Muffler Acoustics - Application to Automotive Exhaust Noise Control

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Lecture – 16
Muffler Performance Measures: Insertion Loss

Welcome back to the NPTEL course on Muffler Acoustics, this is week 4 and lecture 1. So, the objectives of this week's lecture today's lecture will be to understand what are the different metrics by which we understand the muffler is performing well or not that is to say muffler is doing its required job or not.

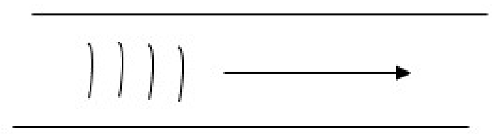
So, **Muffler Performance Matrix;** that will be our goal that will be our learning objective for today's lecture.

So, how do you decide whether the muffler is performing nicely or not? So, we have three parameters. And in fact, let me tell you something these three parameters are not just very sacred to the muffler to the muffler theory, but also in general these are used very widely in noise control engineering and in some way in different applications as well.

- **INSERTION LOSS (IL) dB**
- **TRANSMISSION LOSS (TL)**
- **LEVEL DIFFERENCE (LD)**

So, these are insertion loss also known as IL measured in dB everything is measured in dB. Transmission loss also expressed or measured in decibel or dBa if you want to put the filtering thing. And finally, level difference, level difference is relatively lesser of the lesser used quantity as compared to the above ones LD. So, all of them are measured in dB decibel.

So, insertion loss, transmission loss IL, TL and LD; we are going to understand we are going to study, what is meant by these three parameters. That is to say how the muffler theory is, how to evaluate the muffler performance based on these three parameters? Alright. So, before we go about doing that let us or probably just review the different expressions used for evaluating the acoustic power.



$$\tilde{p}, \tilde{U}$$

So, we know that if you have a duct and if you have say wave propagating along this direction and you have general say the pressure$\tilde{p}, \tilde{U}$.

So, we saw from I guess from the last weeks lecture week 3 the initial 1st or 2nd lecture not sure. So, we saw that the intensity is given by

$$I = Re(Z)\tilde{U}_{RMS}{}^{2}\frac{M_{Re}^{2}(Z)}{2\rho_0}$$

And another equivalent expression is

$$= \frac{\tilde{p}_{RMS}^{2}}{|Z|}cos\theta$$

So, well these are the two expressions that are used quite often, but then we do not know what U RMS is what p RMS is in directly in our front manner. Theta of course, being the phase between the pressure p tilde and U tilde, but an equally important representation that is used and from that we can derive a lot of expression is basically equals.

Let me write it in another.

$$I = \frac{1}{2}Re\{\tilde{p}\tilde{U}^{*}\}$$

So, here your $\tilde{U}^{*}$ is the acoustic particle velocity, not the volume velocity, not the mass velocity is just the acoustic particle velocity $\tilde{U}$. So, you get that from the Euler equation of course, and U what does the star mean what does this particular thing mean here? It means complex conjugate. So, p in we get the solution in terms of complex exponential.

So, $\tilde{U}^{*}$is the complex conjugate of U. So, let me make this thing clear here; U^{*} is the complex conjugate of U^{*}that is what it is. So, using this expression we can actually show this of course, is equivalent to

$$\frac{1}{2}Re\{\tilde{p}^*\tilde{U}\}$$

And you can keep manipulating to get different expressions.

So, we can show that the intensity for periodic signals over a given cycle can be expressed also in terms of

$$I = \frac{1}{tang}\int_0^{tang} Re\{\tilde{p}\}Re\{\tilde{U}\}dt$$

So, $\tilde{p}$ is obviously, ZU. So, we I mean if you simplify this thing further you will get omega by

$$I = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} Re\{Z\tilde{U}\}dt$$

So, well the point I am trying to make here is that, when you do all the math you will find the starred expression same as this one or probably triple star on this is the same.

So, the idea is that either we figure out the real part of pressure and velocity and integrate that and then find out the intensity or simply multiplying taking half of real part of p into U star complex conjugate will give us this. So, half happens because we are integrating over a period of time t average which is nothing but 2π/ω and the half factor actually comes because when you take the RMS you get,

$$\frac{1}{\sqrt{2}}|\tilde{p}|\frac{1}{\sqrt{2}}|\tilde{U}|$$

So, the half factor is basically for the time thing. So, I mean it can be shown I am not going into the details. So, once we get basically the idea is to make use of either the double star or the triple star expression shown here.

For evaluating the muffler performance, we will probably use this one and this thing then needs to be multiplied by the cross-sectional area to get the acoustic power. So, we will use quite frequently acoustic power carried by

$$\text{wave front} = \frac{1}{2\rho_0}\ Re\{\tilde{p}\tilde{U}^*\rho_0 S\}S$$

If we multiply this by the density and divide by density rho naught and then multiply this by cross sectional area and also divide the denominator by S that is a cross sectional area. So, we are going to get the acoustic power and this obviously, has to be multiplied by S further because Sis we are trying to do get this thing in terms of the mass velocity. So, basically this guy is the intensity, and this is the area cross section area. Well, we saw that the acoustic power carried by a wave front then is this thing.

So, what we probably can do is that; represent this guy here we recognize that this all these terms combined represents the acoustic mass velocity. So, we will simplify that eventually and these things and this thing cancel because you know acoustic intensity times the area is your power.

$$P = I.S = \frac{1}{Z\rho_0} Re\{\tilde{p}\tilde{U}^*\}$$

$$TL$$

So, this is what it is real part of this thing.

So, we are going to use this expression as well as the some of the previous expressions for evaluating the muffler performance insertion loss or transmission loss. Especially this is the one that I guess we will be making use of and another one the specially for doing the TL calculations and also these 2ones. Or basically the insertion laws actually this can be simplified once, we recognize that

$$\tilde{p}_{RMS} = \frac{1}{\sqrt{2}}\tilde{p}$$

$$-\frac{1}{2}\frac{|\tilde{p}|^2 cos\theta}{|Z|}$$

So, once we do that, we get this, similarly we can also get

$$\frac{|U|^2}{2\rho_0} Re(Z)$$

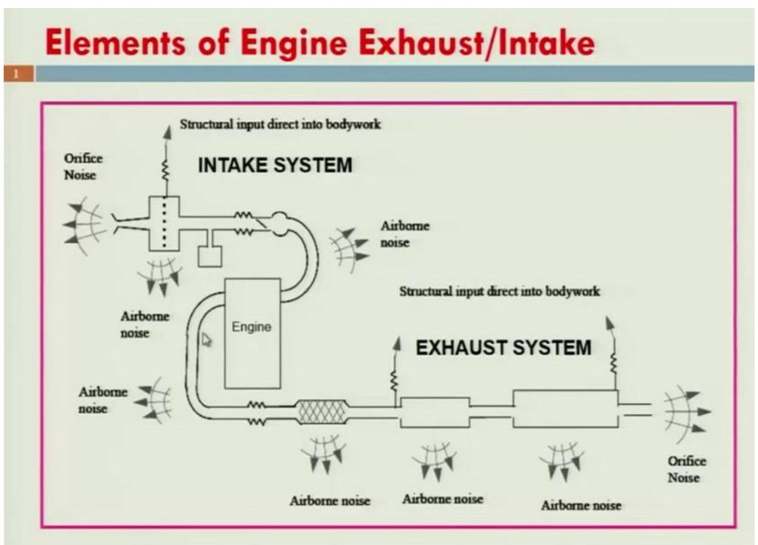
here denominator can be density and S will be multiplied in the numerator to; so this can be actually the mass velocity. The point is we will use these expressions for the insertion laws and particularly this one for the transmission loss computation so, with this background or recap of the acoustic power relations with pressure and velocity.

I guess we are in a good position to talk about the insertion loss the development of expressions for computing the insertion loss transmission loss and so on. So, let us begin with the IL first. So, what is IL?

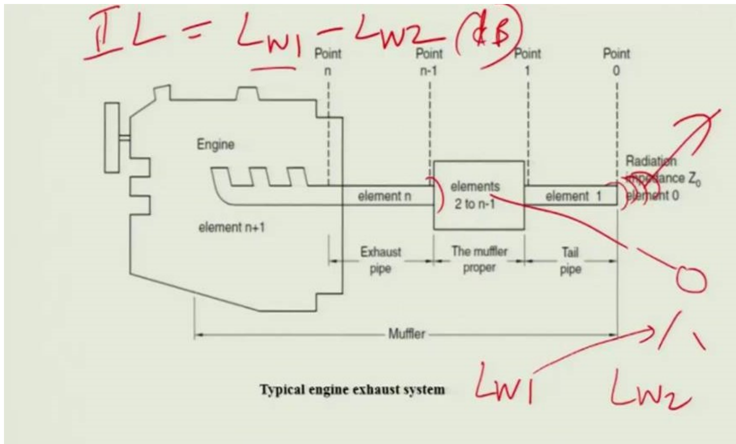
$$IL \rightarrow \ ?$$

So, insertion loss is defined as the acoustic power radiated without any filter and with the filter. So, is the difference of the logarithm of the acoustic power that is radiated without the filter and when the filter is in place.

So, basically this is your say in the engine block and we saw that, we saw the slide a while back.



So, this is just a thing with this. So, this is your engine block another representation can be perhaps this one. So, we see this is the engine which is making a lot of noise and this is the exhaust system or the muffler system; let us say this is not in place.



Typical engine exhaust system

Or I would say this entire thing what we are seeing here it is not there. Only the engine is making noise, and this is going away. And it is almost unbearable because this without the muffler engine does produce a lot of noise and that is why the mufflers are in place.

So, then we decide to put the muffler on that is your have all the complicated elements it could be much more complicated things and the noise emission is reduced that is to say. Now, when you are standing here and listening. So, now, it is you can manage you can it is still tolerable. So, basically then you say that well the muffler is doing its job it is well designed muffler and you are happy. How do you define that how, what is the quantity?

Suppose when this was not there this was not there and only this part was there it was making lot of noise. So, we will call this part as the L W_1. That is basically the acoustic power remember W; W was used to denote the acoustic power and SPL was used to denote the sound pressure level.

So, L W_1 means when the muffler is this thing is crossed order it is not there. So, L1 is the acoustic power that is being emitted out and going off in the atmosphere without this thing. Now, when you have got the muffler in place the acoustic power comes out the waves come out here and you record this note that this L W_1 is to be recorded at the same place.

So, when the muffler is not there, then you record it at a certain distance from this thing and. When the muffler is there you still should record the noise at this point and then it is given by LW_2 when the muffler is on. So, now, basically the difference of the insertion loss then is defined as

$$IL = L_{w_1} - L_{w_2}$$

So, when the muffler is on it produces certain power and when the muffler is not there it is off, then it produces certain power L W_1 and when the muffler is on when it is put on it is doing what is supposed to do.

Then it reduces a different sound acoustic power level and that is giving by L_{W2}. So, the difference of that is called insertion loss L_{W1}- L_{W2} alright this of course, is in measure in dB it may. So, turn out that for certain frequency spectrum insertion losses is a significantly large quantity.

That means, in that frequency spectrum the muffler is doing its job nicely and in some frequency range insertion loss can be negative also. Which means that the muffler is counterproductive over that frequency is creating more noise. So, insertion loss at the point to be noted here is that the insertion loss can be negative also.

So, basically IL is then given by,

$$IL = L_{w_1} - L_{w_2}$$

$$= 10\, log_{10} \left|\frac{w_1}{w_2}\right|$$

So, because W was used to denote the acoustic power radiated,1 and 2 means with and without the filter or the muffler filter means the muffler.

FILTER = MUFFLER

$$IL = 10\, log_{10} \left\{ \frac{\rho_{0,2}}{\rho_{0,1}} \frac{R_{0,1}}{R_{0,2}} \left| \frac{\tilde{p}_{n+1}}{Z_{n+1} + Z_0} \right|^2 \right\}$$

Let me also clarify muffler is defined filter is defined as the muffler right. So, now, we will make use of the expressions that we talked about.

So, I guess we probably have to go back to the presentation and actually incidentally to the; to this form and talk about the acoustic power radiated. So, when this guy is not there you will use the pressure form or the Theremins' form and what is the acoustic power radiated when this guy is not there.

So, that is so when the muffler is not there then the circuit just becomes p; $p_{n+1,}$ it goes like this then there is a load here and it goes like this alright. The point I am trying to make is that what is the current passing through that or what is the mass velocity.

$$\tilde{V}_n = \frac{\tilde{p}_{n+1}}{Z_{n+1} + Z_0}$$

so that is what it is. Now, what is the power? Since we know this so we just derived an expression for the acoustic power radiated or carried away by the wave fronts, when you know the amplitude of the velocity. In terms of the acoustic mass velocity is given by

$$\frac{1}{2\rho_0}\left|\frac{\tilde{p}_{n+1}}{Z_{n+1} + Z_0}\right|^2 R_{0,1}$$

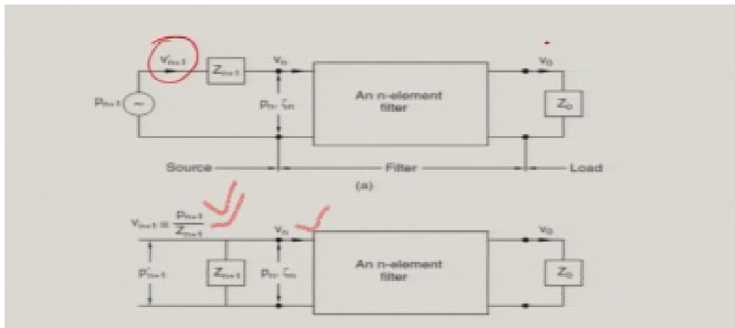
So, this is the velocity V is something like in the factor of 2 comes because of the RMS sign 1 by root 2 multiplied by 1 by root 2.

And rho naught is a density because you are working with a mass velocity and S of course, will be intrinsic to that it is multiplied and we need to take the real part of the impedance. So, real part of the impedance means its $R_{0,1}$.

So, what does $R_{0,1}$ means? $R_{0,1}$ is there a component, or the resistive component please keep reminding yourself real component of the impedance is the resistance that will that is always going to dissipate the energy or if you evaluate that over a cycle it is always going to be nonzero of the real part or the resistive part is nonzero.

Real part real component of the radiation impedance or the atmospheric impedance radiation impedance $Z_{0,1}$. So, $Z_{0,1}$ is the same as Z_0; well, we are just introducing terms like or terminologies like comma 1, basically to talk about the fact that the muffler is not there and with 2, when the muffler is there like I have been emphasizing on that. So, $\frac{1}{2\rho_0}$ just thing this is the acoustic power that is radiated when the muffler is not there just read it to the atmosphere.

And it is a simple thing then to know or to appreciate that when the muffler is there, we need to figure out the velocity that goes on. So, when the muffler is there then let us say we need to make use of the this forms we need to make use of the Norton's form to evaluate the velocity that goes on that comes here. So, when using the Norton's form, we can evaluate the velocity that goes on in this part and we can figure out that.



A filter with its terminations. (a) Pressure representation for the source. (b) Velocity representation for the source

When the muffler is there then we use the Norton's form and to find out V_0 making the shunt impedance thing. We know p_{n1} and we know is the source impedance source characteristics which are p_{n+1} and Z_{n+1} quite difficult to evaluate analytically or through it has to be done through experiments.

But if you want to find out predict the insertion loss; you first need to know a priori what the source characteristics is we are going to mention about that. So, V_{n+1} is known because this is known this is known. And now V_{n+1} is this thing and then V_n can also be figured out because the this thing gets split into this part and this part.

And then from once we know V_n we can evaluate V_0 and. So, that was actually done previously. So, what we probably can do now is to just simply make use of the expression of once we know assume the things are known in terms of V_0. So, the power that is radiated when the muffler

$$w_2 = \frac{1}{2\rho_{0,12}} |V_0|^2 R_{0,2}$$

So, $2\rho_{0,12}$ might be slightly different based on the temperature of the exhaust gases, but we can assume them to be the same for just for simplifying our analysis and this is square times $R_{0,2}$.

So, where again R_{02} is the radiation impedance or the atmospheric impedance here, when the muffler is misplaced. So, write down the expression for the insertion loss which is nothing but

$$IL = 10\ log_{10}\left\{\frac{\rho_{0,2}}{\rho_{0,1}}\frac{R_{0,1}}{R_{0,2}}\left|\frac{\tilde{p}_{n+1}}{Z_{n+1}+Z_0}\right|^2\frac{1}{|V_0|^2}\right\}$$

So, we get this particular expression for the insertion loss.

So, here we get this thing I guess probably I was missing a term here. So, in the denominator we probably have your V naught square.

$$= 10\ log_{10}\left|\frac{W_1}{W_2}\right|$$

And so once we simplify this expression what we are going to get is

$$IL = L_{w_1} - L_{w_2}$$

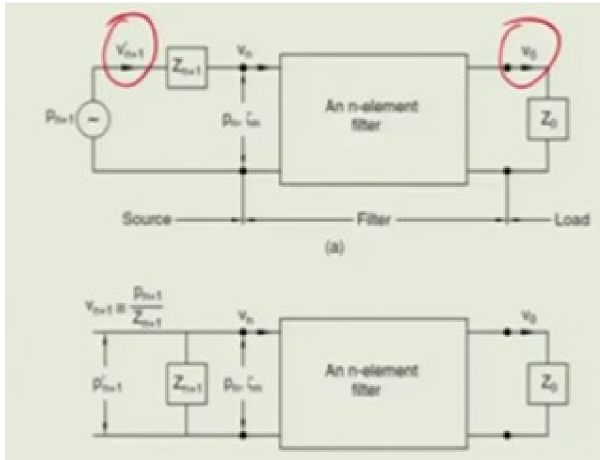
$$20\ log_{10}\left[\left(\frac{\rho_{0,2}R_{0,1}}{\rho_{0,1}R_{0,2}}\right)^{\frac{1}{2}}\left|\frac{Z_{n+1}}{Z_{n+1}+Z_0}\right|\left|\frac{\tilde{V}_{n+1}}{\tilde{U}_0}\right|\right]$$

So, this is square was if you recall I mean this there was a square here. So, we are taking the square there and, but the important thing is that

$$\tilde{p}_{n+1} = Z_{n+1}\tilde{V}_{n+1}$$

So, this is what we did. So, we replace this thing using the velocity representation or the Norton's form this was possible using that, we made use of Norton's representation.

So, the specialty or the peculiar thing about this expression is that it relates the insertion loss in terms of what is popularly known in classical muffler analysis is the velocity ratio. So, V_{n+1} is the velocity acoustic mass velocity right at the upstream.



A filter with its terminations (a) Pressure representation for the source. (b) Velocity representation for the source.

$$\left|\frac{\tilde{V}_{n+1}}{V_C}\right|$$

So, V_{n+1} so suppose if you have no matter how complicated muffler you have all you need to do is that find out the ratio of V_{n+1} and V_0. These things once you once you can evaluate that using some analytical planar wave modeling. We are going to do that ensuing lectures; then I guess one can easily figure out the insertion loss.

So, what we need to do is that we need to know often this assume that

$$\rho_{0,2} = \rho_{0,1}$$

$$R_{0,1} = R_{0,2}$$

We can assume that without too much of without making the systems too much complicated. So, as a result what happens is that you this entire term is evaluated to be 1. And insertion loss can simply be written as

$$IL = 20\, log_{10}\left[\left|\frac{Z_{n+1}}{Z_{n+1}+Z_0}\right|\left|\frac{\tilde{V}_{n+1}}{\tilde{V}_0}\right|\right]$$

So, this is what we are going to get and this is called this is known as VR_{n+1} or simply VR which is a velocity ratio.

So, now, in the hypothetical case where you have 0 temperature gradient. I am saying hypothetical because well, as a gas hot gases flow throughout out of the exhaust system its temperature drops. But for you know most of the analysis in muffler is carried out for 0 temperature gradient, we assume a constant or uniform sound speed.

If you recall sound speed was root over R $\gamma.R_0$into temperature in the absolute Kelvin scale. So, for hotter gases sound speed increased and for as the gas is cooled down towards the tail pipe its temperature reduces and the so does the sound speed is directly proportional to square root t.

So, but of course, lot of analysis assume a constant sound speed. So, that is what in fact, we are going to do in this course without complicating matters. And area S and we often assume that the cross-section area of the exhaust pipe and the tail pipe are the same.

$$S_n = S_1$$

So, with all these assumption it can be shown that if you have these things in a constant pressure source.

$$Z_{n+1} \rightarrow 0$$

$$\rightarrow Z_{n+1} + Z_0$$

So, when this term tends to 0 the source impedance tends to basically tends to 0. So, under such a condition we can still combine Z_{n+1} and Z_{n+1} and this term can be combined here. So, basically what happens is that the insertion loss becomes

$$IL = 20\, log_{10} \left| \frac{\tilde{p}_{n+1}}{\tilde{p}_0} \right|$$

atmospheric pressure and this is what we are going to get.

$$10\, log_{10} \left| \frac{\tilde{p}_{n+1}}{p_0} \right|^2$$

So, in fact, we will see and this actually can be written in yet another manner which is nothing great about it is just another representation the square 1of course, square times.

So, basically for a constant pressure source that is if the source impedance is 0. If you just measure the acoustic pressure without the muffler on and with the muffler on at a given location. So, p_{n+1} and p_0 and well take this ratio. So, we are going to get the actually this will be n, I am so sorry.

So, then we are going to get the insertion loss and this will be shown to be equal to the level difference in the limiting case. Actually, another thing is that if the source impedance in this case insertion loss will be equal to level difference which I am going to talk about soon and transmission losses yet another quantity.

So, before we will probably talk about TL and LD in the next lecture; lecture 2 of week 4. But one thing a couple of comments about insertion loss is that in unlike transmission loss insertion loss is easier to measure.

Basically as will soon appreciate is for insertion loss we just need to measure the pressure without the muffler on at a given location put the muffler on and measure the pressure again at that point and then taking the ratio of that for a constant pressure thing we can evaluate. We can actually experimentally get the insertion loss.

However, I experimentally measured IL in dB or dBa. However, if we were to analytically or well through predict through some numerical finite element scheme once we know the characteristics of the muffler. If we can relate the upstream and downstream variable through some numerical modeling and well evaluate the four pole parameters, we will talk about that sometime later.

And once we know that and we want to evaluate the insertion loss through some numerical scheme predict that. Then it is a much more complex task for the simple reason because this we need to know the source characteristics p_{n+1} and Z_{n+1} or p SZS.

So, if we basically want to start from the analytical principles or some numerical thing not only we need to do the muffler wave propagation modeling within the muffler. We also need to evaluate or know the source impedance and the source pressure. Fortunately, lot of work has been done in this field.

And there are some empirical expressions that are well known to us you which we can use in the codes to numerically predict the insertion loss. So, another quantity is the transmission loss which will turn out that it is much more difficult to experimentally measure compared to insertion loss.

But easier to predict using set of routines you know analytical modeling or numerical FE based modeling and so on. It is best that we talk about TL and possibly about LD level difference in the next lecture. So, next is a transmission loss what we are going to talk about in the next class. Next week that is lecture 2 of week 4; so, stay tuned for that.

Thanks.