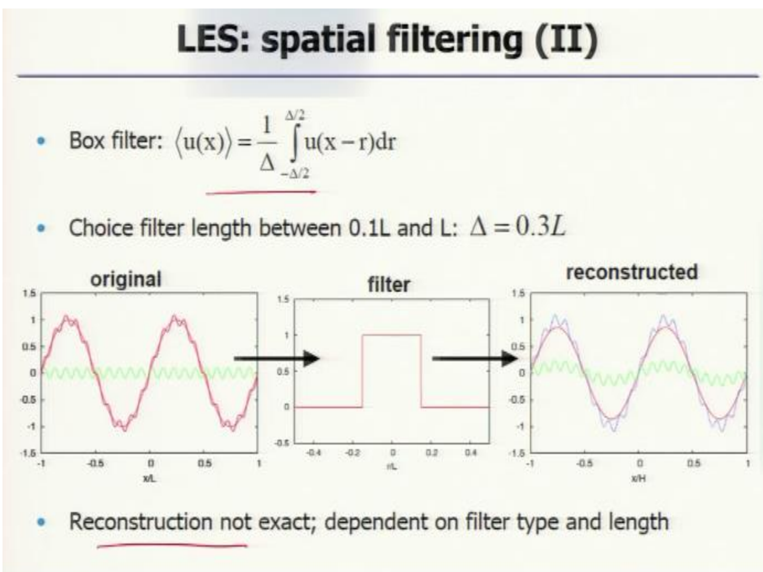


Now, one hand you had so this is an intermediate solution between RANS and DNS, because these DNS you resolved for all the scale, RANS you model for all the scale, in LES what do you do you model for large scale structure a resolve for large scale structure and only model the small scale structure. So, essentially, theoretically LES can go towards DNS, if you are modeling part. So, there are 2 parts, 1 is the resolve part another is the model part.

Now, if the model part goes to 0, so, this LES system should replicate the DNS behavior. Now, the large scale structure or the macro structure, these are most energetic structure they will only contribute to mixing and also they are an anisotropic in nature, whether if you look at the small scale or the micro structure, they are not energetic. So, they do little contribution to mixing and more or less isotopic or universal structure.

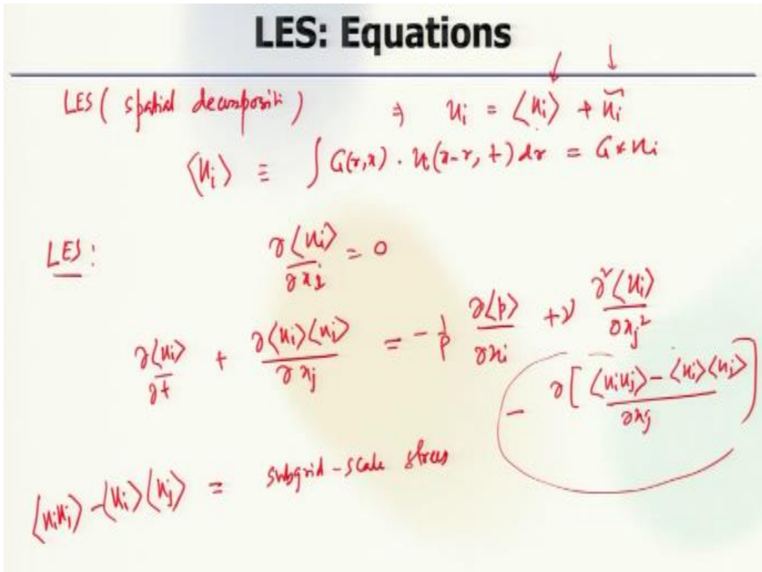
So, the whole idea is that in LES, you solve for this large scale structure and models for part the models scale structure. So, as these small scale structures are universal in nature so that is why LES actually this decomposes into 2 components, 1 is resolved component and the model component. So, this is the modeling of the small scale structure and this is the result of the mostly large scale and the energy content in structure so this is done through some spatial filtering.

(Refer Slide Time: 11:49)



So, there is a different kind of filters which are available, one can see there is a box filter then originally the signal looks like this then if you put the filter then it comes like that. Construction obviously not exact, but it depends on filter type and the length which has been used.

(Refer Slide Time: 12:20)



Now, this is a decomposition for LES again it's spatial decomposition and where

$$u_i = \langle u_i \rangle + \widetilde{u_i}$$

So,

$$\langle u_i \rangle = \int G(r,x).u(x-r,t)dr = Gu_i$$

So, our LES equations become

$$\frac{\partial \langle u_i \rangle}{\partial x} = 0$$

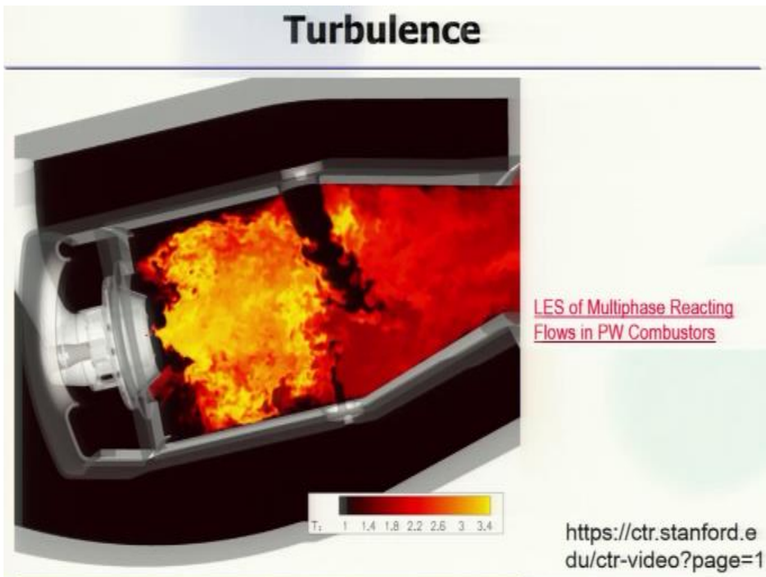
$$\frac{\partial \langle u_i \rangle}{\partial t} + \frac{\partial \langle u_i \rangle \langle u_i \rangle}{\partial x_j} = -p\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 \langle u_i \rangle}{\partial {x_j}^2} - \frac{\partial [\langle u_i' u_j' \rangle - \langle u_i' \rangle \langle u_j' \rangle]}{\partial x_j}$$

So, this is a filtered component and this is the fluctuating component rather after species building this is mean and this is in fluctuating component.

Here this particular term $\langle u_i' u_j' \rangle - \langle u_i' \rangle \langle u_j' \rangle$ this is known as sub grid scale stress from this comes from the small scale terrible motion. So, only if you look at the equation system, this is continuity momentum, this is the term which remains actually unclosed. So, the closure problem is required for these.

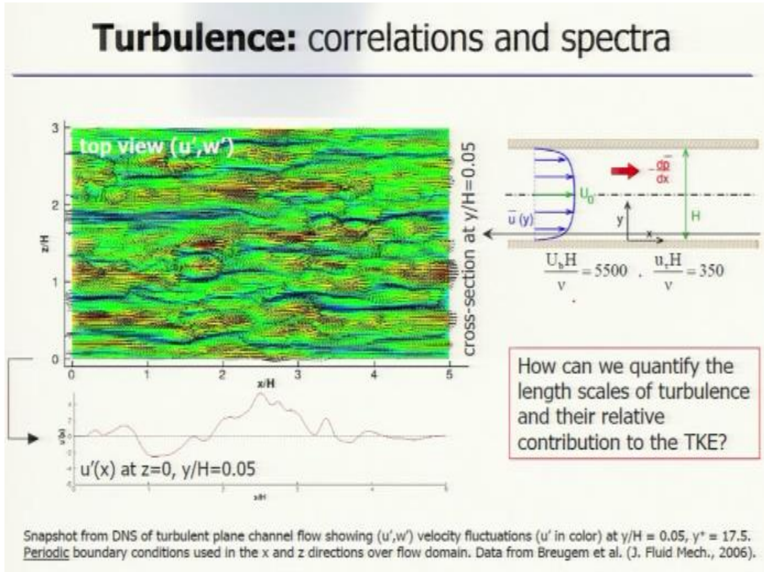
And this was proposed by a very popular model by the smugglers key which is initially developed in 1963 and then later on it was modified by different people.

(Refer Slide Time: 14:51)



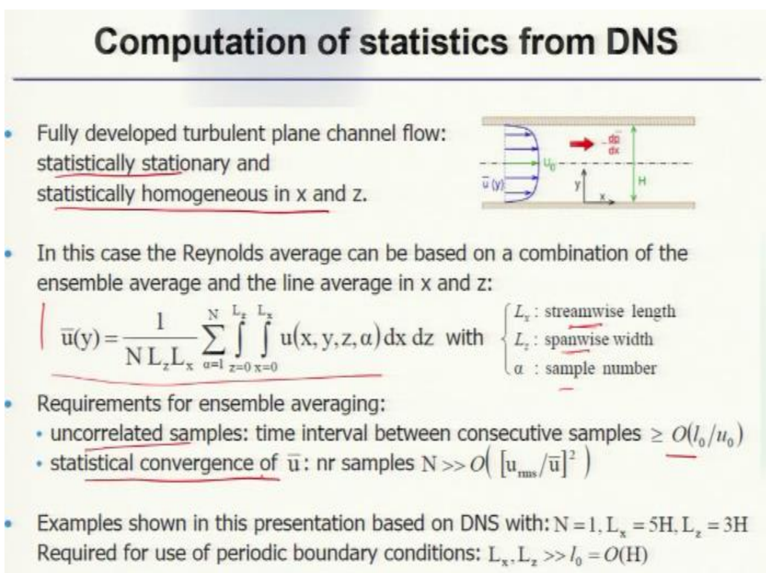
And so, this will give you an idea about when you use LES in multiphase reacting system. So, this kind of combustion going on, whether it is in real gas turbine combustion where your liquid fuel is injected here. Here comes through swelling condition and this is the unsteadiness what one can capture.

(Refer Slide Time: 15:16)



So, now, we go to the last part of the discussion is the correlation and some spectrum before we talk a little bit more about the modeling aspect and all these things. So, this is an image of a DNS calculation for channel flow, where this is the Reynolds number and this is the turbulent.

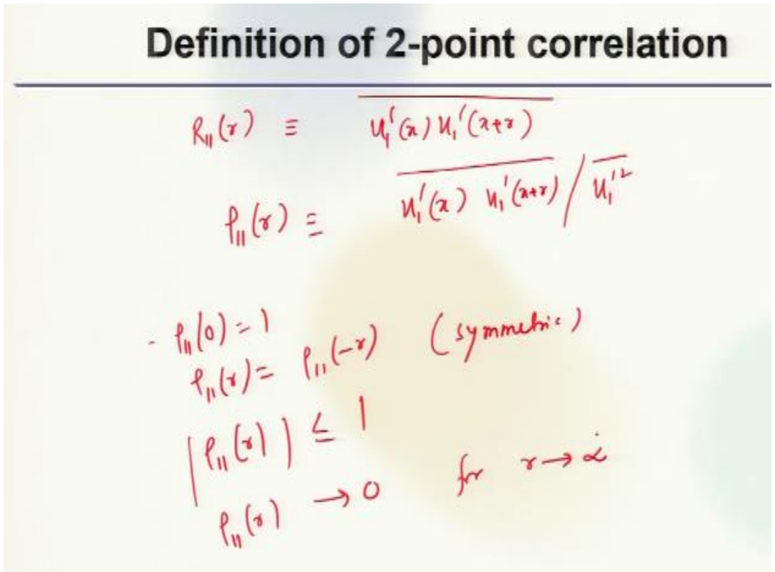
(Refer Slide Time: 15:45)



So, this is fully developed channel flow. So, this will have be statistically stationary and also statistically homogeneous in both X and Z direction. So, the Reynolds average can be based on a combination of ensemble average and linear line average like this, where L_z is span wise length, L_s is stream wise length, alpha is sample number. So, the requirement for the ensemble averaging if they are uncorrelated sample, then the time interval between the consecutive scale.

Which, should be higher than the order of a l not by u not or statistical convergence to reach for u prime, so, the sample size could be quite large. So, essentially when you do this kind of sampling, you need to.

(Refer Slide Time: 16:39)



Then one can find out 2.1 time auto covariance for statistically homogeneous turbulence

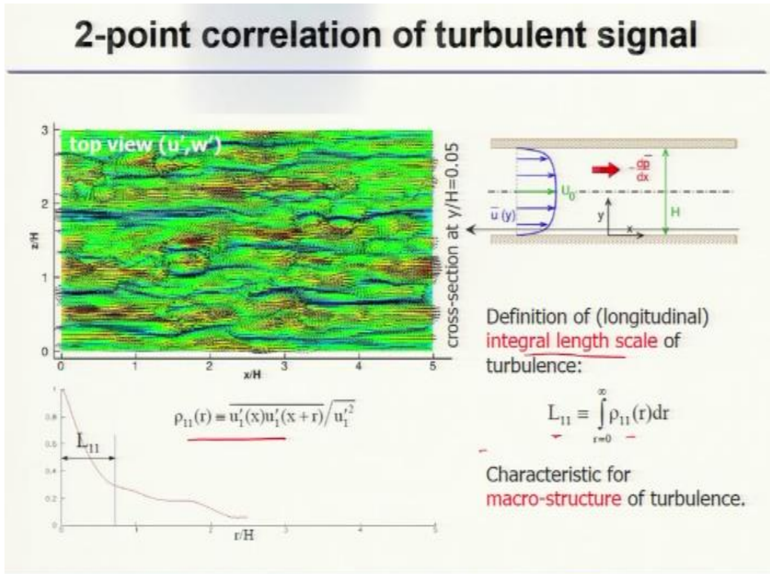
$$R_{11}(r) = \overline{u_1{}'(x)u_{\iota}{}'(x+r)}$$

Now, if you normalize with the value at R 0 they need to get you is

$$\rho_{11}(r) = \overline{u_1{}'(x)u_{\iota}{}'(x+r)}/\overline{u_1{}'^2}$$

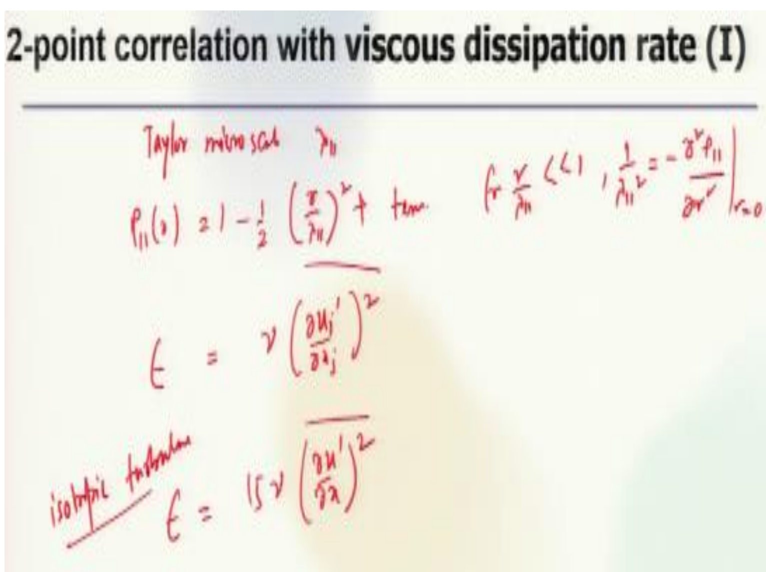
So, the properties that $\rho_{11}(0)$ is 1 $\rho_{11}(r)$ is $\rho_{11}(-r)$ this is symmetric then $|\rho_{11}(r)|$ the magnitude should be less than 1 and this tends to 0 for r tends to infinity.

(Refer Slide Time: 17:52)



For the said 2.1 time auto covariance for statistically homogeneous terminals, again if you look at the same calculation of the channel flow, you can see if you plot this normalized variable, this is how it actually vary and from here one can define the integral length scale, the integral length scale would be the integration of rdr. So, this is a characteristic for macro structure of turbulence where you get an estimate of your integral landscape.

(Refer Slide Time: 18:26)



Now, we can define another micro scale which is called Taylor microscale which is λ_{11}. Now, $\rho_{11}(r)$ is

$$\rho_{11}(r) = 1 - \frac{1}{2}\left(\frac{r}{\lambda_{11}}\right)^2 + term \qquad for \ \frac{r}{\lambda_{11}} \ll 1, \frac{1}{{\lambda_{11}}^2} = -\left.\frac{\partial^2 \rho_{11}}{\partial r^2}\right|_{r=0}$$

So, we can estimate the viscous dissipation rate which will be

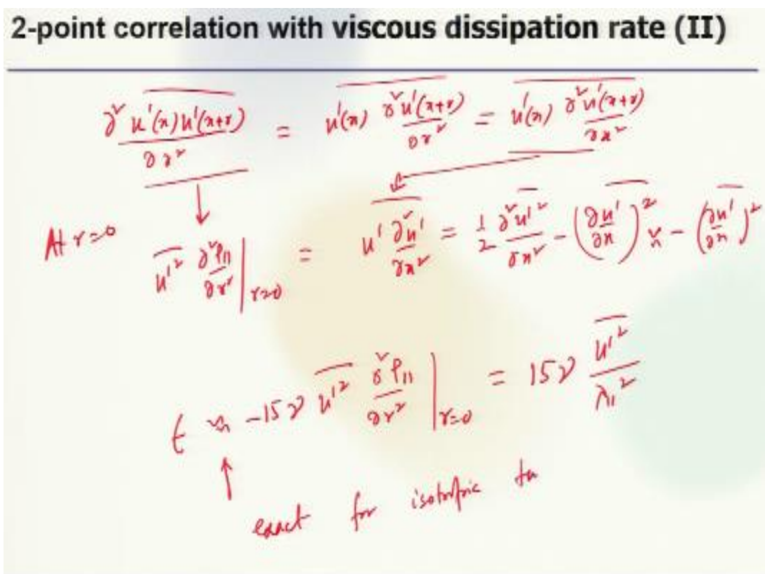
$$\varepsilon = \nu \left(\frac{\partial u_i'}{\partial x_j}\right)^2$$

Now, for isotopic turbulence, it is shown that the for isotopic turbulence it is shown by Pope that

$$\varepsilon = 15\nu \left(\frac{\partial u_i'}{\partial x}\right)^2$$

Now, the since dissipation occurs mostly at the small scale, so, they remain highly isotopic, which is as per the hypothesis. So, this is also valid for other turbulent flow as long as requisite high that means, it is in the turbulent journey.

(Refer Slide Time: 19:54)



Now, we can take the second order derivative of 2 point auto covariance. So, what that gives me

$$\frac{\partial \overline{u'(x) u_\iota'(x+r)}}{\partial r^2} = \overline{u'(x) \frac{\partial^2 u_\iota'(x+r)}{\partial r^2}} = \overline{u'(x) \frac{\partial^2 u_\iota'(x+r)}{\partial x^2}}$$

So, now at r = 0 these term would become

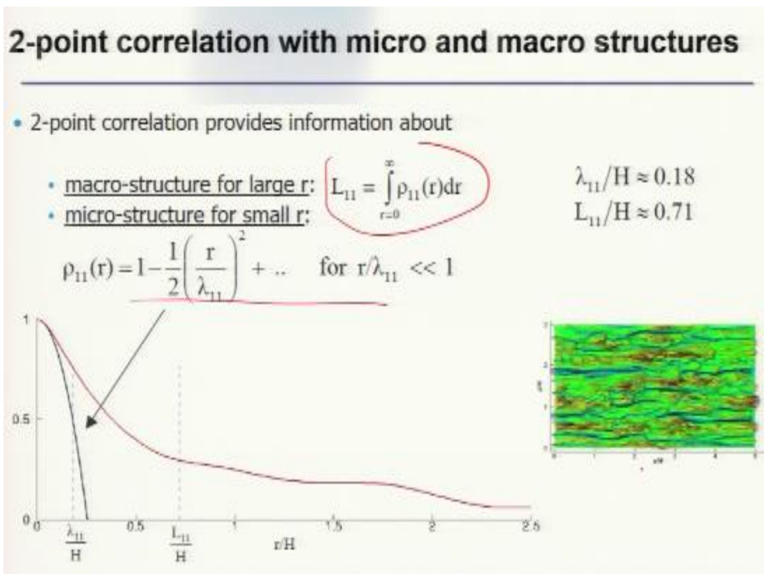
$$\overline{u'^2} \frac{\partial^2 \rho_{11}}{\partial r^2}\bigg|_{r=0} = \overline{\frac{\partial^2 u_\iota'}{\partial x^2}} = \frac{1}{2}\overline{\frac{\partial^2 u_\iota'}{\partial x^2}} - \overline{\left(\frac{\partial u'}{\partial x}\right)^2} \approx -\overline{\left(\frac{\partial u'}{\partial x}\right)^2}$$

Which one can approximate as the del u prime by del x square. So, which follows that dissipation is approximated as

$$\varepsilon \approx -15\nu\overline{u'^2}\frac{\partial^2\rho_{11}}{\partial r^2}\bigg|_{r=0} = 15\nu\frac{\overline{u'^2}}{\lambda_1^2}$$

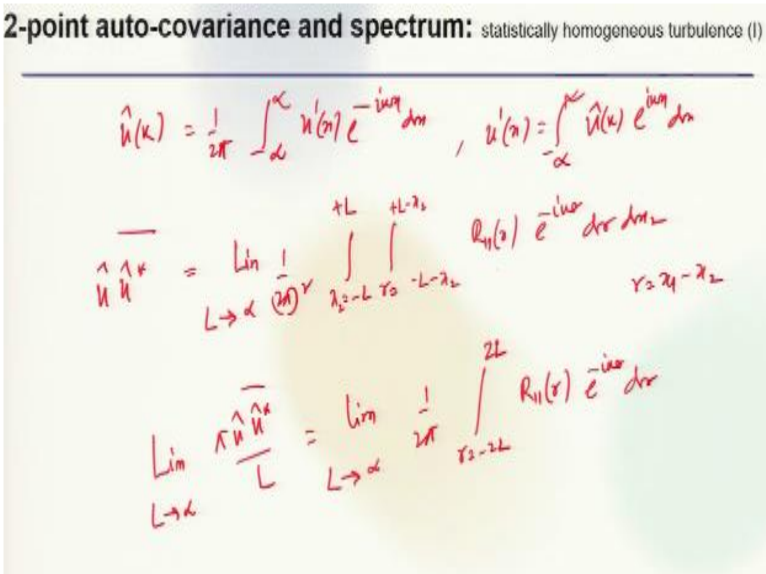
So, this is actually this approximation becomes exact for exact for isotopic turbulence so, this is what one can get.

(Refer Slide Time: 21:44)



Now, when you look at the 2-point correlation so, this provides the information about macro structure for large L which is the integral length scale and then microstructure are small r which is like this. So, if you plot these things, this is what it buries, this is again for that channel flow that we had this is I mean one can do this for.

(Refer Slide Time: 22:16)



Now, if you have a statistically homogeneous turbulence, so, we can look at the fourier transform pair of the velocity fluctuation

$$\hat{u}(k) = \frac{1}{2\pi}\int_{-\propto}^{\propto} u'(x)e^{-ikx}dx$$

$$u'(x) = \int_{-\propto}^{\propto} \hat{u}(k)e^{ikx}dx$$

Now, this, once multiply this fourier transform with the complex conjugate and use that flow for statistical homogeneous turbulence what we get actually

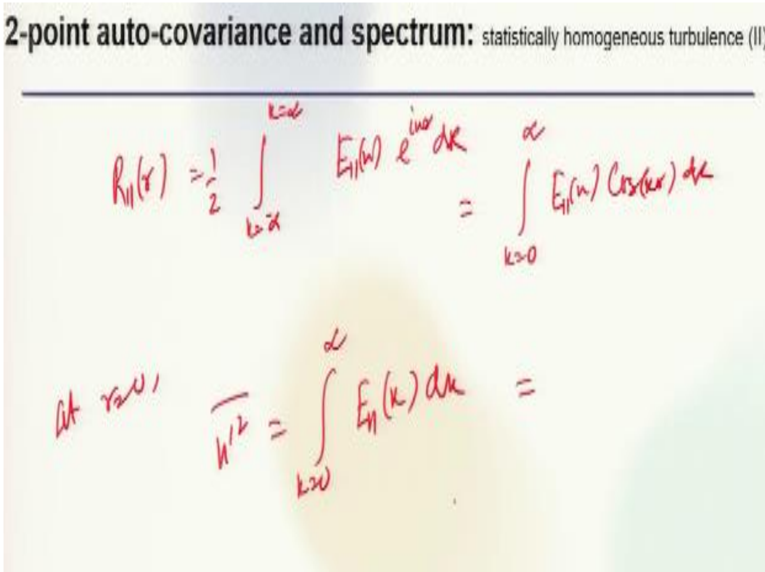
$$\overline{\hat{u}\hat{u}^*} = \lim_{L\to\propto}\frac{1}{(2\pi)^2}\int_{\lambda_2=-L}^{+L}\int_{r=-L-\lambda_2}^{+L-\lambda_2} R_{11}(r)e^{-ikx}dr\,dx_2$$

where r is $x_1 - x_2$.

So, we can use that R_{11} is symmetric and this can vanish so, our limit actually becomes when

$$\lim_{L\to\propto}\frac{\pi\overline{\hat{u}\hat{u}^*}}{L} = \lim_{L\to\propto}\frac{1}{2\pi}\int_{r=-2L}^{2L} R_{11}(r)e^{-ikx}dr$$

(Refer Slide Time: 24:25)



Now, the inverse furious transform what do we get

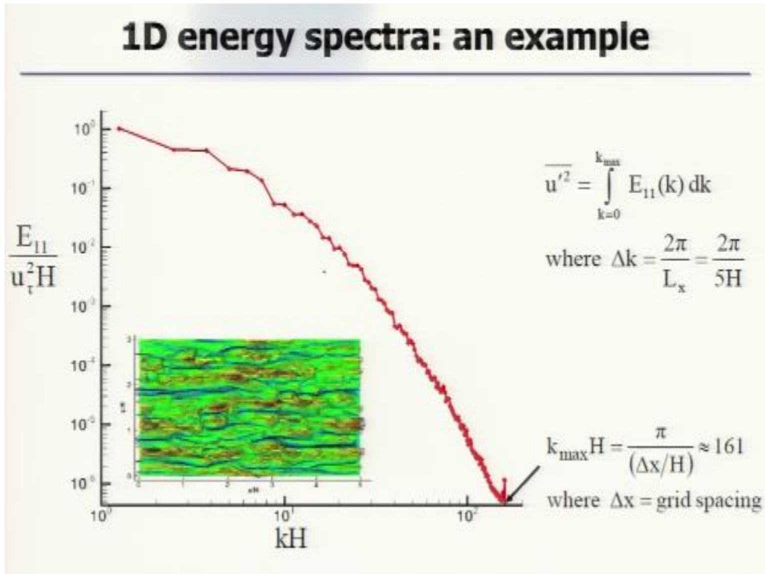
$$R_{11}(r) = \frac{1}{2}\int_{k=-\propto}^{\propto} E_{11}(w)e^{ikr}\,dk$$

which will get us back

$$\int_{k=0}^{\propto} E_{11}(u)\cos(kr)\,dk$$

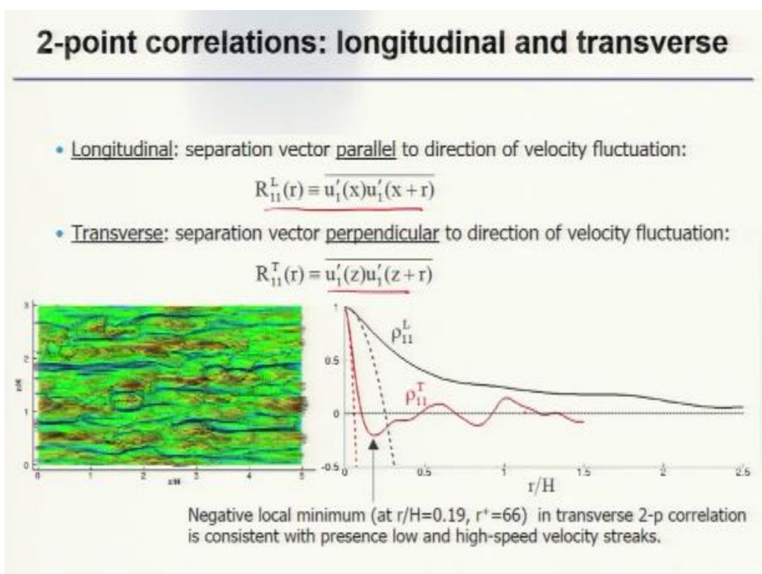
So, this is an energy spectrum E_{11} replace the distribution of energy over wave number space. So, now one dimensional energy spectrum is twice the fourier transform of the 2.0 auto covariance.

(Refer Slide Time: 25:18)



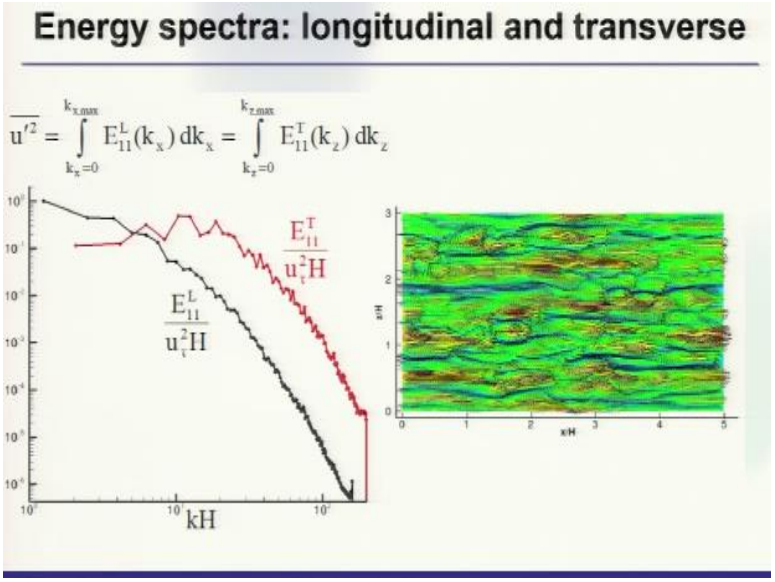
And that one can plot it for this channel flow and this will look like this. So, these are the estimate of that thing what we just derive u prime square would be the delta k will be 2 pi by length.

(Refer Slide Time: 25:33)



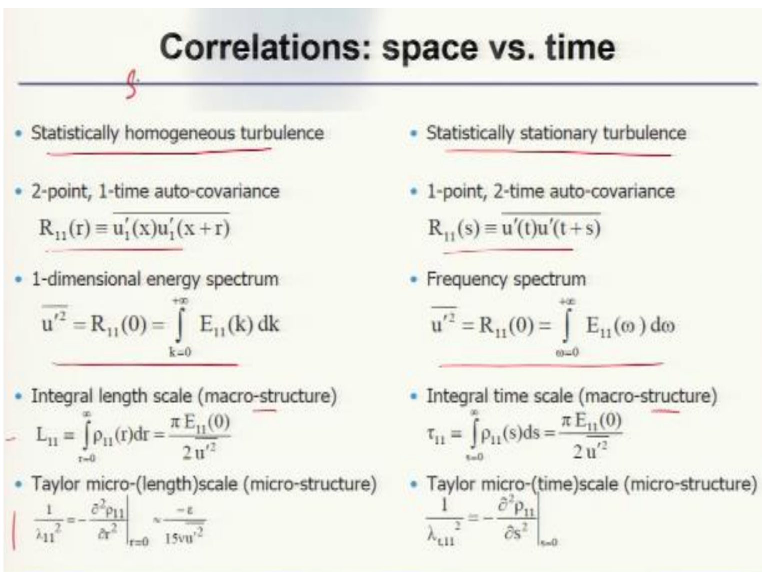
Now, similarly, you can have the longitudinal and the transverse variation. So, the longitudinal separation vector parallel to the direction of the velocity fluctuations where R_{11} is defined like this and then go to transverse this will be transfer which is perpendicular to the flow and there variation are different the longitudinal variations is like this where the transverse one initially quite steep and then it decays.

(Refer Slide Time: 26:10)



So, similarly for that particular case one can plot and look at it.

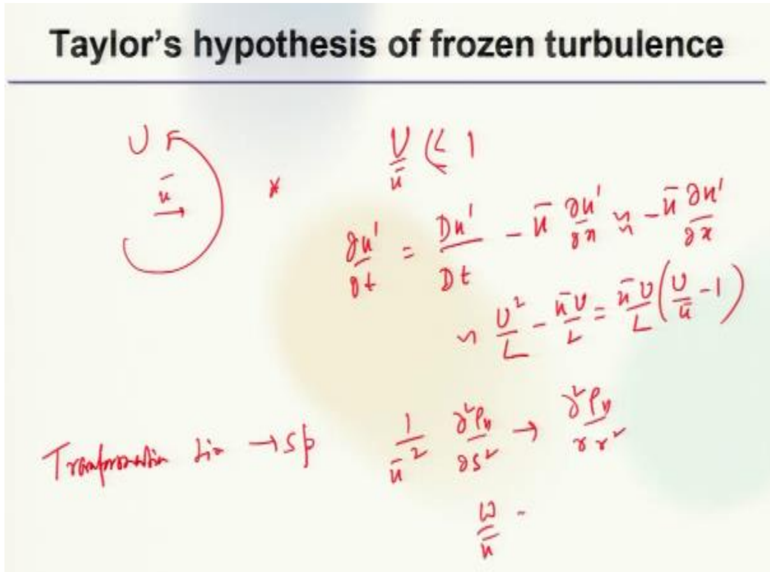
(Refer Slide Time: 26:18)



So, this if you combine things here so, one case you have a statistically homogeneous turbulence, you can have statistically stationary turbulence your 2.1 time covariance is calculated like this, 1.2 time covariance is calculated like that, because this is statistically stationary turbulence. Then one dimensional energy spectra will get you back the u prime square this frequency spectrum which will get you.

Then we can estimate the integral length scale which is a length scale of the macro structure and this is the time scale which will get where this is and then the Taylors micro scale and Taylor micro timescale. So, this is what it is in space and this is what it is in time. So, in space and time how we actually

(Refer Slide Time: 27:11)



Now, already we have the taylor hypothesis where you can have eddy size like that, where u bar is coming and this is of the large scale, then one can approximate the space time transformation. So, for low turbulent intensity where U by $\bar{u}$ is less than one you can get

$$\frac{\partial u'}{\partial t} = \frac{Du'}{Dt} - \bar{u}\frac{\partial u'}{\partial x} \approx -\bar{u}\frac{\partial u'}{\partial x}$$

$$\sim \frac{U^2}{L} - \frac{\bar{u}U}{L} = \frac{\bar{u}U}{L}\left(\frac{U}{\bar{u}} - 1\right)$$

So, in other words one can think that adequately admitted by mean flow. So, this is the measurement location where we look at says that he does not have time to change. So, the implication is that major time series is in fact a special series. So, the transformation time is to the space so,

$$\frac{1}{\bar{u}^2}\frac{\partial^2 p_o}{\partial s^2} \rightarrow \frac{\partial^2 p_v}{\partial r^2}$$

$$\frac{\omega}{u'} \rightarrow k$$

So, stop here and finish this one in the next lecture and look at the modeling aspect. Thank you.