Muffler Acoustics - Application to Automotive Exhaust Noise Control

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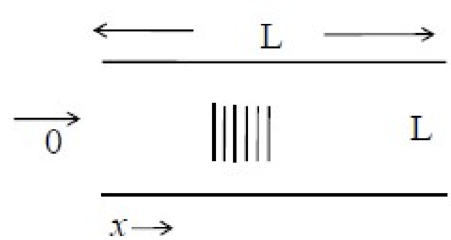
Lecture – 18
Lumped analysis of a Tube, Simple Area Discontinuity and Transfer Matrices

Welcome to lecture 3 of week-4 of the course on Muffler Acoustics NPTEL course. Here in this lecture, we are going to talk about basically three different things we are gradually entering into the elements that constitute a muffler. And the last lecture if you recall we discussed things like insertion loss, transmission loss, and level difference.

So, the objective of this lecture is basically analyze the transmission loss performance of constituent elements, particularly thissimple sudden area discontinuity also known as simple area discontinuity what I have written also known as sudden area discontinuity and simple expansion chambers.

- Lumped-Element Representation Of A Tube
- Sudden Area
- Simple Area Discontinuity
- Simple Expansion Chambers

But, before we do so, there is another thing to bestudied and that is a lumped element representation of a tube that is basically this bit. So, let us start off with this thing. And the idea is to basically discuss different implications of the lumped system analysisthat is basically valid at low frequencies, and how do we; how do werepresent that in an through an electro acoustic circuit orelectro acoustic analogy, and how this is applicable to the sudden area discontinuities and so on. So, we will build continuity from there.



So, if we recall for a tube something like this from our last several lectures back, I guess probably week-3– few lectures in week-3– probably the third or fourth. If we had a tube of length say L, and this was *x*, and this is something

$$\xi(0) = \frac{\xi(l)\cos k_0L + jY_0\, sink_0L}{j\{\xi(l)/Y_0\}\, sink_0L + cosk_0L} \qquad (1)$$

So, what this expression means is basically your let us call this equation (1).

You are looking at the tube from this end, you are trying to relate the

$$\zeta(0)\ and\ \ \zeta(L)$$

are related through some expression which is basically this expression is nothing but obtained by some algebraic manipulations of the one-dimensional wave propagation equation.

So, what happens when you have a very low frequency approximation? So, what do you mean by low frequency? What is the term that decides whether we are in the low frequency or whether lumped system analysis approximation can be made or cannot be made? So, this is the term argument of the sin or trigonometric function. So, $k_0L \rightarrow$ Helmholtz number, let meput this again here Helmholtz number.

So, when $k_0 L << 1$, then obviously, we can make a lot of simplifications, and then your lumped system analysis will apply. So, we can have different scenarios,of course, like we discussed either the frequency is very small, or the length L is also quite small.

So, in either case we will see how we go about it.

$$\zeta(0) \simeq \frac{\zeta(l) + jY_0k_0L}{j\frac{\zeta(l)}{Y_0}k_0L + 1}$$

And what do you get here? You will also get again k_0L + 1. So, this is only an approximaterelation when you have a very low frequency approximation.

So, we saw that the expression of impedance at they are related by the following expressions. So, let us make some approximations, obviously with understanding Y_0 is C_0 by S and this will happen when you have

$$Y_0 = \frac{C_0}{S}\ sink_0L\ \simeq\ k_0L$$

$$k_0L \simeq 1$$
$$\cos k_0L \ll 1$$
$$SL = V$$

So, it is basically a very low frequency approximation as we are discussing here. So, this all these simplifications are possible because of this thing. And when we further put this thing here,

$$\zeta(0) = \frac{\zeta(l) + j\frac{C}{S}\frac{\omega}{C_0}k_0L}{j\frac{\zeta(l)S}{C_0}\frac{\omega}{C_0}L + 1}$$

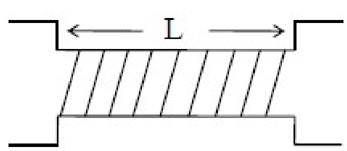
So, basically what happens now is that this will

$$= \frac{\zeta(l) + j\frac{\omega S}{S}}{j\zeta(l)\frac{V\omega}{C_0} + 1}$$

Remember S into L is the volume of the cavity.

$$SL = V$$

So, let us say the cavity is like thiswith wave propagation likealong these directions of, V= S2 to S into L that is what we are going to get. So, under such a situation, you get we can combine S into L as volume of the tube. And here we getomega and C naught square. So, all these terms, we should recall. We are seeing all these familiar faces.



$$\zeta(0)\ \frac{\zeta(l) + Z_m}{\zeta(l)\ /Z_C + 1}$$

So that was clearly our inheritance. So, if you recalltheinheritance for a thing like this for a structure like this, for a small tube sandwich between two large cavities, so the impedance or the inheritance was given by

$$Z_M = j\frac{\omega L}{S}$$

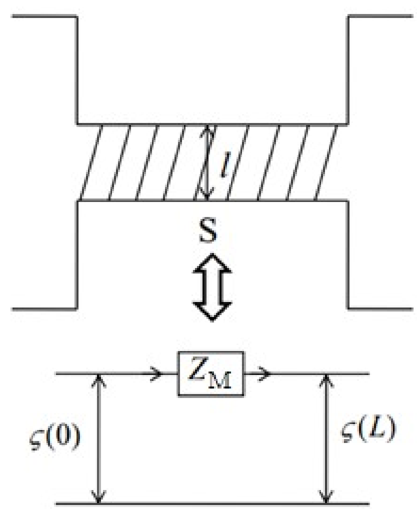
Similarly, what is this particular term? It is Z_c. So, again recall that

$$Z_C = \frac{C_0^2}{j\omega V}$$

So, basically what is happening then that

$$\frac{1}{Z_C} = \frac{j\omega V}{C_0^2}$$

And then if you simplify this expression,

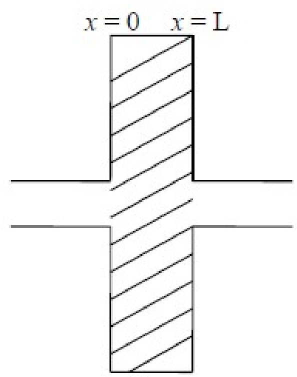


So, I probably should have written this as L and L here and of course, you have these two terms, this oneand this one. So, now, the thing is that what do we do with this expression, where do we go now from here? So, basically what is happening here that we can probably further simplify based on the condition that we know.For example, what is the, we, all we know is that there is just a tube here, but what additional thing that we need to know is that what lies upstream and downstream of the tube.

So, if you recall this figure, so this figure tells you thatif you have a tube of whose diameter or cross sectional area is much much smaller than this one ok, so when you have such a thing, then of course this will be dominantly the volume of such a tube is mostly much much smaller than the volume of this one and this one, alright. So, the compliance will be much much larger.

$$\zeta(0) = \zeta(L) + Z_m$$

So, your basically what is going to happen then is that for such a situation Z_m will be basically whatever we will be much much greater than Z_l, in other words or ςL, is basically something that is much much smaller than the compliance of the tube. So, one can safelyignore the underlying term.



So, under such a situation what we actually get

$$\zeta(0) = \zeta(l) + Z_m, \quad Z_m = \frac{j\omega L}{S}$$

So, what it means is that if we know additionally or beforehand that what lies upstream and downstream of the tube, so if you have a small tube sandwich between two much larger cavities.

Then impedance transfer or the impedance at this point and this point is related to just the addition of the inheritance, that is impedance at x = 0 xi at is the impedance here plus the inheritance of the tube at very low frequency.

So, basically then this thing is I probably should have, let me make some space here. And, so this then is equivalent to your circuit the following electroacoustic circuit. So, here you have Z_M and here you have this thing. So, this is the electric or the acoustic velocity or the current, andthis is $\zeta(0)$, and $\zeta(L)$.

So, clearly whatever the impedance we seewe are seeing at these two points and whatever we are seeing here, they are related by the relationlike I just wrote down. So, the same as next pagewhat I have been writing. So, the thing is that impedance for a tube which is sandwich between two large cavities, we just need to find out the inheritance at very low frequency, and this is the electro acoustic circuit for that.

Now, if we have some the reverse situation, now let us say the tube is no longer if the cross sectional area of the tube is no longer very short or very small. It is basically much larger than the upstream and downstream elements, so something like this. So, recall the situation here. And recall this thing here, it is much much larger.

So, and of course, $x = 0$ is measured from here $x = L$ here, and this is what we are going to get. So, basically what happens in such a case? In such a case, you have

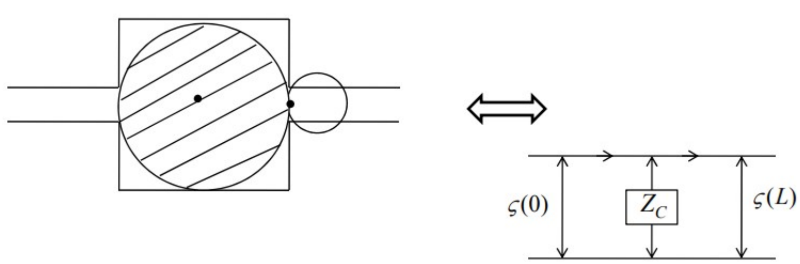
$$Z_m << \zeta(l)$$

So, if you recall this expressionwhen this, this guy this particular impedance that is basically the inheritance that is much much smaller than your impedance here.

So, then you can ignore this term, and just retain this thing in such a case what we will get is basically the equivalent impedance is basicallyzeta

$$\zeta(0) = \frac{\zeta(L)Z_C}{\zeta(L) + Z_C}$$

This is what we are going to get, so that means, the element basically has a compliance kind of a behavior and that acts like a shunt element.



So, basically if I draw the electro acoustic circuit for the figure this one, so we will have situation like this. So, this is represented by this thing Zc. And here you have this thing.

So, what happens is thatthe current or the acoustic mass velocity gets split into two parts, one that goes within the compliance and the one that propagates further downstream.

So, basically, this compliance or the large cavityor the cavity with a large volume much larger than the tubes that are upstream or the downstream part. If you have such a situation, then the element acts like a shunt element and the compliance can be considered as a shunt element.

So, then basically when you have such an element, so this element can then be represented or this expression rather what it means it can be represented by the electroacoustic circuitin which the compliant acts like a shunt element Z_c.

So, the acoustic mass velocity gets split into two parts – one that goes within the compliance and the one that propagates downstream. So, you basically that is how this zeta naught is related to zeta L. So, this expression obviously, can be written in the form

$$\frac{1}{\zeta(0)} = \frac{1}{\zeta(L)} + \frac{1}{Z_C}$$

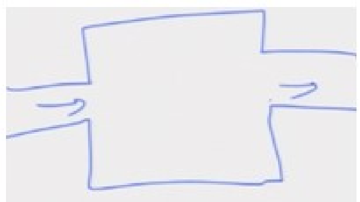
So that means, the impedance or the compliance this entire thing and the impedance at this part just at this section here, they are acting in parallel. So, that is how you get this representation.

So, what is the meaning of all this discussion is that it is important to note that what really matters is that how Z_m and Z_c compare with zeta at x is equal to L that is the, in other words the lumped element behavior of a tube is then a function of what lies downstream of it. So, just to summarizethis discussion if the tube in question is much much smaller and thin and sandwich between two tubes of much larger cross section that is too larger cavities, then it behaves like an inheritance.

On the other hand, if you have an acoustically small tube $k_0 << 1$, and sandwiched into two tubes of much smaller cross section that is to say the element with a much smaller length that itself is having a diameter much bigger than the elements on the upstream and downstream then, that element behaves like a compliance with a shuntimpedance.

So, for typical acoustic filters that is mufflers, silencers or mufflers, they are also called acoustic filters low pass filters, because they generally have a low performance or not so good performance it very low frequencies, because they allow most of the acoustic power at low frequencies to pass through, and they generally beyond at a certain higher frequencies.

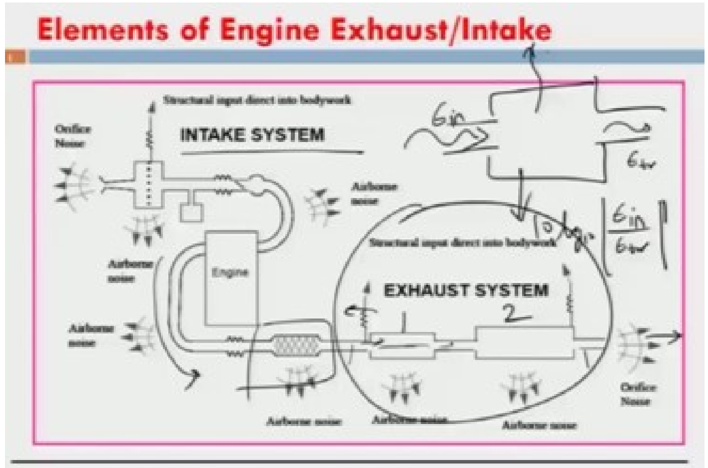
So, they are also called low pass filters. And lumped element approximations they hold at very low frequencies. But just tounderstand the basic behavior of mufflers, all this will be useful, so that is the reason that we talked about the lumped element behavior of a tube.



Because eventually a muffler can be considered in the most simple muffler is a simple expansion chamber. So, in many which ways we can by studying lumped analysis of such systems, we can get to know the basic behavior of a muffler at very low frequencies. So, basically now we intend to move to the elements that constitute the muffler.

And for thatlets basically first look at what are the elements that constitute a muffler, and we can go to the slide or the presentation. Well, now we probably need to enter into the simple area or sudden area discontinuities of a muffler, and also simple expansion chambers.

With this background, I guess it is time that we move to the elements the very fundamental elements that constitute a muffler. And but before we do so, I guess it is prudent to have a look at the entire structure of a muffler which is basically yourthing like this. Now, this is a schematic or a block diagram of a typical layout of a engine exhaust system. So, what we probably need to worry about is basically this part – the engine exhaust system.



So, let me however, very basically I will tell you very briefly about the engine exhaust system. So, you have your the intake system here, and there is orifice noise, their number of filters upstream of the muffler, and there is kind of a resonator to kill the noise here. And through number of manifolds, it comes to the engine the engine sucks in air.

And after the combustion takes place, the products of combustion are then let off through the exhaust pipe. And here you have something like a bellows kind of a thing and there is an airborne noise.Finally, this is your exhaust pipe; this is your tail pipe. So, this is the muffler entire exhaust system that we intend to study.

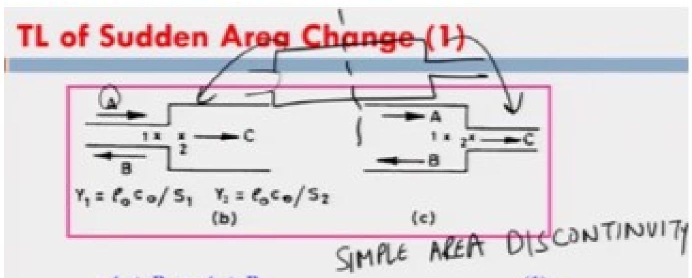
So, if you see here muffler 1, muffler 2, you can have you can it is not very sacred, you can have very different arrangements. You can have double outlet mufflers also which is popular. So, basically what I wanted to tell you about is that these are the elements that constitute the muffler.

You see a port an inlet port through, which air comes or the acoustic power is incident, and some part is reflected back some part is transmitted; again some part is reflected back, some part is transmitted. And this process is repeated for the muffler 2. And eventually very less amount of acoustic power is transmitted in the atmosphere, so that at least for the frequency range of interest that is how you design a good muffler.

And most of the acoustic power then is transmitted upstream, and then there is lot of power that is radiated out it is called structural acoustic noise or the breakout noise. So, we are not going to worry about that part at least in this course. We just going to focus on things like if you have a muffler like this, if you have a certain element like this, what is and if you havecertain wave incident here.

So, what is the ratio of the energy incident and energy transmitted, so we are trying tostudy that and log of that. So, basically the transmission loss or thealso known as the axial transmission loss or basically considering the muffler shell to be rigid,of course nothing is rigid muffler shell will vibrate.

But we are just trying to study the effects of axial transmission loss or the energy or the acoustic power that is incident on the muffler to the acoustic power that is transmitted downstream, we are just going to study that without regards or without consideration to the noise that breaks out from the transverse surface.



$$A_2 + B_2 = A_1 + B_1 \quad (1)$$

$$(A_2 - B_2)/Y_2 = (A_1 - B_1)/Y_1. \quad (2)$$

$$\zeta_2 = \zeta_1 = Y_1, \quad (3)$$

So, basically before we begin the analysis of this system and this system we need to worry about the constituent elements that I have been talking about so far. So, this schematic shows you a sudden area change or simple area discontinuity, also known as a simplearea discontinuity. We will derive carefully how these equationscame, simple area discontinuity.

So, figure b or figure c, basically what it tells is that there is an incident wave of amplitude A that is incident at a cross section points marked as 1 and 2 and some part of the wave is reflected back, and some part is propagated downstream which is C. So, naturally there will be some energy that will be transmitted into the atmosphere. So, this is a system where there is a sudden area increment or increase.

And in on the other hand, in this point, there is a sudden area decrement or sudden area reduction. So, if you consider a simple expansion chamber like this, what this figure does is that it breaks a simple expansion chamber into sudden an area expansion where the expansion is where thearea change is positive, or the area has increased.

And in this thing, there is a contraction, or the area change suddenly is reduced. So, the idea is to have a impedance mismatch, at these ports, so that most of the acoustic power or significant acoustic power is reflected back. So, let us see how it works and how the things are related. So, we will probably worry about that in the coming lecture and stay tuned for that. Thanks a lot.