Muffler Acoustics - Application to Automotive Exhaust Noise Control
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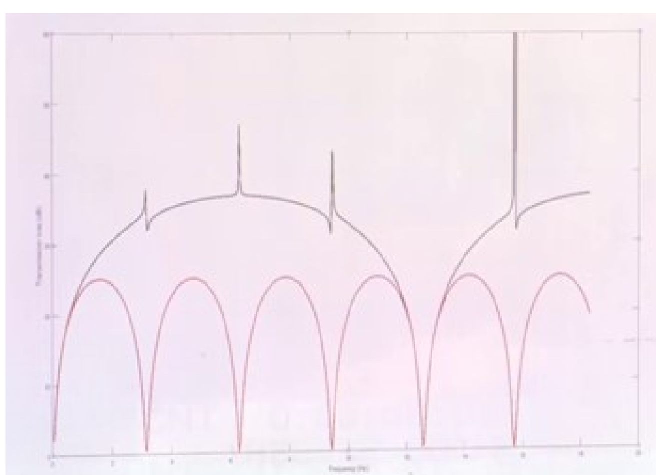
Lecture - 25
TL analysis of Side-Inlet and Side-Outlet Muffler using Transfer Matrix method

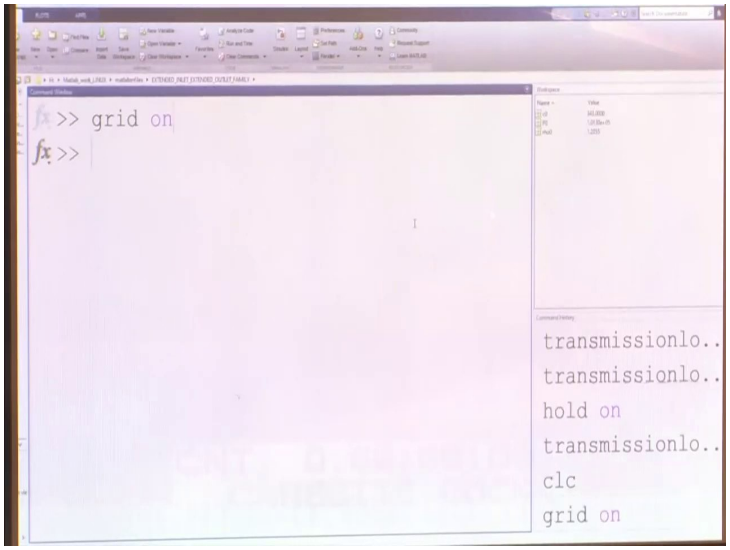
Welcome to lecture 5, the final lecture of week 5 of this NPTEL course on Muffler Acoustics. So, what we are going to do in this the final week is that, there is in the last week we last lecture I am sorry, we analyzed in detail the transmission loss performance and did some parametric studies in MATLAB and try to understand some expressions in maple using symbolic computer package of the T matrix parameters.

Now, what we would probably like to do in this final lecture is basically try to understand a very related kind of a muffler configuration known as side-inlet and side-outlet or end-inlet and side-outlet. So, these are as far as one dimensional plane one dimensional planar wave propagation is concerned, they are pretty much equivalent compared to an extended inlet and outlet system.

Now, before we move on to the discussion of an end inlet and side outlet system or a side inlet and a side outlet muffler we will also have a look at the photographs how do they look like and why we should be using them, why should we should be analyzing them. We it is also good idea to talk something about the muffler configuration that we just analyzed in the last class.



Let me briefly show you a nice figure transmission loss plot. So, what we are seeing here let me increase the thickness of these lines. Let it be 2 and let it be also 2.



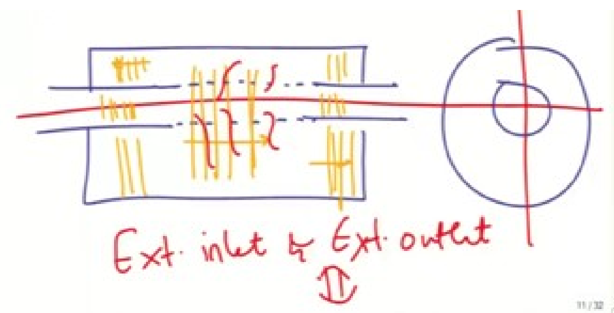
And, this is grid on. What does this figure mean? Of course, we are talking about an extended inlet and an outlet system. So, with extensions of the extended inlet l by 2 it is able to cancel out the trough at the first and the third resonance frequencies and extension is equal to l by 4 it is able to cancel out the trough at the other at the second resonance frequency.

Now, if we compare the transmission loss performance which is such a tuned extension extended inlet and outlet system with that of a simple expansion chamber in which there are no extensions we would see the following comparison. The red is a simple expansion chamber; the black colored curve is the tuned extended inlet and outer system.

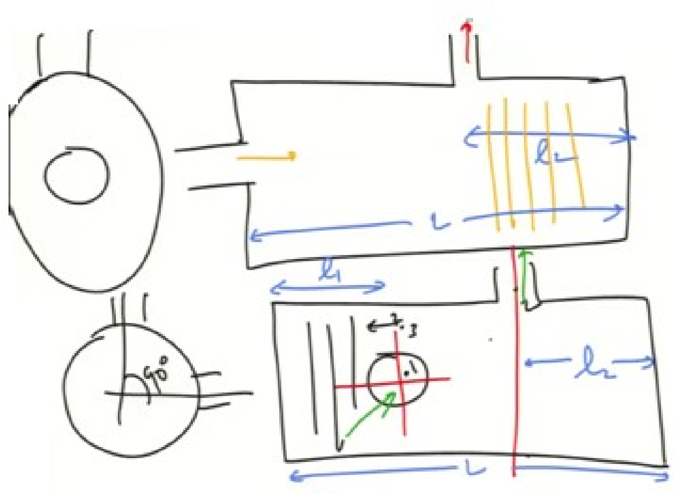
So, we see a phenomenal a dramatic increase or improvement in the transmission loss performance. Not only have the troughs been uplifted and they have been replaced by attenuation peaks, overall you know where we are getting maximum transmission loss of say you know 25 dB at the domes we are able to achieve much more. The curve you know increases significantly and we are able to accomplish a much better performance.

Overall this kind of broadband hump is seen which basically, which almost guarantees you 20 to 22 dB of transmission loss from the range as low as say k naught l is equal to 0.1 or 0.2 to all the way up to k naught l is equal to maybe 11 or something like that, that is roughly from 100 hertz to about maybe 1500 hertz or 12, 1400 hertz or so on. So, it is pretty impressive just a small.

So, what is the moral of the story? So, the moral of the story is that just by cleverly you know choosing the muffler configuration by simple extensions of the inlet and outlet and tuning the extension lens we are able to achieve such a dramatic improve in the performance even without the use of any absorptive materials or perforates and all that sort of a thing. No, it is a fairly simple configuration.



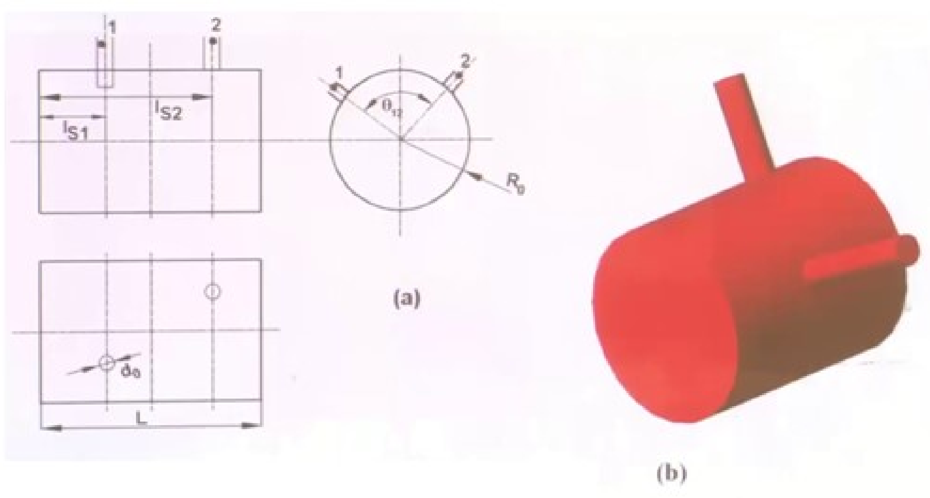
So, basically now if we focus our attention on the schematics so, this is what we did and what I am claiming now is that we have this kind of a configuration and what I was mentioning to you guys at the beginning of this lecture was that this extended inlet and extended outlet is equivalent to and of course, a plane wave propagation occurs here. Plane wave propagation occurs here.



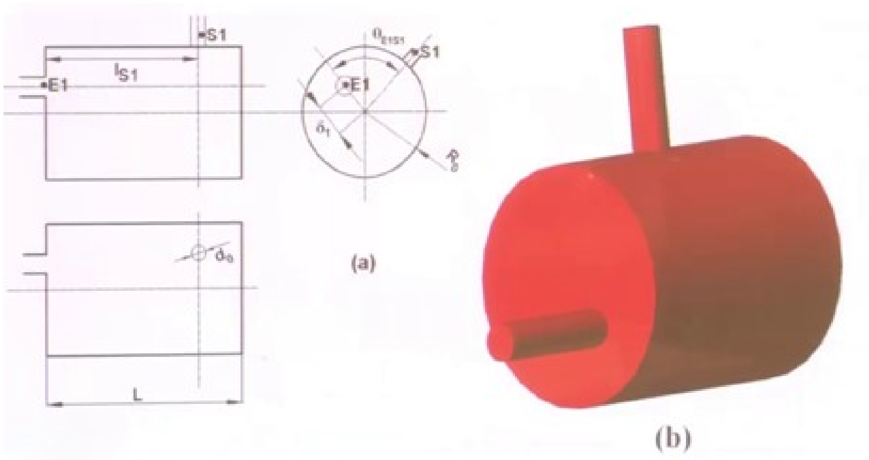
What it means is that this configuration then is equal to your end inlet and side outlet that is basically suppose, if you have a thing like here and thing like this the side view, the front view or maybe you can have a configuration like this you know you look it look at this from a front view maybe you can see something like this. This is your length l_2, this is the length L and this is the length l_1.

The muffler configuration at the top that is this one is called an end inlet and a side outlet. And, when you have the ports both on the side surfaces you know inlet can be here, outlet can be here. The angle included angle need not be 90 degree always as is shown here, it could be something else as well. But, the fact remains that if you have for side inlet and a side outlet configuration.

If you have an angle of 90 degree you would be much better of as far as increase in the broadband attenuation band is concerned. We will probably worry about all those things when we consider 3-dimensional wave propagation within the chamber analytically or through some numerical schemes like finite elements, but what these configurations show you let me show you nice schematic figures somewhere here for the side inlet and a side outlet.



Angle between them can be generally you know theta and with respect to say one of the ends it is it can be lS_1 or lS_2 here I have drawn it to be l_1 and l_2 and the total length of the chamber being l in the bottom figure that of the schematic that I just drew. So, this is a side inlet and a side outlet configuration.



Another configuration is basically end inlet and a side outlet. So, note that in the schematic that had just drawn you know the port was concentric, the end port was concentric it need not be. However, for design purpose is best to have a concentric inlet port all these things like I said will be taken up when we do 3D analysis.

But, as far as planar wave propagation is concerned, the angular location of the port on the end surface is sort of immaterial. It does not matter whether it is concentric or it is a it is located at an offset or things like that. Basically, all that matters all that bothers is that we are concerned with only planar wave propagation. So, regardless of wherever the center is it does not matter.

So, in this configuration we see the waves entering where the inlet flow is entering here and it is outlet at the side port and the included angle is generally θE_1, S_1, where E and S stands for the end and side ports respectively and inlet and outlet ports have the same diameter.

Well, the idea is that the side inlet and a side outlet is equivalent to a extended inlet and an extended outlet expansion chamber muffler as far as planar wave propagation is concerned. Because the transfer matrices that relate point say 1, I am sorry not here point 1 and point 2 here.

So, the transfer matrices will be of the same form exactly the same form as that of the extended inlet and outlet chamber we will show that how because of the cavities that are formed they are like your quarter wave resonators at the cavities that are formed between the port and the end phase.

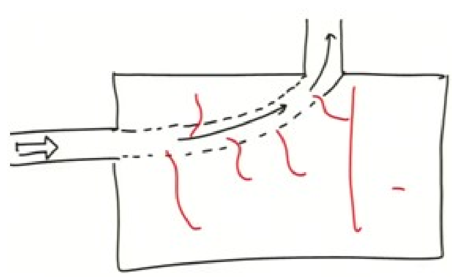
And, the cavities that are formed here in the chamber that is shown on the top this also acts like a quarter wave resonator and this is a plane wave propagation path. So, this system acts like expansion chamber muffler with just an extended outlet or extended inlet in case if this is the inlet and this is the outlet.

So, what we will do in the next few minutes is possibly derive the transfer matrices that will be all for this class let us see how we go about it. So, let us consider the muffler configuration shown below that is the side inlet and side outlet configuration. And, before we move on actually I forgot to mention, why are these configurations important? Why do we study that? Because it is not always necessary that the ports have to be located on the end phase.

So, if you have a muffler like this, so, we have two end phases of a cylindrical chamber muffler, but the thing is that due to logistic reasons, we are forced to you know place or locate the center of pipes on the side of the curved surfaces. In such a case, the inlet can be on the side surface or the outlet can be on the side surface as purely due to logistics.

And, actually it turns out that sometimes these logistic arrangements are good as far as the attenuation is concerned, because we will see that the cavities that are formed in the space between the port and the nearest end phase they act like the quarter wave resonator.

So, their function is like I said similar to the extended inlet and outlet chamber. However, this obviously, comes at a cost. What is the cost? Although it does not it is not included in the mathematical model here, but for all practical purposes generally people do not simply put this kind of a configuration.



What they do is basically let us say if you are using a end inlet and a side outlet configuration. So, one good practice is not to allow the flow to gradually flow to just simply expand into the chamber. So, when the flow is coming something like this you know it is it expands out and all that creates lot of shear layer noise and it might be counterproductive.

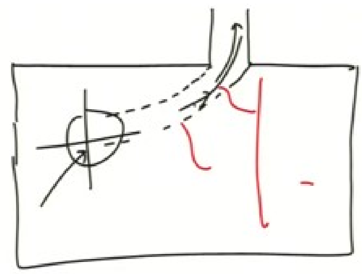
So, one good practice to have some kind of a bend, which connects gradually this thing and what we actually do is basically you know have some sort of a perforated surface you know all throughout. So, this entire cavity then is accessible. So, you have something like.

So, what happens is the flow is nicely smoothly guided from the inlet and outlet and the waves are able to go through the cavity here and interact and basically you know this is like a quarter wave resonator, this is your expansion chamber kind of behavior and similar logic would probably apply for your side inlet and side outlet chamber.

And, in fact, let me go back to this figure. It is very rarely that we simply in fact, we do not use simply an extended inlet and outlet chamber we actually connect this by a perforated bridge. Again, the same reason because all situations will have flow and if you were to just connect the inlet and outlet pipe and do not have anything between them, the shear layers will expand and will create a lot of noise. So, that will be counterproductive.

So, the flow has to be nicely guided, and the waves can then interact with the annular cavity. They can interact with annular cavity and acoustically they will be the same they can be highly porous we will probably talk about that when we consider things like perforated elements very crucial parts of muffler analysis. Since we are discussing this I guess it was important to mention that systems like this they are often connected through a perforated bridge a highly porous perforated bridge.

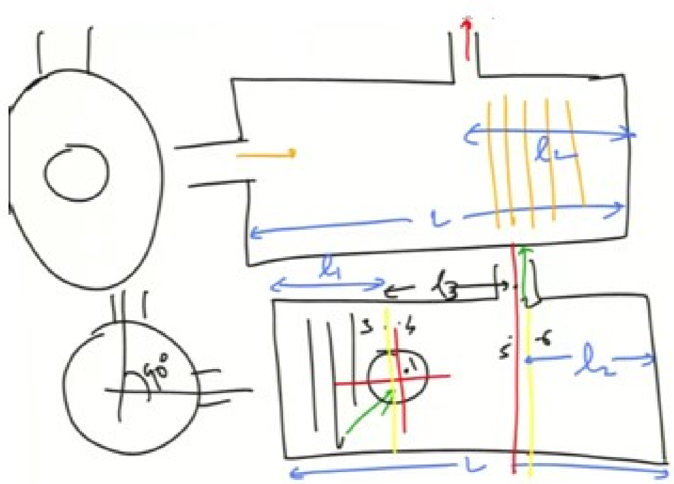
What it means that mechanically the flow is allowed to be guided smoothly through the pipe at the same time the porosity allows the waves to very freely interact with the annular cavity as if the perforated tube the bridge was not there. So, we have porosity as high as high 30 percent or so which means it is acoustically transparent, but mechanically the flow is guided



Similarly, we can have such a thing for this thing or possibly you know things like this for systems here. So, the flow comes here and it goes out and that sort of a thing. So, we have all sorts of things. We can connect it by a fully perforated bridge with highly porous thing.

In this lecture, let us quickly wrap up our transfer matrix modeling we will consider this system when the wave enters here it sees this as a cavity. So, what is the p_1 and v_1 relation? And let us also talk about the electro acoustic circuit for this.

So, for Side-Inlet and Side-Outlet Chamber, let us say this is p_1 and the wave that goes here let us say this point is 2 and the this point immediately within this part is 3 or probably since we have already used 2.



So, let me name it something different 3, 4, this is 5 and this is 6.

$$\tilde{p}_1 = \tilde{p}_3 = \tilde{p}_4$$

$$\tilde{V}_1 = \tilde{V}_3 = \tilde{V}_4 = \frac{\tilde{V}_3}{\tilde{p}_3}\, p_4 + \tilde{V}_4$$

$$\tilde{V}_1 = \frac{1}{Z_1}\, \tilde{p}_4 + \tilde{V}_4$$

So, when we write this is a transfer matrix what we will get

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{V}_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_1} & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_4 \\ \tilde{V}_4 \end{Bmatrix}$$

$$\begin{Bmatrix} \tilde{p}_4 \\ \tilde{V}_4 \end{Bmatrix} = \begin{bmatrix} C & jYS \\ \frac{jS}{Y} & C \end{bmatrix} \begin{Bmatrix} \tilde{p}_5 \\ \tilde{V}_5 \end{Bmatrix}$$

$$C = cosk_0 l_3$$

$$S = sink_0 l_3$$

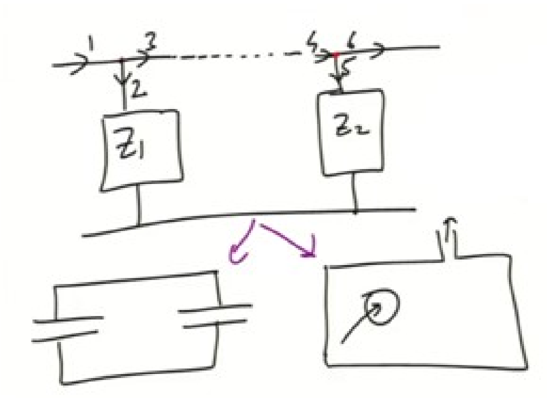
$$l_3 \quad = L - l - l_2$$

Remember, this was l_1 this was l_2 and so, the intermediate length is l_3 which is what I just wrote now l_3. Similarly, the final transfer matrix between now, this transfer matrix is not mathematically it is the same as that of the extended inlet and outlet. The only difference between being the port is not, you know extended in into the chamber it is basically flush mounted on the side surface.

$$\begin{Bmatrix} \tilde{p}_5 \\ \tilde{V}_5 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_2 \\ \tilde{V}_2 \end{Bmatrix} \; Z_2 = \frac{\tilde{p}_6}{\tilde{p}_6}$$

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{V}_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_1} & 1 \end{bmatrix} \begin{bmatrix} C & jYS \\ \frac{jS}{Y} & C \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_2 \\ \tilde{V}_2 \end{Bmatrix}$$

So, this is what we are going to get the transfer matrix relation for the side inlet and side outlet chamber as we discussed here, exactly the same as extended inlet and outlet.



Now, for both such systems side inlet and the side outlet and extensions system with extensions at the inlet and outlet the following electro acoustic circuit will hold. So, this is your Z_1 and this is the your transmission line or the propagation path, this is a shunt impedance at the outlet and then this is here.

So, basically one the velocity that comes here is goes into the shunt and the other goes here and then this is your 4, this is 5, this is 6, this splits into 2 parts.

$$Z_1 = -jY_C \, cot k_0 l_1$$

$$Z_2 = -jY_C \, cot k_0 l_2$$

So, you get such a thing this is this models both your system here shown here as well as system that is shown here where you have your thing like this in inlet outlet, side-inlet side-outlet.

So, this basically models both as far as plane wave propagation is concerned. Now, if for example, if you are dealing only with a end inlet and a side outlet case that is this one or probably better represented by this one, you will have only the matrix given. Let me sort of write it for you again. It is given by

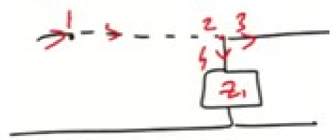
$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{V}_1 \end{Bmatrix} = \begin{bmatrix} C & jYS \\ \frac{jS}{Y} & C \end{bmatrix} \begin{Bmatrix} \tilde{p}_2 \\ \tilde{V}_2 \end{Bmatrix}$$

$$\begin{Bmatrix} \tilde{p}_2 \\ \tilde{V}_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_1} & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_3 \\ \tilde{V}_3 \end{Bmatrix}$$

So, now is just a matter of multiplying. So, what I will do, I will just get rid of this thing and will pull it here.

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{V}_1 \end{Bmatrix} = \begin{bmatrix} C & jYS \\ \frac{jS}{Y} & C \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_1} & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_3 \\ \tilde{V}_3 \end{Bmatrix}$$

So, basically you get this. You get this sort of a thing for end-inlet and side-outlet and needless to say the electro acoustic circuit will be something like this.



So, it comes here gets split into two parts. So, this is say 1, 2 well you can call this is 4, this is 3, this is say Z_1 electro acoustic circuit.

And, there is one more thing finally, at the at this point that I want to emphasize that you know all this while we did not consider the mean flow effect. So, it was a reciprocal system. So, what does acoustic reciprocity means? So, basically what it means is that if you change the positions of inlet and outlet. That is basically if you excite a system at a certain point and measure the outlet another point a response at the other point and keep the response aside.

And do the reverse that is the receiver point now becomes the source point and where the original source was there that becomes a response point. You see the response function in each of the case are identical or the transfer functions of the acoustic transfer function as it is called they are identical. So, basically so, this is the acoustic reciprocity.

Basically, what it says that if in if an acoustic system if the positions of the source and receivers are interchanged that the acoustic response function does not change. So, why am I telling you all of this suddenly because you know tip in if you change the positions of inlet and outlet, the transmission loss of such a thing will not change.

So, and incidentally using another representation named as impedance matrix which will cover in the probably week 7 or so, we will talk about all these concepts in a perhaps in a little bit greater detail. So, basically what it means if there is a flow then there is a certain direction there is a preferred direction for the wave to propagate so, but in the absence of flow, there is no direction.

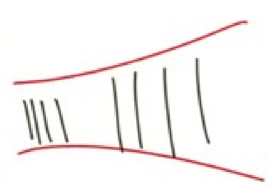
So, basically if you have a source at a certain point and receive another point that is take the measurement another point things will remain the change if you do if you basically reverse the system.

So, as a result if you have a side-inlet and end-outlet system although the transfer matrix the sum there will be some changes to the sins or maybe some terms will be inversed and all those sort of things because if you have a end-inlet and if you relating the outlet with the inlet then we have to invert the matrix.

So, some terms might be inverted that is fine some signs might also change based on the matrix multiplication rules and all that that is ok, but the transmission loss that will not change. That will be the same whether it is a side-inlet and an end-outlet or end-inlet and a side-outlet.

Similarly, if right now we talked all this file in the last lecture we talked about the extension being inlet extension being l by 2 and outlet extension being l by 4. If you do the opposite that is l by 4 for the extended inlet and l by 2 for the extend outlet things would not change provided that there is no flow. If flow is there then it obviously, complicates the matter and this quite a few things that could happen.

So, I guess these concluding remarks I would like to end here and, but before I do. So, just a brief summary of what you should be expecting in the next week that is week 6 set of lectures, lectures 1 to 5 of week 6 is that we will be introducing the concept of gradually varying area ducts.



So, basically we will be focusing on things like wave propagation through a gradually you know varying troughs. So, we will still assume planar waves, but we will see how you know, what is a wave propagation equation for such a duct what is a transfer matrix or simple shape like exponential horns, conical flares and those sort of things.

Because these systems are used you know in the inlet side of the muffler inlet portion of the muffler and especially when we go for a special kind of a perforated muffler. So, next set of lectures in week 6 or at least the first few lectures will be dedicated to a detailed analysis of the development of such equations and then transfer matrices.

And possibly the transmission loss of such system using only planar wave propagation; this is the thing that we will do before we move to much more complicated elements. So, with this I guess I will probably stop here. And I will see you in the next video.

Thanks.