Muffler Acoustics - Application to Automotive Exhaust Noise Control

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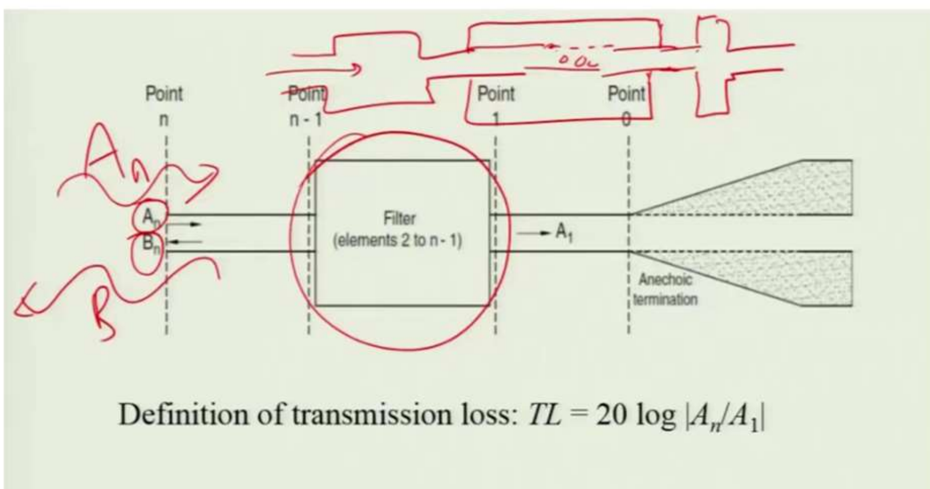
Lecture – 17
Muffler Performance Measures: Transmission Loss & Level Difference

Welcome back to the NPTEL course on Muffler Acoustics this is week 4, lecture 2.

Well the contents for today's lecture, this lecture 2 is

- **TRANSMISSION LOSS (TL)**
- **LEVEL DIFFERENCE (LD)**

The development of expressions to evaluate transmission loss and level difference for a general muffler system. To facilitate our discussions for the transmission loss I would like to go to the presentation just a schematic here, where you show the figure here.



So, what do we see here? Well, we see a wave, that is coming well towards the muffler directed towards the muffler or coat and coat incident on the muffler, which is A_n and the wave front that propagates in the opposite direction.

That is basically reflected from the muffler back into the engine system, which is your B, B_n. Now, this is the point n-1, this is 1 and this is your muffler proper. This is called the muffler proper, which can be well symbolically it could have any number of elements, which are sequentially connected something like this.

You can have inlet extended inlet and outlet, something like this. And, you can have, you know perforates as well. And, you can have any in short chambers or pointers is sequentially connected, that is a there is a unique or uniform or single direction in which the acoustic power propagates on the one end of the engine to the other end, the atmosphere is there. Any ways what I am going to do now is basically develop the generalized expression for the transmission loss.

So, when a wave is incident on the muffler some part of course, is reflected back. And, then transmission loss is defined, it is independent of the source parameters that is the source characteristics and the impedance. What is transmission loss? As the name suggest it is the acoustic power it is defined, as the ratio of the acoustic power that is carried by the wave fronts that are incident on the muffler to the acoustic power that is transmitted into the anechoic termination.

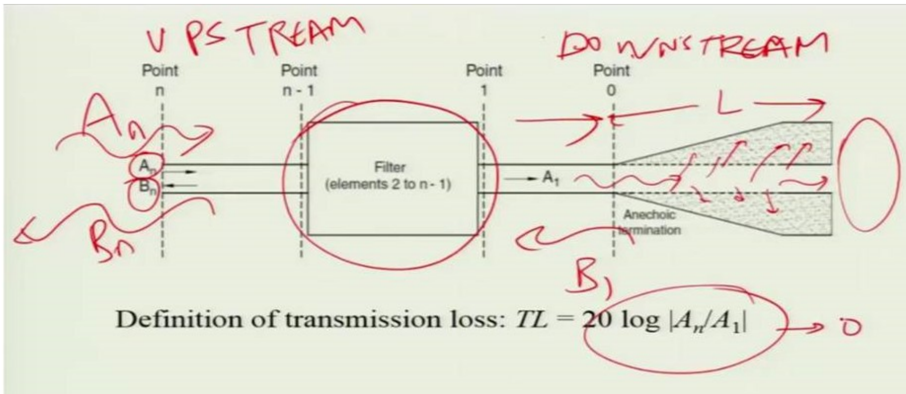
Now, here lies the crucial difference between the transmission loss and insertion loss. So, in insertion loss the tube the outlet pipe was simply exposed to the atmosphere. So, the radiation impedance was in play. Right now by anechoic termination it is a special kind of a termination.

As we are seeing as we discussed in our probably the very first lectures of the course, what we saw that, there is a wave this A_1 wave it continues to travel infinitely you know along this direction, there is no reflection, that goes on here there is no reflection.

So, why is this is happening? Because you see this special kind of a arrangement or a device that is fitted. So, this is more like a I do not have much space here but imagine it to be the cross section view something like this. And the annular part is filled with some sound absorbing or a dissipative material.

So, and of course, basically what is happening here is that, you have a conical pipe, which is more like a frustum of a cone and it extends for sufficiently large distance. And, it is filled the annular material here is filled with dissipative or sound absorbing material and they are backed by a perforated tube.

So, basically the as the waves acoustic energy acoustic power traverses in this uniform pipe, it gets a chance to interact or escape through the holes to the annual cavity, where the acoustic power is completely dissipated. So, the larger the length the more will be the attenuation as it goes at this end. And at some end of course, it has to meet the atmosphere.



So, but the trick here is that to make it of sufficiently large length. So, that by the time the acoustic power reaches here most of it is dissipated and there is nothing left to be reflected. It is sufficiently dampened or the amplitude has died down a lot.

So, this anechoic termination basically it is a kind of a conical arrangement frustum of a cone arrangement filled with the annulus of which is filled with dissipative material with the aim to absorb sound. And, so, that there is no reflections by the time the wave reaches the other end.

So, it is job then is to prevent the wave's B_1 that would have propagated or reflected back into the muffler from the open end as was the case when the termination was exposed to the atmosphere. So, this anechoic termination serves the purpose of allowing only a unidirectional or one this direction propagation of waves and not allowing the waves to come inside. But, on the upstream so, this is the downstream part downstream thing this is the upstream.

So, it basically the idea is to ensure that, whatever waves are incident some part was reflected back and that is crucial, if nothing is reflected back and everything is propagating downstream, then the muffler is useless. Because, it is not doing it is job properly. On the other hand if B is nearly the same as A and A_1 is tending to 0, in such arrangement then muffler is very effective at that frequency, it is doing it is job very well.

So, and; obviously, in the first case that I have discussed where A_1 is nearly the same as A_n then of course, transmission loss will tend to 0, the muffler is acoustically transparent or useless. We will see how we got to this stage.

$$P = I.S = \frac{1}{2\rho_0} Re\{\tilde{p}\tilde{V}^*\}$$

$$IL \quad \rightarrow \ ?$$

We will now make use of ourthis expression for transmission loss. So, this is our friend we will use it.

$$TL = \frac{1}{2\rho_0} Re\{\tilde{p}\tilde{V}^*\}$$

$$\tilde{p} = \left(\vec{A}_n e^{-jk_0x} + B_n e^{jk_0x}\right) e^{j\omega t}$$

Now, we are interested in finding out the acoustic power that is transmitted by this wave. So, what is that?

$$\frac{1}{2\rho_0}\{A_n e^{-jk_0x}$$

And, what is we will just consider this circle term. And, what is the mass velocity, let us first derive it. So, mass velocity from Euler equation if we do it, we are going to get

$$\rho_0 j\omega U = -jk_0\, A_n e^{-jk_0x}$$

$$A_n^* e^{jk_0x}$$

$$\tilde{V}^* -$$

$$\rho_0 \tilde{U}^* S$$

$$= \frac{S}{C_0} A_0^*$$

$$e^{+jk_0x}$$

So, when we write down this thing, it will become or probably let me use

$$\frac{1}{2\rho_0}\{A_n e^{-jk_0x}\frac{A_n^* e^{jk_0x}}{Y_0}$$

And remember we have the mass velocity here. Also, have to do is that multiply this thing by ρ_0 and multiply further by S of the cross-sectional area so we get S here. So, this would mean this is nothing, but v tilde star is equal to this.

$$\tilde{V} = \qquad Y_0 = \frac{C_0}{S}$$

$$\rho_0 \widetilde{U}^* S$$

$$= \frac{1}{Y_0} A_n^* e^{+jk_0x}$$

So, here in the denominator we are going to get this.

$$W_{inc} = \frac{1}{2\rho_0 Y_0}|A|^2 \; |A|^2 = A.A^*$$

$$W_{reflecte} = \frac{1}{2\rho_0 Y_0}|B|^2$$

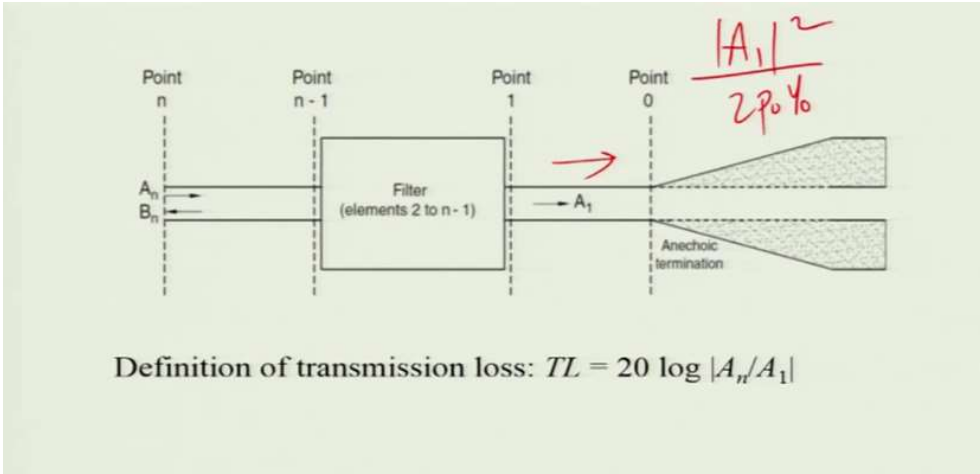
Now, couple of things here if we were to directly plug on the expression for p and derive the complex well a little bit formidable expression for particle velocity and get the mass velocity expression where have it has both the terms A and B. Then we do all this business of half of rho naught times p into v star complex conjugate.

So, what we probably would get something like this. The same expressions like these ones, but we will get something like this thing.

$$\frac{1}{2\rho_0 Y_0}\{|A|^2 - |B|^2\}$$

That the power that is carried away by the forward propagating wave and the acoustic power that is reflected back. And the by doing some algebra complex analysis, the cross terms will cancel out, well this is just to verify our computations.

Similarly, I mean if you let me just go back to the presentation and we see here the wave that goes here the acoustic power is nothing.



So, here of course, the duct area assumes to be constant they need not B, but what we can do now is get back here.

$$W_{inc} = \frac{1}{2\rho_0 Y_n}|A_n|^2 \; |A|^2 = A.A^*$$

$$W_{inc} = \frac{1}{2\rho_0 Y_n}|B_n|^2$$

And, write down the expression for power transmitted in the downstream, which is this is let me write this

$$W_{trans}\frac{|A_1|^2}{2\rho_0 Y_1}B_1 = 0$$

So, by the definition of transmission loss let me write in big fonts, because that is what we are going to use for a large part of our course to evaluate the performance of a muffler I am going to talk about that shortly. We defined as acoustic power incident divided by acoustic power transmitted.

So, it is pretty simple to then figure out that, this then becomes rho naught rho naught gets cancelled and what we are essentially,

$$S_n = S_1 = 10 \; log_{10}\left|\left|\frac{A_n}{A_1}\right|^2 \frac{Y_1}{Y_n}\right|$$

left with is A_n by A_1 mod square and in the numerator you will probably have so, this is. So, generally we often have the thing that the inlet and outlet pipes are the same.

So, $Y_n = Y_1$because the cross-section area the of inlet upstream pipe is the same as the downstream pipe. In such a case

$$TL = 20\ log_{10} \left|\frac{A_n}{A_1}\right|$$

$$A_n = A_1$$

So, this is as simple as that transmission loss there is 20 times the logarithm of the incident wave amplitude, ratio of the incident wave amplitude to the amplitude or the wave that is transmitted downstream into the anechoic termination.

So, computationally there is no difficulty in the sense that if we were to do a proper analysis of this muffler part by some you know 3D consideration,1D considerations or numerical 3D schemes, or even experimentally. Experimental is a different story I am going to talk about that briefly. But, if you were to do it analytically or numerically, then transmission loss is perhaps the easiest to evaluate. There is a insertion loss is the most difficult, this is what it is.

Now couple of things here so, is there any limiting condition under which transmission loss and insertion loss are related. Well it turns out, that well in the limit where p_1 is equal to the pressure in the downstream port, where you have anechoic termination.

$$p_1 = A_1 = Y_1 v_1$$

$$Z_0 = Y_1$$

that is the atmospheric impedance is the characteristic impedance of the tube; this is a very important condition. And, the upstream part that is the upstream wave that is incident is given by simply

$$An = \frac{p_n + Y_n V_n}{2}$$

So, then what happens is that, the equation that we just derived, now here let me call it star TL would become in the limiting case

$$TL = 20\ log_{10} \left|\frac{p_n + Y_n V_n}{2 Y_1 V_1}\right|, \qquad Z_0 = Y_1$$

using this relation this is simple to see, how do we get it. So, basically what we will get is this is this thing.

$$TL = 10\ log_{10}\left|\frac{W_{inc}}{W_{trans}}\right|$$

So, as the progressive wave does not undergo a change in the amplitude, while moving across a uniform tube A_n and A_1 can be measured anywhere, in the respective pipes. So, basically this thing is can also be written as

$$20\ log_{10}\left|\frac{p_n + Y_nV_{n-1}}{2Y_1V_1}\right|$$

So, Z_0 is of course, given by this thing. So, we will see that transmission loss then is the limiting case of the general case of insertion loss when the atmospheric impedance is replaced by your anechoic termination. So, now, the thing is like I was mentioning transmission loss is probably the easiest to compute of the 3 parameters, I L,LD and TL and, but experimentally it probably is the most difficult to do, why? Because of the necessity of anechoic termination.

Before, even we begin to attempt the measurement of transmission loss, it is a good idea to talk about the development of a suitable anechoic termination facility. That is a basically a tube with which would attenuate wave. So, it has to be filled with like I said rock wool glass with some dissipative stuff, and it is reflection coefficient needs to be checked, absorptivity should be high, for a given length.

So, then we can use that anechoic termination and use certain experimental techniques to figure out the acoustic waves that is transmitted downstream. And, but suppose if even if you do not have anaerobic termination it is still possible to measure transmission loss. But it will require the development of more complicated well more advanced experimental techniques names as 2 source method or 2 load method.

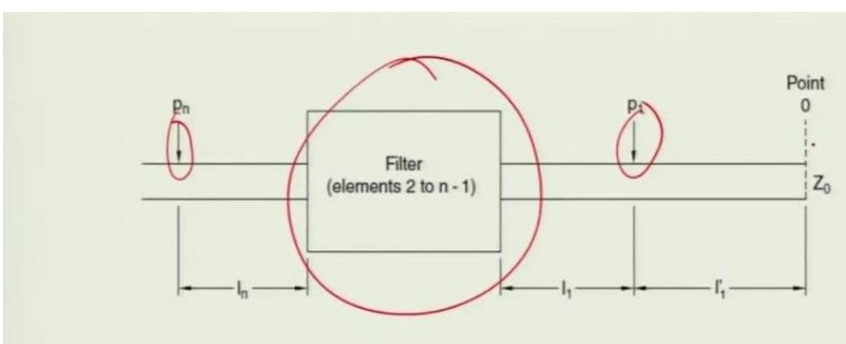
I am not going to discuss about that now just yet, but just sort of mentioning that TL is a little bit more difficult to measure experimentally ok. That is about the transmission loss. But one thing is their TL really tells you, how good or how bad a muffler is. Unlike insertion loss transmission loss can never be negative, why? Because, it is really the ratio for at least for a conservative system, there are no sources within a mufflers chamber then transmission loss is always positive.

In the worst case or in the minimum thing TL can be 0. Now, this is something that I did not discuss I should have this when this quantity is equal to 1, that is $A_n = A_1$ so the TL, then becomes or tends to 0, that is the muffler is transparent I was as I was briefly mentioning. And for a large A_n/A_1 value TL is high; that means, very good performance.

Now, the thing is transmission loss is probably one parameter that is completely independent of the source characteristics ps and zs. So, if you have a certain source and if you want to design a muffler without worrying much about the source characteristics and you want to compare you know 2 or 3 or more different muffler sets, which one is the best suited as far as noise attenuation is concerned.

So, one can actually evaluate I mean develop some computational schemes or analytical schemes to calculate the transmission loss performance. And, compare across different models different configurations. So, that would give us a good idea of the sound reflecting capability of a given muffler or a sound absorbing capability, in summary the noise attenuation capacity of a muffler without worrying much about the source characteristics.

Now, finally, we come to the another part of the top level difference. So, level difference or the noise reduction is also known as noise reduction. That is probably the most perhaps the simplest to experimentally evaluate. Basically is the difference in the sound pressure levels at two arbitrarily selected points.



So, if we have again a complex muffler element and if we were to take a microphone and measure the pressure inside the duct at certain point and certain point here. So, the basically just LD, that logarithmic difference LD is

$$LD = 20\ log_{10} \left|\frac{p_n}{p_1}\right| dB$$

That would give us the level difference.

So, unlike transmission loss the definition of level difference does not make use of standing wave pressures, and does not require a anechoic termination. And, for the purpose of calculations; however, $p_1$may not be known directly. So, we can develop some suitable relationships and evaluate this p_1 in terms of the acoustic pressure, somewhere measured right here.

So, you know it turns out that after some algebra a level difference then may be expressed in the terms

$$LD = 20\ log \left|\frac{\frac{p_n}{p_0}}{(cosk_0 l_1 + \frac{jY_1}{Z_0}\sin k_0 l_1)}\right|$$

$$l_1' \rightarrow 0$$

$$20\ log_{10} \left|\frac{p_n}{p_0}\right|$$

So, then level difference is a limiting for a constant pressure source as we discussed constant source that is your ps is tending to 0, level difference and insertion loss are the same.

So, level difference is a limiting value of insertion loss for a constant pressure source. So, level difference like I was mentioning it is it does not require the source impedance, and source characteristics basically acoustic source pressure and source impedance and also does not require anechoic termination.

So, therefore, is easiest to measure and also calculate perhaps, that is using may be *fe* codes and analytical things. And, it is become a very popular measure to do some experimental corroboration. But, what really matters? What really matters is the ability of a muffler; it is neither transmission loss nor LD. Those are good for a LD transmission loss is good for basically you know let me just summarize this in the table for the benefit of the students here, in this course.

That tabulated comparison of the pros and cons

TL	IL	LD
Experimental difficult	Relatively easy	Very easy
Prediction-analytically- numerical much easier	tough	Easy
physical significance we can compare a good idea of a given configuration	Matters most	can give us, a good idea of the effectiveness

Now, what is the physical significance? That so, physical significance we can compare, we can get a good idea of a given configuration; without even bothering whether what happens in the muffler is put in actual operating conditions.

So, you can compare different mufflers. This is what matters the most, insertion laws, some matters most; is something like to put it very bluntly or grossly. You would if you pay to an acoustic consultant you know lacks of rupees or millions of dollars whatever it is, that you ultimate objective is to basically tell us that whether the muffler or the noise control measure is good or not.

You would eventually want that without the muffler. Whatever the noise is there it should be it should really come down; when the muffler or the noise control measure is put in it is place. So, the idea is that these insertion losses like I said like I have been mentioning. Simply put if you remove that thing the noise is too large, if you put that thing on the noise is too less. So, it works you are happy.

So, that is what matters the most without even bothering about, you know complicated things like anechoic termination or not it is just your atmosphere is there. So, whatever impedance it imposes it does.

So, experimentally easy to do and that is what matters the most, computationally the toughest part to evaluate. Level difference well is the easiest to measure experimentally when that is what I have written and can give us, a good idea of the effectiveness of a muffler system. At the end of the day insertion law should be good, then you have done a good job.

So, I guess with this background we can just stop for the day and we will probably resume in the next class where we talk about performance of different muffler configurations. The simple constituent elements like a sudden area expansion and extended inlet and outlet things and simple expansion chamber of course, and the similar thing. So, that will be very interesting to see, what happens when you just consider a plane wave analysis and go about evaluating the performance of such mufflers.

If, the idea is; obviously, the plane wave like, I said it is too simple or will break down, but it can give us a very good feel of the system. So, that is what our aim should be, and we are going to talk about that in the next few lectures.

Probably in the next few lectures are going to be devoted to a range of elements that constitute simple elements that constitute a muffler, and I will show some sketches and talk about the basic philosophy how mufflers operate. With this I guess; I will stop here. Stay tuned.