Muffler Acoustics- Application to Automotive Exhaust Noise Control
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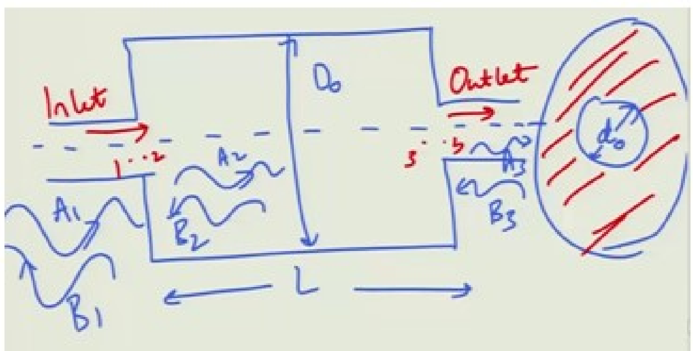
Lecture – 20
Simple Expansion Chamber Analysis Using Transfer Matrix Method

Welcome to, lecture 5 of week 4 on this NPTEL course on Muffler Acoustics. We will do things.

- Simple Expansion
- Chamber which is really the most fundamental muffler element that is used.

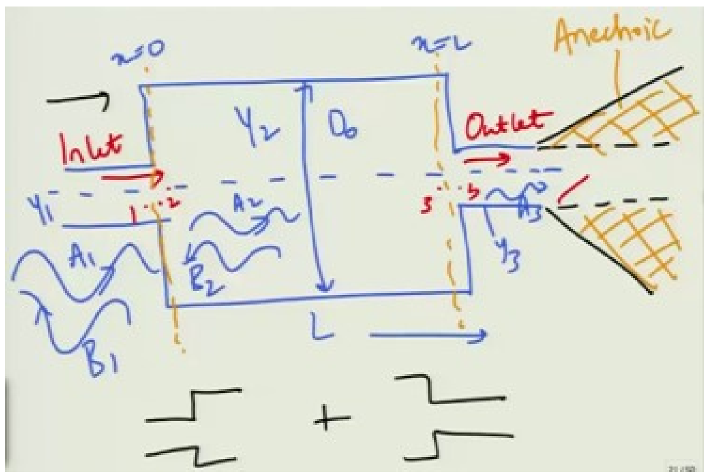


So, basically as you can now guess that a simple expansion chamber something like this you have your thing like this and you have and here you have a thing like this. So, the section can be elliptical this section it can be elliptical or a circular thing pressures are of circular cross section. They can also be elliptical, but circular other ones that are more preferred because they have less break out noise have more rigidity like I was mentioning.

And this is the inlet port and this is the outlet port I will call this as this point as 1 just at the interface, but within the inlet pipe and there is the point 2 which is just in the chamber, but at the interface of the port in the chamber this is point 3, this is point 4 inlet outlet ok.

So, now what is our job and of course, I forgot to mention the length is L the diameter let say the diameter is d and let say this dimension or diameter is d_0 and this is D_0 small d naught big D_0. So, what is our job then? To analyze a simple expansion chamber muffler, it is called simple because there is really nothing in the chamber it is an empty chamber it works on the principle of reflecting a significant portion or part of the acoustic energy back into the system.

So, when a wave is incident somewhere in here some part is reflected back A_1,B_1 here it gets to A_2 and here it is B_2 and some part is A_3 and here there will be some part that will be we let it back of course, if we have anechoic termination this will not be the case. So, let us assume that this is discharging into an anechoic termination. So, I will kind of rub this thing.



And what I am going to do is that put a anechoic termination as we discussed. So, things like this anechoic termination. Now what do we do? If you recall from the last weeks lecture. We wish to combine the sudden expansion plus the sudden contraction considering the finite length effects of a chamber this no longer infinite.

And at least a chamber and we are trying to determine how the transmission of performance will be. That is to say if a certain amount of acoustic power is incident on the muffler from here and certain amount is going towards anechoic termination, what is the performance of the muffler look like across the frequency spectrum?

To answer this question I guess it is time that we, now start talking about the field continuity conditions at the point 1 and 2 and 3 and 4 and relate it with the standing or the progressive wave variable.

$$p_1 = A_1 e^{-jk_0 x} + B_1 e^{jk_0 x}$$

$$V_1 = \frac{A_1 e^{-jk_0x} - B_1 e^{jk_0x}}{Y_1}$$

So, pressure at 1 is given by we will fix the coordinate system So, this is characteristic impedance Y_1,Y_2 and Y_3. So, of course, putting x is equal to 0 we get this and we get this, we get,

$$\tilde{p}_1 = A_1 + B_1$$

$$\tilde{V}_1 = \frac{A_1 - B_1}{Y_1}$$

So, this is what we going to get and similar,

$$\tilde{p}_2 = A_2 + B_2$$

$$\tilde{V}_2 = \frac{A_2 - B_2}{Y_2}$$

So,

$$at \qquad x = 0$$

Now,

$$\begin{aligned} \tilde{p}_1 &= \tilde{p}_2 \\ \tilde{V}_1 &= \tilde{V}_2 \end{aligned} \qquad (1)$$

And mass velocities if you recall they are also the same what we derived in the last lecture. So, what do we do after this? We start putting well the entire idea then is to actually formulate a set of equation then eventually we would need everything in terms of A_1. So, there will be A_1 A_2, A_1 B_1, A_2 B_2, A_3 and B_3 will be 0, if you assume anechoic termination nothing is coming back. So, all the parameters that is to say let,

$$A_1 + B_1 = A_2 + B_2 \qquad (1)$$

we will get this, but it will be pretty cumbersome we can sure form the formulate the equations, if we get equation let,

$$\frac{A_1 + B_1}{Y_1} = \frac{A_2 + B_2}{Y_1} \qquad (2)$$

So, we have got this equation and then we will keep this thing aside here and we will probably have to worry about the things that happen somewhere here at the section 3 and 4the same field continuity conditions will apply. So, what are that let us figure out that part.

$$B_2\,, A_2, B_1, A_3 = f\ (BA_1)$$

So, here,

$$\left.\begin{aligned}\tilde{p}_1 &= \tilde{p}_2\\ \tilde{V}_1 &= \tilde{V}_2\end{aligned}\right]\ x = L$$

$$A_2 e^{-jk_0 x} + B_2 e^{jk_0 x} = A_3 \qquad (3)$$

$$\frac{A_2 e^{-jk_0 L} - B_2 e^{jk_0 L}}{Y_2} = \frac{A_3}{Y_3} \qquad (4)$$

Now the thing is that here you will have L why because it is the pressure field.

So, the idea is that what you have to put if you are putting your coordinate system somewhere here x is equal to 0 here it will be x is equal to L.

And you will get things like this thing. So, this will happen and then you will get and for other chamber you can just put it the coordinate system again translate it for this port you can define the local coordinate system somewhere. So, equations is 2, 3 and 4.

Eventually like I said we want to figure out everything in terms of in terms of A_1. So, we have got 4 equations 1 2, 1, 2, 3 and 4 and we want to find out things in terms of A_1.

So, it is going to be a little algebraically a bittious. So, I guess now it is a time to start working in terms of the transfer matrices which will make our life very convenient. So, let us do that you will soon see that why all these things that I am talking about is important. So, when we have this condition, we can write this set of equation something like here.

$$\begin{Bmatrix} p_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \begin{Bmatrix} p_2 \\ v_2 \end{Bmatrix}_{x=0}$$

$$\begin{Bmatrix} p_2 \\ v_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \begin{Bmatrix} p_3 \\ v_3 \end{Bmatrix}_{x=L}$$

Now, what happens between x is equal to 0 and x is equal to L, between the points here 2 and 3 what happens? How do you relate A_2 and B_2? So, right now we are in terms of the standing wave variables that is p and v we are not use the progressive wave variables in the matrix presentation shown in the slide. We would want basically I guess it is time now to derive also the transfer matrix for a tube with planar wave propagation.

And that would really help us relate things with point 2 and 3 here. So, to that end what we will probably do is we will probably relate use this set of equations as well as this set of equations and from that we will kind of eliminate A_2 and B_2 to relate things at section 2 to section 3 and then we can just simply multiply the matrices.

So, let us do that now that we have got the transfer matrix. We have written down the relation between the upstream and downstream state variables at the sudden area expansions and the contractions as we see in slide number 21 between 1, 2 , 3 and 4.

Now, what remains is to derive basically relate the transfer the state variables between point 2 and point 3 that is pressure and velocity at point 2 with those at the point 3. So, how do we do it?

Basically our objective should be to basically derive the relation a transfer matrix relation between the pressure and velocity at the dotted section here at the interface and I mean at this section to that at the section 3. So, how do we go about doing that? So, let us make use of this thing. So, we have here p at x is equal to L we have this kind of a thing.

Now, this should be pretty straightforward because we know from the last equations that pressure 1 is equal to pressure at the point 2 and volume velocity at the point 1 is equal to the volume velocity at point 2. So, this is straightaway putting the argument of the complex of exponential to be 0. So, we get this now at x is equal to L it is the matter of just actually using the length of the chamber.

So, we substitute that to get A_2 into A_2 exponential $e^{-j\,k0t}$ L+ B_2 into e to the power jk_0 L and so on for the acoustic pressure. So, basically we tend to use this thing for the acoustic pressure and the below expression for the volume velocity at the section 3.

$$\tilde{p}_{x=0} = A_2 + A_2$$

$$\tilde{V}_{x=0} = \frac{A_2 - A_2}{Y_2}$$

So, what happens now is that this is something that we are aware of, we are aware of these things and we now introduce because the to simplify the algebra. Let us introduce the convention that complex exponential that is **Euler's formula**.

$$e^{j\theta} = C + jS = cos\theta + jsin\theta$$

$$\tilde{p}_{x=L} \;\; = \;\; A_2 e^{-jk_0 L} + B_2 e^{jk_0 L}$$

People who have some background in engineering mathematics would immediately recognize this is the Euler's formula. So, very convenient to write cosine and sine functions together. So, here we have p acoustic pressure at section x is equal to L is given by this relation. So, now, we simplify we kind of expand using the Euler formula.

$$= A_2\,(C - js) + B_2(C + js)$$

$$= C\,(A_2 + B_2) - js\,(A_2 + B_2)$$

Now, what we do is regroup the terms. So, basically what happens is that your this thing is the pressure and we immediately recognize that this would be the velocity thing.

$$\tilde{p}_{x=L} = C\tilde{p}_{x=0} - js\,Y_2\,\tilde{v}_{x=0}$$

$$\tilde{v}_{x=L} = \frac{1}{Y_2}(A_2 e^{-jk_0 L} - B_2 e^{jk_0 L})$$

$$\tilde{v}_{x=L} Y_2 = \; A_2(C - js) - \; B_2(C + js)$$

$$= C(A_2 - \; B_2) - js\,(A_2 + \; B_2)$$

So, what happens is pressure at the section x. So, in place of that we are using this thing here. And we using this part here. So, once we do that we immediately recognize that p acoustic pressure within the chamber right at x = L section, is cosine times the acoustic pressure

$$\tilde{v}_{x=L} Y_2 = \; CY_2 v_{x=0} - js\tilde{p}_{x=0}$$

Now volume velocity similarly can be expressed as 1 by Y_2 times A_2 into complex exponential minus B_2 times another complex exponential. As usual we go about multiplying just to simplify terms we multiply both sides by Y_2 and expand the Euler formula expand these terms using Euler formula.

And once we do that we will rearrange the terms A_2 into j times sin k_0L. Similarly for the terms underlined here and then we regroup the terms A2 - B2 minus j sin k naught L A_2 plus B_2 and this is nothing but your pressure at x is equal to 0 and this is nothing but velocity at x is equal to 0. So, we get another relation this one and this one.

$$\tilde{v}_{x=L} - C\tilde{v}_{x=0} - \frac{-js}{Y_2}\,\tilde{p}_{x=0}$$

$$\begin{Bmatrix}\tilde{p}_{x=L}\\ \tilde{v}_{x=L}\end{Bmatrix} = \begin{bmatrix} C & -jsY_2 \\ \dfrac{-js}{Y_2} & C \end{bmatrix}_{2\times 2} \begin{Bmatrix}\tilde{p}_{x=0}\\ \tilde{v}_{x=0}\end{Bmatrix}$$

$$Y_2 = \frac{C_0}{S_c}$$

So, now we can clearly very easily write that in the matrix form, but hang on just before that what we will probably do is, obviously divide both sides by Y_2 and so, in the end we will get something like volume velocity at x = L is equal to cosine times volume velocity at x is equal to 0 minus j sin k_0 L divide by Y_2 into acoustic pressure at x = 0.

Where obviously, Y_2 is your characteristic impedance of the chamber which is given by C_0 times the cross sectional area of the chamber. So, when you write this entire thing in a compact nice 2 by 2 matrix form, what are we going to get?

We are going to get a 2 cross 2 matrix pressure, but hang on we are able to relate in a matrix form pressure at the section x is equal to L with the pressure and volume velocity at x = 0. That is this is the D downstream, down stream variable this is the upstream variable, upstream variable.

So, once we do that what we are going to get is basically relation between this point and this point, this point and this point. So, now, what we would transfer matrix means something like if you recall our last lectures we would be trying to relate the upstream variable to the downstream variable that is.

The variables at the point x is equal to 2 or point 2 or x is equal to 0 to the variables that x is equal to L right now it is just the inverse. So, let us focus on how do we go about doing that we just need to the idea is fairly simple, we just need to take the matrix inverse that is let say this is the matrix say T 1.

$$\begin{Bmatrix} \tilde{p}_{x=0} \\ \tilde{v}_{x=0} \end{Bmatrix} = \begin{bmatrix} C & jsY_2 \\ \dfrac{js}{Y_2} & C \end{bmatrix}_{2\times 2} \begin{Bmatrix} \tilde{p}_{x=L} \\ \tilde{v}_{x=L} \end{Bmatrix},$$

$$C = cosk_0L, \qquad Y_2 = \frac{C_0}{S}$$

$$S = sink_0L,$$

So, if you just invert the matrix that is

$$[T] = [T_1]^{-1}$$

So, we will get this form we can actually multiply this matrix the one underline with the matrix somewhere here and you see that we will get an identity matrix its fairly simple to see. So, I am not going to derive it here this is left as an exercise for the students here, basically the idea is that try to invert this matrix. So, when you invert this matrix whatever matrix you are getting is shown here.

Now, the upstream variables, upstream variables and downstream variables are related by a nice clean transfer matrix which is your this thing. So, this is nothing, but let me simplify things for you or make it more clear.

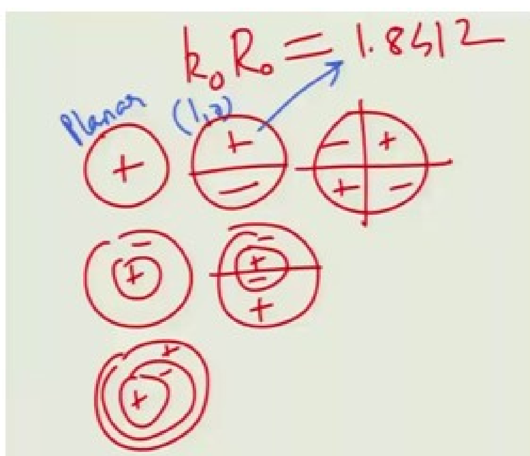
This is cos so is this and yourj into sin of k_0 L by Y_2 j into sin of k_0 L by Y_2. So, well we have our 2 cross 2 matrix which relates upstream variables to the downstream variables. So, we will see this is the probably one of the very important fundamental results in muffler acoustics and we probably had to wait till the last lecture of week 4 to arrive at this.

But this was all we kind of building momentum, building enough background to arrive at this fundamental result. So, you can pause the screen reflect and have a look at this transfer matrix in a perhaps in a more with a much more attention, there are lots of properties what can be revealed just by looking at this like. I will probably not be discussing all those things, but probably something about conservative system reciprocal systems and symmetric systems and those details are there lot of things can be done.

Determinant one thing one couple of things that I would like to point out determinant of such a transfer matrix is always unity, that is determinant of T is unity read more about what does it mean in terms of energy conservation or reciprocity of the stuff. So, since there is one more thing that because the element is really symmetric.

We have a uniform tube like this we are relating 1 to 2 or whatever terminology you are using. So, if we were to invert the matrix like we have seen here apart from the difference in the minus sign everything else is the same. And actually by changing the sin conventions of mass velocity, we can actually see that the inverse would be the same as the T matrix. So, that is characteristics of the fact that this is a it is a uniform tube. So, if you even if you reverse the element the transfer matrix would be the same

Probably later in the course we will come up with a thing like impedance matrix not now, but later on when we dealing with the network analysis and then again try to correlate this, but for now let us would be the transfer matrix. Now, that we have derived the transfer matrix for uniform tube. Now this is valid whether it is a chamber of diameter capital d naught big diameter or a small diameter as long as plane wave propagation is there.



So, this transfer matrix is there. So, what is the limit if you recall the lectures from week 2 what for a duct of radius R_0 or diameter D_0 the cut on frequency for the 1 0 mode 1 0. You know if you recall our discussion this was the plane wave mode, this was the first circumferential mode and so on. Then this was the first radial mode second radial mode and so on. So, like this there will be a cross mode plus minus, minus plus and so on.

So, we are talking here about the cut on frequency of this mode that is a lowest first higher order mode to propagate. This is the, this is the planar wave mode always propagates for a rigid duct, this is the cut on frequency of the first higher order mode also known as the 1 0 mode.

The lectures from week 2 the reason that I am telling you is this because if the frequency

$$\frac{2\pi f}{C_0} = \frac{1.8412}{R_0}$$

$$f_c = \frac{C_0 \times 1.8412}{\pi D_0}$$

$$f_c = \frac{C_0}{D_0}\left(\frac{1.8412}{D_0}\right)$$

So, I guess we discussed this briefly, but it is probably worthwhile to mention this thing again here it is probably,

$$f_C \simeq 0.5861\frac{C_0}{D_0}$$

$$f_C \leq 0.5861\frac{C_0}{D_0}$$

So, point I am trying to make is that this transfer matrix is the valid one if you are well within or probably less than the frequency gained by this thing because at the such frequency is only the planar waves that propagate. So, this cut on frequency obviously, will change for an elliptical duct.

Something that we again discussed long time back and for a rectangular duct it will be a different expression, but so the idea is find out the propagation frequencies of the lowest order mode. And then check if you are above or below that frequency if you are below then the transfer matrix for uniform tube will apply.

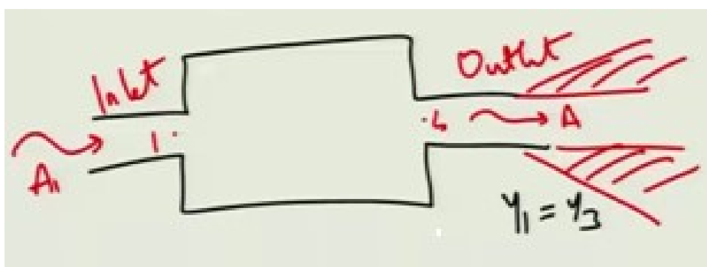
Now quickly getting back to our, I guess we digressed quite a bit, but that was important. We will use this thing this matrix here along with what we saw probably in the slide here and gradually kind of multiply.

So, the thing is that we can comfortably now see that if you are considering the muffler system like what we discussed in slide number this one. So, we know relation between 1 and 2,2 and 3 is was just derived that transfer matrix 3 and 4 we know. So, we can relate what happens to 1 and 4 ok.

$$\begin{Bmatrix}\tilde{p}_1\\ \tilde{v}_1\end{Bmatrix} = [I]\begin{Bmatrix}\tilde{p}_2\\ \tilde{v}_2\end{Bmatrix} \quad \begin{Bmatrix}\tilde{p}_3\\ \tilde{v}_3\end{Bmatrix} = [I]\begin{Bmatrix}\tilde{p}_4\\ \tilde{v}_4\end{Bmatrix}$$

$$\begin{Bmatrix}\tilde{p}_2\\ \tilde{v}_2\end{Bmatrix} = \begin{bmatrix} C & jsY \\ \frac{js}{Y} & C \end{bmatrix}\begin{Bmatrix}\tilde{p}_3\\ \tilde{v}_3\end{Bmatrix} \quad I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}_{2\times 2}$$

So, once we do that, So, now, it is just a matter of sequentially multiplying the elements.



So, let me write it down like this p_1. So, identity matrix times this thing is just matrix and again this matrix times identity matrix

$$\begin{Bmatrix}\tilde{p}_1\\ \tilde{v}_1\end{Bmatrix} = \begin{bmatrix} cosk_0L & jsink_0L\, Y_2 \\ \frac{j}{Y_2} sink_0L & cosk_0L \end{bmatrix}\begin{Bmatrix}\tilde{p}_4\\ \tilde{v}_4\end{Bmatrix}$$

So, we get finally, when we have such a muffler. So, pay attention to the fact that the point 1 is just in this pipe outside the chamber and point 4 is just outside the chamber in the outlet pipe.

So, we know all those things and now what do we do? We would basically want to basically relate further relate the incident wave variables at the inlet port to the thing that was transmitted remember we have an anechoic termination. So, we know it is A_1 here and we know it is A_4 here or what was it? I guess it was probably let me just recall was it A_3 or just the point was 4.

So, A_4, but it is A_3 the point obviously is 4 but it is A_3. So, what we would probably be doing is that let me just go back and verify my algebraic simplifications and now what we would probably be doing is that

$$\left.\begin{aligned} p_1 &= A_1 + B_1 \\ Y_1 v_1 &= A - B_1 \end{aligned}\right|_{Y_3} \begin{aligned} \tilde{p}_4 &= A_3 \\ \tilde{v}_4 &= A_3 \end{aligned}$$

Let us assume the diameter of inlet and outlet ports are the same. So, we will put those things in here and proceed further.

$$\begin{Bmatrix} A_1 + B_1 \\ \frac{A_1 - B_1}{Y_1} \end{Bmatrix} = \begin{bmatrix} C & jS\,Y_2 \\ \frac{jS}{Y_2} & C \end{bmatrix} \begin{Bmatrix} A_3 \\ \frac{A_3}{Y_1} \end{Bmatrix}$$

$$A_1 + B_1 = CA_3 + jSY_2 \frac{A_3}{Y_3}$$

$$A_1 - B_1 = \frac{jS}{Y_2} Y_1 A_3 + CY_1 \frac{A_3}{Y_3}$$

This is what we are going to get assuming anechoic termination. Of course, now things should be relatively much more simple. So, A_1 plus B_1 let us expand this thing let us consider the first equation of the matrix thing.

$$A_1 - B_1 = \frac{jS}{Y_2} Y_1 A_3 + CA_1$$

And. So, I will rather not write this term and just bother with writing A_3 ok.

Now, I guess you guys should be able to figure out just adding these two equations, that is your equation star and double star. So, if we add these two equations what will we get 2 A_1 is equal to we will take A_3 common throughout, throughout. So, let me write this A_3 somewhere here assuming that we needs that much space. we will get contribution from cos k_0 again here.

$$2A_1 = \left\{ \begin{aligned} & 2cosk_0L \\ & +jsink_0L\,\frac{Y_2}{Y_1} \\ & + jsink_0L\,\frac{Y_1}{Y_2} \end{aligned} \right\} A_3$$

$$\frac{A_1}{A_3} = \left\{cosk_0L + \frac{1}{2}jsink_0L\left(\frac{Y_2}{Y_1} + \frac{Y_1}{Y_2}\right)\right\}$$

$$\frac{Y_2}{Y_1} = \frac{S_1}{S_2} = \frac{1}{M}\ Y_1 = \frac{C_0}{S_2}, Y_1\frac{C_0}{S_1}\ \frac{Y_1}{Y_2} = M$$

So, we get this form. So, now, let us see what happens after this we need to worry about that as well we need to worry about that.

$$TL = 10\ log_{10}\left|\frac{A_{inc}}{A_{wave}}\right|^2$$

$$= 20\ log_{10}\left|cosk_0L + \frac{jsink_0L}{2}\left(\frac{1}{M} + M\right)\right|$$

$$a + jb$$

$$cos^2k_0L + \frac{sink_0L}{4}\left(M + \frac{1}{M}\right)^2$$

$$a^2 + b^2$$

Now, transmission loss if you recall was defined as 20 log 10 by A_1 incident wave by transmitted wave, is not it?

So, it will pretty much convenient to write it like this. So, then it will be Y_2 by Y_1 we just derived its 1 / m + m, we get this and it will be like this. So, we could possibly simplify the terms and probably worry about that. So, let us do the simplification now we need to simplify this thing. So, we immediately recognize that like well this is of the form,

$$Z = a + jb$$

$$|Z| = \sqrt{a^2 + b^2}$$

So, that is what we are going to get. Now they are clearly of the form a square plus b square.

$$1 - sin^2 k_0 L + \frac{sin^2 k_0 L}{4}\left(M + \frac{1}{M}\right)^2$$

$$1 + sin^2 k_0 L \left\{\frac{1}{4}\left(M^2 + \frac{1}{M^2}\right) + \frac{2}{4} - 1\right\}$$

$$1 + sin^2 k_0 L \left\{\frac{1}{4}\left(M^2 + \frac{1}{M^2}\right) - \frac{1}{2 \times 2}\right\}$$

Now, it is the simple fairly straightaway straightforward algebraic trigonometric simplification is one we expressed cos in terms of sin and you know rearrange the terms sin k naught square L is taken common.

This term is there and so what we do basically expand this thing out m square plus 1 by m square 1 by 4 factors always there and 1 by 4 into 2 by 4 and minus m is there. So, what happens this is now is that this is further a simplified half minus 1 that is minus half. So, once you multiply this by 2 and 2.

$$1 + sin^2 k_0 L \left\{\frac{1}{4}\left(M^2 + \frac{1}{M^2}\right) - \frac{1}{4}\right\}$$

$$1 + sin^2 k_0 L \left\{ M^2 + \frac{1}{M^2} - 2M\frac{1}{M}\right\}$$

So, what we get is basically 4, so 2 times 4.

So, we take we get the following m minus 1 by m whole square we get this. So, then the transmission loss can be written very conveniently in terms of the following expression

$$= 1 + sin^2 k_0 L \left(M = \frac{1}{M}\right)^2$$

Remember,

$$TL = 10log_{10}\left\{1 + \frac{1}{4}\left(M - \frac{1}{M^2}\right)^2 sin^2 k_0 L\right\}$$

$$TL = 10\ log_{10} \left|\frac{A_{inc}}{A_{wave}}\right|^2$$

So, this is the finally, the much coveted expression that we aim for at the beginning of this lecture 5 of week 4. It is a, it is a nice beautiful compact relationship and probably, we will worry about the consequences of or probably the implications of this relation in the next lecture, that is week 5, lecture 1.

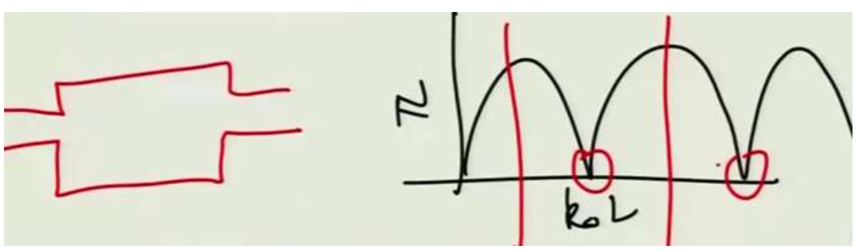
$$= 20\ log_{10} \left|cosk_0L + \frac{jsink_0L}{2}\left(\frac{1}{M} + M\right)\right|^2$$

$$cos^2k_0L + \frac{sink_0L}{4}\left(M + \frac{1}{M}\right)^2$$

Fairly detailed, we can probably like to do a fairly detailed analysis of how the transmission loss varies with frequency for a simple expansion chamber. So, basically just as a teaser for a result just to keep your curiosity alive. So, if this is the frequency axis or non dimensional frequency axis non dimensional with respect to the length L k_0 L and this is your transmission loss.

$$= 1 + sin^2k_0L\left(M - \frac{1}{M}\right)^2$$

$$TL\ 10\ log_{10}\left\{1 + \frac{1}{4}\left(M - \frac{1}{M}\right)^2 sin^2k_0L\right\}$$



So, you will get domes at frequency attenuation domes and how does this vary with m, that is expansion chamber ratio. And what are the frequencies at which this your crest or the maximum or the transmission loss happens and where does your trough happens? We will see all these later, stay tuned.

Thanks a lot.