Muffler Acoustics-Application to Automotive Exhaust Noise Control
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Lecture - 44 and 45
Plug Mufflers, Three-pass Perforated Element Muffler (Commercial Configurations) – MATLAB

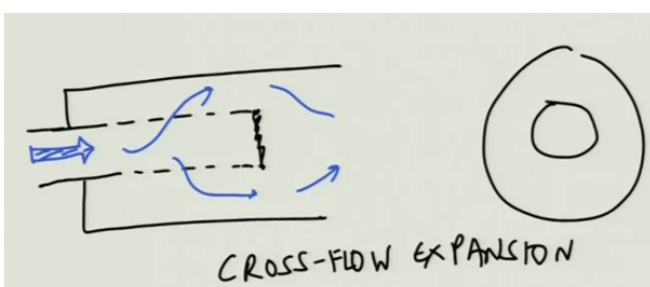
Welcome to lectures 4 and 5 of week 9, as like the previous lectures here also we will be combining you know the lectures two lectures into lectures, 4 and 5. And you know the focus of this week was really to you know analyze in detail even more realistic perforated muffler configuration, so we did cross flow configurations all right.

So, today also we will be we will be dealing with couple of more configurations pertaining to cross flow type. And in and in one of the configuration the second one that I am going to talk about today.

We are going to talk about where we have 3 perforated pipes and an outer jacket or outer shell that is housing all these things. So, that will be you know quite realistic and that is used quite a lot in the commercial vehicles.

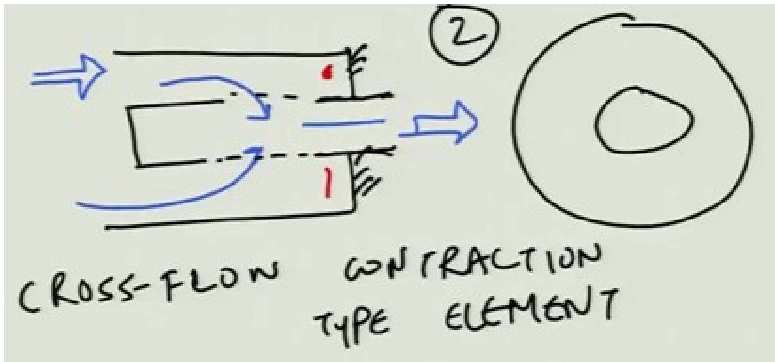
You will probably have a look at some of the only skim through some of the papers published of late regarding such you know double reversal and three perforated muffler configurations, and also possibly share with you some images. Now, before we sort of go into that, let us you know get back to our old friend that is basically a configuration this.

So, today is week 9, and a lectures 4 and 5 of this week .



This particular configuration a muffler and suppose we have such a kind of a thing and this is expanding into a bigger chamber.

So, this can be circular shape or elliptical shape or whatever it is. So, we have a flow, mean flow that is coming in from this part. So now, so this is a dead end, the flow really has no option but to kind of navigate through these perforated holes and go through this. So, what is this element called? This is called a cross-flow type, cross flow expansion ok.



We make another such configuration which is something like this, you know, and so it kind of goes like this. You can sort of draw this thing like this, and this can be extended over a certain thing, so this part is like a kind of resonator or something like that.

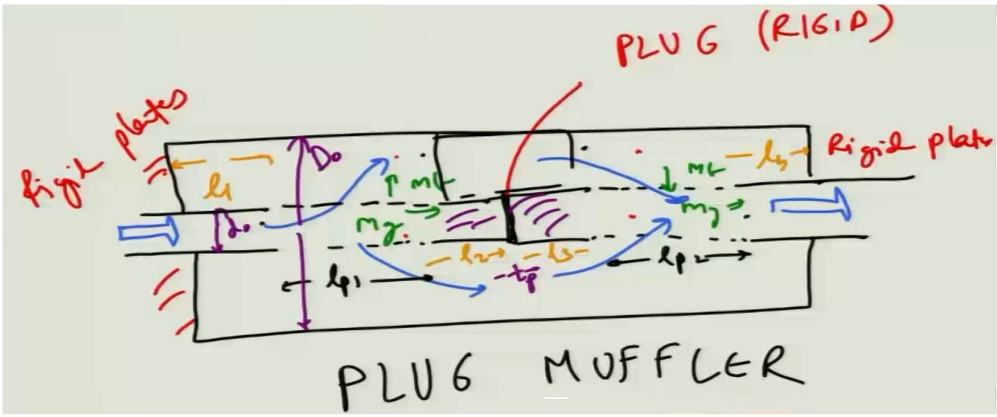
Anyhow, the flow you know comes through the annular surface or annular portion and it has no other choice, because this is a, this guy is a dead end, here like the side end plates of a expansion chamber.

It has flow has no other choice, but to go inside the duct, the perforated duct through this perforate, so it basically goes in something like here and then it finally, leaves the chamber like this. So, what is this configuration called? Very similar, this is called a cross flow contraction element, cross flow contraction type of element ok cross-flow contraction.

So, one question that comes to our mind is that what do we do with this element, how do we use them in practical muffler configuration. So, any so any guesses about that?

So this is not quite, this is an online class so we have to we cannot be really interactive, but there is something that you should probably have a look, I mean something try to you should try to think about different ways in which this configuration this one, let us say 1 and 2 can be combined to form to constitute one muffler, and we are trying to take advantage of this cross or dissipative losses that happens when occurs when flow has to basically cross through the chamber through the perforated pipe enter the chamber and again enter into this thing. So, one such configuration which is quite popular is called the is basically synthesized something like this ok.

So, let me just sort of draw this thing in a greater detail. So, this will be like this, a flow goes through this thing and has again forced to expand or again enter back ok.



And this is the, this guy is called the plug, rigid plug and these are rigid blades, these ones alright. This is called a plug muffler a plug muffler ok.

Let us sort of analyze or label the different geometrical dimensions of the plug, so we will we will basically you know call let us start naming this thing. So, this we will call this as l p_1 from here to here, and this is l p_2, these means the length of the perforated section 1 and 2. So, the flow has no choice, but to leave the chamber from here and then come back here.

So, you know as we can recall from our last lectures, only for the sake of simplicity we assume that the racing flow is sort of uniform all across here and then it is there is some uniform bias flow across this thing, and there is a convective uniform convective flow across this thing and then some uniformity is there everywhere.

But, you know if you do a proper computational fluid dynamic analysis of such flows in such bounded media in these mufflers, you will realize that you know this grazing flow gradually sort of decreases and becomes almost 0 here, and the bias flow tends to increase or more towards this side.

And then there is convective flow which is also non uniform across along the space, and then the grazing flow inside this duct tends to increase as we go towards the end side. So, we can do a proper CFD analysis, but then there are segmentation approaches also which assumes some sort of a linearly decreasing mean flow profile which is more kind of complicated.

What possibly we can do is that, we can assume for the purpose or the sake of this course and not for research purpose, what we can do is that we can just assume things to be uniform alright. So mg, the grazing flow is uniform here, it is uniform here, and there is a bias flow that is leaving the duct perforated duct and entering the chamber and it is here leaving the chamber and entering the duct mb.

So, this is assumed to be uniform and there are some relations that we derived previously we can use them. Now what about the other things, you know. So, let me use some other color, so this is l_1 this guy, this is l_2, this is l_3, this is l_4 ok. So, the net or the complete length of the chamber will be capital L = l_1,

$$L = l_1 + l_{p1} + l_2 + l_3 + l_{p2} + l_4 + t_p$$

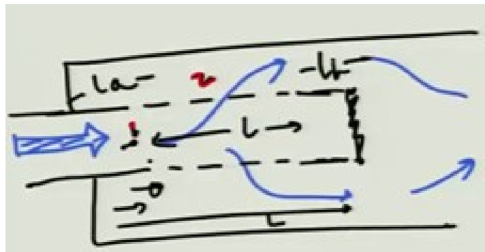
and if we consider this as the chamber length d_0, this is D_0.

And we of course, we sort of neglect the length of this thing, but you can actually consider the thickness of this thing to be t, you can you might just want to add t thickness here of the plug t_p.

You will see the transmission loss of such a thing would be better than your simple expansion chamber of course, and it would be sort of somewhere intermediate compared to the ECTR that is your extended concentric tube resonator, but you know I say intermediate because you know it definitely has more the amount of attenuation that is produced is much more compared to an ECTR, but at the same time there are more number of dips, the number of troughs and dips will be a bit more.

So, let us let us sort of analyze such a configuration. So, this is clearly a two duct configuration you know, this one and this one and this one and this one. So, I need not write down the equations again for this thing, you can have a look at the previous slides in the same two duct interacting thing. However, what needs to be done is possibly going back to configuration 1 and 2 and writing down the appropriate boundary conditions for such a thing ok.

So, writing an appropriate boundary condition is a must. So, we will see how do we will go back to 1, configuration 1 and start sort of writing down the appropriate boundary condition.



So, there is one small thing that I need to do, so you know this is let us call this length Lb, un-perforated length, this is La and this is L ok, so this is L. What we will do is that we will write down? Relevant boundary condition in this figure itself. So let me sort of just rub, put one here. What happens that the section let us call this as 0 and this is L, so this is duct 1, duct 2, let me write it with red color so this easy.

So, you have $Z_2(0) = \frac{p_2(0)}{-U_2(0)}$, because we have to reverse the sign.

$$Z_2(0) = \frac{p_2(0)}{-U_2(0)} = j\rho_0 C_0 Cot k_0 la$$

$$Z_1(L) = \frac{p_1(0)}{U_1(0)} = j\rho_0 C_0 Cot k_0 lb$$

So, this is what we sort of get, ok for this cross flow expansion chain type of a configuration. These are the boundary conditions. So, one can once you apply that you can relate the variables you know at this section, 0 you know somewhere in the duct here to the variables here, and then once you relate the transfer matrix when you get the transfer matrix at this point to this point, you know let us you know put

$$(p_1)_0 = [T](p_2)_L.$$

And then similarly, if we go back to if you go to such a contraction type element you know let me sort of rub this.

So, what are the boundary condition? This is again your Z = 0, Z = l. So, what are the boundary conditions? So this is duct 1 duct 2 as usual. So this will be Z_1.

$$Z_1(0) = \frac{p_1(0)}{-U_1(0)} = -j\rho_0 C_0 Cotk_0 lb$$

$$Z_2(L) = \frac{p_2(L)}{-U_2(L)} = -j\rho_0 C_0 Cotk_0 la$$

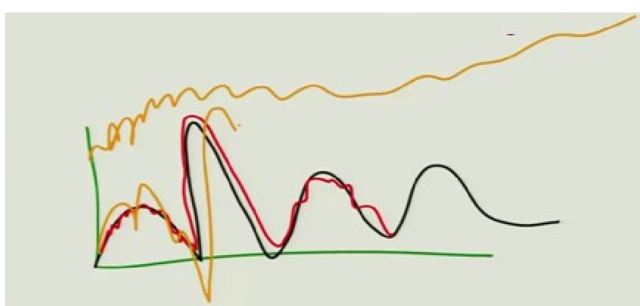
So, these are the two boundary condition, and at the end of the day, what we really need to do is that sort of you know relate the transfer matrices, basically you know for a reversal chamber kind of a configuration, what we need to do is that basically get the transfer matrix from you know this point right up to this point ok. So, we can write things like p_2, what was the previous one?

The previous one was ok this one actually there should be p u_1 here also and u_2 stuff here also. So, that is understood, but still. So let me call this as trans [T^2] matrix ok, and

$$\begin{pmatrix} p_2 \\ U_2 \end{pmatrix}_0 = (T^2)\begin{pmatrix} p_1 \\ U_1 \end{pmatrix}_L.$$

And, and then the intermediate one, now the thing is that once having obtained the transfer matrix using these boundary conditions you know from this point really to somewhere here and from this point really to somewhere here, the intermediate transfer matrix you know this one also has to be obtained and right and these are the anyways the cavities that are formed.

So, we this I mean this particular stuff, this is already taken care of when you apply this boundary condition. So, all we need to do is basically for a plug type of muffler which is very practical implementation of such a thing.



$$\begin{Bmatrix} p_1 \\ \rho_0 C_0 U_1 \end{Bmatrix}_0 = (T^1)(T_{\ Interm})(T^2)\begin{Bmatrix} p_1 \\ \rho_0 C_0 U_1 \end{Bmatrix}_L$$

We just simply need to multiply you know let us say p_1 U_1 of course, there will be $\rho_0 C_0$ stuff here at 0.

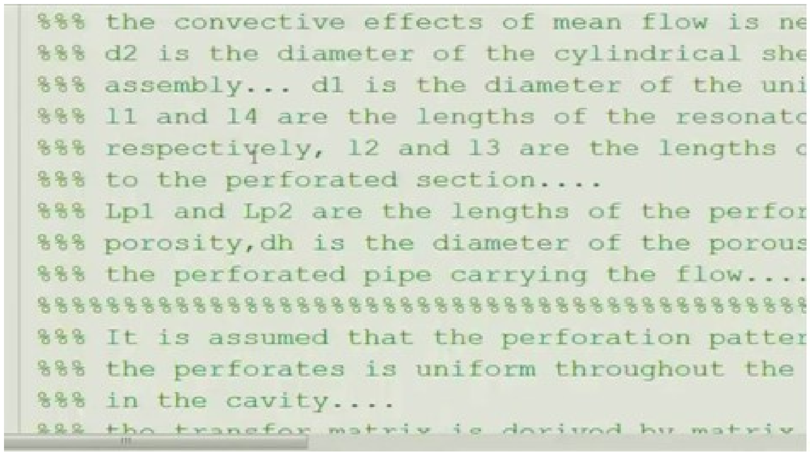
So, this will be T_1 matrix and T intermediate, that is basically nothing, but expansion chamber kind of a stuff will be there, and T_2 will be there and p again p, but at a different location say you know L, you can say L. So, these boundary conditions T_1 and T_2 already incorporate the boundary conditions for the resonator kind of a thing that we sort of see here.

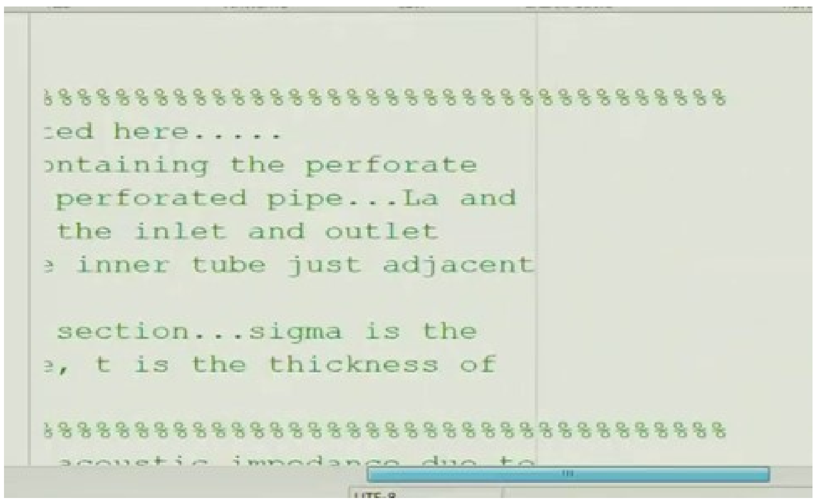
And you know the resonators formed somewhere here and resonators that are formed here and here, in this thing resonator that is formed here and in this part and as well as in sort of this part. So, we just need to simply multiply the transfer matrix to get the plug muffler analyzed and maybe we can consider some end correction also. I have not talked about end correction, but there are certain expressions I just give a glimpse of that later.

But what we need to do now is really basically go to the Matlab code for doing a relevant stuff. So, for such a muffler.

```
function [] =transmission_loss_plot(d1,d2,l1,
tic;
f=frange1:1:frange2;
n1=size(f); n=n1(1,2);
for i=1:n
    Tl(i)=transmission_loss(d1,d2,l1,Lp1,l2,l
end
plot(f,Tl,'k');
grid minor
xlabel('Frequency (Hz) ');
ylabel('Transmission  loss (dB) ');
toc;
```

```
function [Tf_pv]=single_plug_muffler_uniform_
c0=343;
j=sqrt(-1);
omega=2*pi*freq;
k0=omega/c0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% the convective effects of mean flow is ne
%%% d2 is the diameter of the cylindrical she
%%% assembly... d1 is the diameter of the uni
%%% l1 and l4 are the lengths of the resonatc
%%% respectively, l2 and l3 are the lengths c
%%% to the perforated section....
%%% Lp1 and Lp2 are the lengths of the perfor
%%% porosity dh is the diameter of the porous
```





So, let us go to the Matlab code. So, let us first analyze in the commented section. So, this one thing that I want to talk about doing a good coding stuff is that, you should be knowing whenever you code, you should be basically giving a lot of comments, like the ones that you see here, the lot of comments.

So, these this file was written some time back, but I can still exactly know what part of the code is doing what just because I have put appropriate comments. For example, let me very quickly walk you through the code, the convective effects of main flow is ignored is sort of neglected in the big interacting you know two duct interaction matrix. They also have ignored the convective effects as you will see you know from these expressions.

```
A=zeros(4,4);   %%% A is the matrix to be inte
   % zeta=(6*10^-3 + j*k0*( t + (0.75*dh) ) )

     mb = mg*(d1/(4*Lp1));
     zeta = perforate_impedance_singlepipe(k0

    A(1,2)=-j*k0;
    A(2,1)=-( j*k0 + (4/(d1*zeta)) );
    A(2,3)=(4/(d1*zeta));
    A(3,4)=-j*k0;
    A(4,1)=(4*d1)/(zeta*(d2^2-d1^2));
    A(4,3)=-j*k0 -(4*d1)/((d2^2-d1^2)*zeta);
T=expm(A*Lp1);  %%%% integrated the system ma
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

I really have not done b inverse a, is just you know I am happy to have some sort of algebraic simplifications at the expense of a small error in the computations or you know rather than you know breaking my head and you know doing that. So, the dissipative effects as you know are much more important, it lowers the peak and increases the trough raises the trough. So, those effects are seen. Now, let us quickly have a look at the configuration.

So, d_1 and d_2 is the diameter of the cylindrical shell containing the perforate assembly that is your, what am I talking about? I am talking about this D_0. Well, the symbols used in the code are sort of different, but still shell means the entire thing that you see you know the entire thing here ok. So, now, going over to the Matlab code d2 is the diameter of the cylindrical shell, d1 is the diameter of the uniform perforated pipe.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
eglected here.....
ell containing the perforate
.form perforated pipe...
or at the inlet and outlet
of the inner tube just adjacent

rated section...sigma is the
s hole, t is the thickness of

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

So the length is uniform, and forget about this La you know this something I do not know I have written this.

So, save the guy $l1$ and $l4$ are the lengths of the resonators at the ends of the inner tube just adjacent to the perforated section. $l\ p_1$ and $l\ p_2$ are the lengths of the perforated sections, so they are I mean there is a there is a flexibility of choosing different length and as usual d h and t h or t is the thickness of the perforate parameters.

So, I will not explain that so, but basically what is important here l one and $_l4$ are the lengths of the resonators of the inlet and outlet. So, let us go back to the thing.

So, following the same convention as Matlab code $l1$ is the length of the resonator here, $l4$ is here $l2$ and $l3$ are the this guy and this guy are the length of the perforate adjacent to the length of the un-perforated section, that is a solid pipe or resonator formed due to the pipe itself just adjacent to the to the perforated section, is not it.

So, you know just have a look at the other length of the inner tube just adjacent to the perforate section L_p and L_1 are the length. So, if you go back $l\ p_1$ you know and $l\ p_2$. So, you know that is the sort of thing that is there in the code.

```
%%% coding for the first perforated section b
mg=0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A=zeros(4,4);  %%% A is the matrix to be inte
   % zeta=(6*10^-3 + j*k0*( t + (0.75*dh) ) )

     mb = mg*(d1/(4*Lp1));
     zeta = perforate_impedance_singlepipe(kC

    A(1,2)=-j*k0;
    A(2,1)=-( j*k0 + (4/(d1*zeta)) );
    A(2,3)=(4/(d1*zeta));
    A(3,4)=-j*k0;
```

```
    A(3,4)=-j*k0;
    A(4,1)=(4*d1)/(zeta*(d2^2-d1^2));
    A(4,3)=-j*k0 -(4*d1)/((d2^2-d1^2)*zeta);
T=expm(A*Lp1);  %%%% integrated the system ma
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% applying the boundary conditions and rela
A1=T(1,1) + (j*cot(k0*l2)*T(2,1));
B1=T(1,2) + (j*cot(k0*l2)*T(2,2));
C1=T(1,3) + (j*cot(k0*l2)*T(2,3));
D1=T(1,4) + (j*cot(k0*l2)*T(2,4));
mat1=zeros(2,2); %%%% transfer matrix relatir
%%%% conditions.......
den1=(j*cot(k0*l1)*C1) + D1;
```

And, you know I have assumed certain well grazing flow I have set it to 0 here and bias flow of course, 0 is a stationary medium. So, I have used expression which I sort of trust more the modern day expression.

So, you know T matrix for the perforated section is found out and boundary conditions are applied it is going to be pretty tedious so I am not going to walk you through the entire thing, but this is what we get and then this further simplified and some in some inverse matrix is calculated to get it in this form, you know p_2 that is the p this thing.

```
.4));
: matrix relating acoustic variables downstre


*j*cot(k0*l1))+T(3,4) )/den1 )*A1;
*j*cot(k0*l1))+T(3,4) )/den1 )*B1;
*j*cot(k0*l1))+T(4,4) )/den1 )*A1;
*j*cot(k0*l1))+T(4,4) )/den1 )*B1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
(l1) rho0*c0*u1(l1) ] = Tf1*[ p2(l1+Lp1) rho0
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
forated section after the plug ....
atrix to be integrated...
```

```
    B(4,1)=4/(d1*zeta);
    B(4,3)= -(j*k0 + (4/(d1*zeta)));
Q=expm(B*Lp2);  %%%% integrated the system ma
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% applying the boundary conditions and rela
A2=Q(1,1) + (j*cot(k0*l4)*Q(2,1));
B2=Q(1,2) + (j*cot(k0*l4)*Q(2,2));
C2=Q(1,3) + (j*cot(k0*l4)*Q(2,3));
D2=Q(1,4) + (j*cot(k0*l4)*Q(2,4));
mat2=zeros(2,2); %%%% transfer matrix relatin
%%%% conditions.......
den2=(j*cot(k0*l3)*C2) + D2;
mat2(1,1)=Q(3,1) - ( ( (Q(3,3)*j*cot(k0*l3))+
```

```
mat2(1,1)=Q(3,1) - ( ( (Q(3,3)*j*cot(k0*l3))+
mat2(1,2)=Q(3,2) - ( ( (Q(3,3)*j*cot(k0*l3))+
mat2(2,1)=Q(4,1) - ( ( (Q(4,3)*j*cot(k0*l3))+
mat2(2,2)=Q(4,2) - ( ( (Q(4,3)*j*cot(k0*l3))+
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Tf2=inv(mat2);  %%% i.e. [ p2(l1+Lp1+l2+l3) r
%%%%%%%%%%%%%%%%%%%        Tf2*[ p3(l1+Lp1+l2+l
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Tf3=[ cos(k0*(l2+l3)), j*sin(k0*(l2+l3)); j*s
%%%%% Tf3 is the transfer matrix relating acc
%%%%% downstream in the annular region or the
%%%%% i.e. [ p2(l1+Lp1) rho0*c0*u2(l1+Lp1) ]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
 (Q(3,3)*j*cot(k0*l3))+Q(3,4) )/den2 )*A2;
 (Q(3,3)*j*cot(k0*l3))+Q(3,4) )/den2 )*B2;
 (Q(4,3)*j*cot(k0*l3))+Q(4,4) )/den2 )*A2;
 (Q(4,3)*j*cot(k0*l3))+Q(4,4) )/den2 )*B2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
e. [ p2(l1+Lp1+l2+l3) rho0*c0*u2(l1+Lp1+l2+l3
   Tf2*[ p3(l1+Lp1+l2+l3+Lp2) rho0*c0*u3(l1+L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 j*sin(k0*(l2+l3)); j*sin(k0*(l2+l3)), cos(k0
fer matrix relating acoustic variables upstre
e annular region or the jacket......
) rho0*c0*u2(l1+Lp1) ] = Tf3*[ p2(l1+Lp1+l2+l
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f is the final transfer matrix relating acous
```

And now for the coding for the second perforated section after the plug, so we get this.

And then we again have to apply boundary conditions and invert the matrix, and then this is the intermediate transfer matrix relating the ups and downs and even variable and then you simply multiply as I have shown.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Tf3=[ cos(k0*(l2+l3)), j*sin(k0*(l2+l3)); j*s
%%%%% Tf3 is the transfer matrix relating acc
%%%%% downstream in the annular region or the
%%%%% i.e. [ p2(l1+Lp1) rho0*c0*u2(l1+Lp1) ]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Tf=Tf1*Tf3*Tf2;  %%% Tf is the final transfer
%%%%%%%%%%%%%%%%%%%% the beginning of the inn
%%%%%%%%%%%%%%%%%%%%  perforated section in t
%%%%%%%%%%%%%%%%%%%% i.e. [p1(l1) rho0*c0*u1(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
S_inn=(pi*d1^2)/4;
Tf_pv=[Tf(1,1),((Tf(1,2)*c0)/S_inn);(Tf(2,1)*
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
s(k0*(l2+l3)), j*sin(k0*(l2+l3)); j*sin(k0*(l
3 is the transfer matrix relating acoustic va
wnstream in the annular region or the jacket.
e. [ p2(l1+Lp1) rho0*c0*u2(l1+Lp1) ] = Tf3*[
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f3*Tf2;  %%% Tf is the final transfer matrix
%%%%%%%%%%%% the beginning of the inner pipe
%%%%%%%%%%%%  perforated section in the inner
%%%%%%%%%%%% i.e. [p1(l1) rho0*c0*u1(l1)]=Tf*
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
i*d1^2)/4;
f(1,1),((Tf(1,2)*c0)/S_inn);(Tf(2,1)*S_inn)/c
```

```
function [Tl]=transmission_loss(d1,d2,l1,Lp1,
c0=343;  %%%% speed of sound
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Y1=c0/(pi*(d1/2)^2);
Y2=c0/(pi*(d1/2)^2);  %%%% impedances of the
[Tf]=single_plug_muffler_uniform_concenric_tu
%%% computing Transfer matrix between inlet/c
v=abs(     (Tf(1,1)+(Tf(1,2)/Y2)+(Tf(2,1)*Y1)+
Tl=20*log10(v/2);
```

And then you finally, sort of put it in the pV form, rather than p_0, that kind of form. And, this function gives the output of a transfer final transfer matrix which is obtained here and then you get your this thing ok.

```
function [] =transmission_loss_plot(d1,d2,l1,
tic;
f=frange1:1:frange2;
n1=size(f); n=n1(1,2);
for i=1:n
    Tl(i)=transmission_loss(d1,d2,l1,Lp1,l2,l
end
plot(f,Tl,'k');
grid minor
xlabel('Frequency (Hz) ');
ylabel('Transmission  loss (dB) ');
toc;
```

```
] =transmission_loss_plot(d1,d2,l1,Lp1,l2,l3,

1:frange2;
; n=n1(1,2);

transmission_loss(d1,d2,l1,Lp1,l2,l3,Lp2,l4,s

'k');

equency (Hz) ');
ansmission  loss (dB) ');
```

```
function [] =transmission_loss_plot()

d1 = 40/1000;
d2=d1;
l1=0.2;
Lp1=0.1;
l2=0.1;
l3=l2;
Lp2=0.1;
l4=0.2;
sigma=30/100;
dh=3/1000;
t=3/1000;
```

What we have the parameters, ideally you could have done in the file, but what we could do is that this is going to be a little sort of tedious so I sort of would not do that, bear with me for a minute and I make some changes in the code.

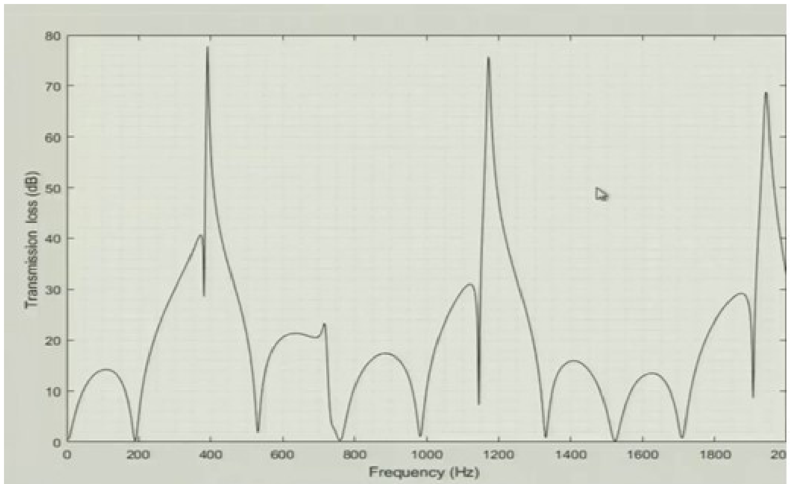
So, what I have done is that, basically I have made some small changes in the code not much. Instead of just entering these one on the same file, whenever we need to do parametric studies we simply need to change these values without having to bug the user.

Before I hit run, so in a perforated duct diameter is 40 and I guess I have made a mistake here. So, this d_2 was d_2 cannot be equal to this thing. So, d_2 the diameter of the cylindrical shell, my mistake.

```
function [] =transmission_loss_plot()

d1 = 50/1000;
d2=150/1000;
l1=0.2;
Lp1=0.15;
l2=0.1;
l3=l2;
Lp2=0.15;
l4=0.2;
sigma=15/100;
dh=3/1000;
t=3/1000;
```

So, what I am going to do, let us say this is about 150 mm and this can be actually a bit more also 50 mm perhaps. And l 1 and l 4 length has the same 0.2 to 100 mm, perforated section is about only about 100 mm, you can make it more, you can make it something like 150 mm.

And the lengths that are formed just adjacent to the perforated pipe in each tube is 100 mm 100 mm each. 30 percent porosity you can use a smaller value, it is known to cause a little bit numerical instability. So, there is lot of scope of research also, we will talk about that in a bit.



So, I have hit run, the code is sort of being executed. This is the you know transmission loss for ignore the peaks, they are sort of a little abnormal in the sense that you do not really have that much attenuation flow will always dampen the things and anyways we are interested not exactly in the amount of attenuation produced at the peaks.

So, you know, but one thing is there that for such a chamber your this thing you know this will be holding good at least for about 1000 hertz or so. I am giving a rough idea of the cut on frequency, you can anyway estimate it using the $k_0\, r_0$ is equal to 1.84 or in this case is because it is axisymmetric is probably $k_0\, r_0 = 3.83$.

So, you can find out giving the by taking the value of r_0 corresponding to 150 by 2, 75 mm and 343 sound speed you can get a decent idea of the cut on frequency the first mode beyond which things will deteriorate. So, until that thing happens, you know what we can sort of do is that we can keep analyze, we can consider the performance transmission loss predictions using this 1 dimensional plane wave theory.

Seriously, only until about 800 to 1000 hertz, beyond which deviations are bound to occur, they will not be that much accurate. Now, we have done this and porosity value was set as 15 percent. Now, what we could do is that you know, we have not really considered mean flow in this thing, let us consider a small amount of mean flow.

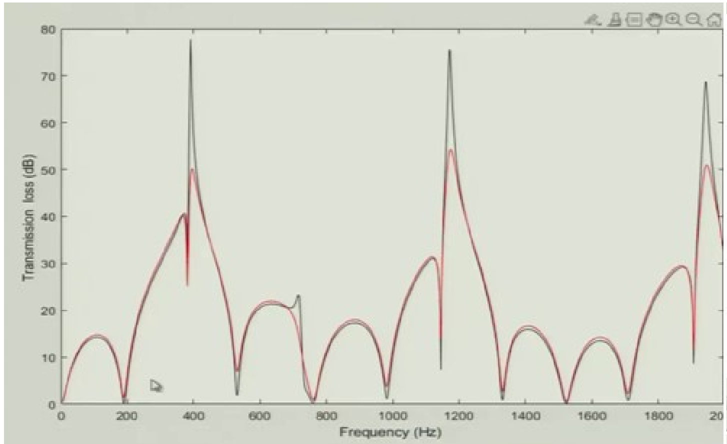
So, you know the transmission and loss expression will not change because 1 plus m i plus 1 plus m o value that will be same, because the diameter of the plugs are the same or the perforated pipe are same.

```
%%% in the cavity....
%%% the transfer matrix is derived by matrix
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% coding for the first perforated section b
mg=0.05;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A=zeros(4,4);   %%% A is the matrix to be inte
   % zeta=(6*10^-3 + j*k0*( t + (0.75*dh) ) )

      mb = mg*(d1/(4*Lp1));
      zeta = perforate_impedance_singlepipe(k0

     A(1,2)=-j*k0;
     A(2,1)=-( j*k0 + (4/(d1*zeta)) );
```

```
Command Window
New to MATLAB? See resources for Getting Started.
>> transmission_loss_plot()
Elapsed time is 8.236115 seconds.
>> hold on
>> transmission_loss_plot()
Elapsed time is 1.159789 seconds.
>> transmission_loss_plot()
Elapsed time is 1.214816 seconds.
fx >>
```

```
frange2=2000;
f=frange1:2:frange2;
n1=size(f); n=n1(1,2);
tic;

for i=1:n
    Tl(i)=transmission_loss(d1,d2,l1,Lp1,l2,
end
plot(f,Tl,'r');
grid minor
xlabel('Frequency (Hz) ');
ylabel('Transmission  loss (dB) ');
toc;
```

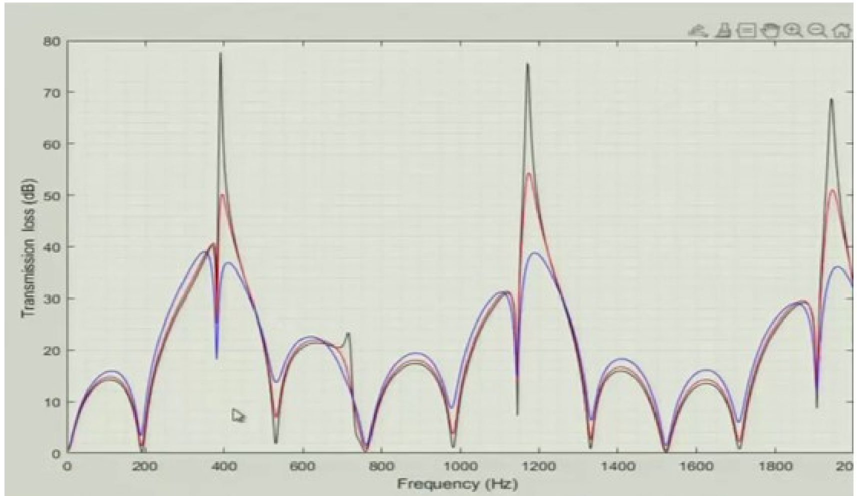
So, let us consider a small amount of mean flow, so there will be grazing bias flow also using this expression. So, I will do hold on hold on and I need to change the color in the code and put r, stands for red.



Peaks have started to come down troughs have raised the same phenomena happening over and over again. So, we will you know sort of give a little higher value let us say we give 0.2.

```
%%% coding for the first perforated section b
mg=0.2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A=zeros(4,4);  %%% A is the matrix to be inte
   % zeta=(6*10^-3 + j*k0*( t + (0.75*dh) ) )

     mb = mg*(d1/(4*Lp1));
     zeta = perforate_impedance_singlepipe(k0

    A(1,2)=-j*k0;
    A(2,1)=-( j*k0 + (4/(d1*zeta)) );
    A(2,3)=(4/(d1*zeta));
    A(3,4)=-j*k0;
```

```
frange2=2000;
f=frange1:2:frange2;
n1=size(f); n=n1(1,2);
tic;

for i=1:n
    Tl(i)=transmission_loss(d1,d2,l1,Lp1,l2,
end
plot(f,Tl,'b');
grid minor
xlabel('Frequency (Hz) ');
ylabel('Transmission  loss (dB) ');
toc;
```



And plot this guy in perhaps blue color. Now interesting thing that has happened that the troughs have raised, there is no you know the peaks are no longer that sharp, they are kind of much shorter and fatter, they spread out, but the troughs have rays. So, mean flow definitely has a leveling effect, we can clearly see that and basically you still get decent amount of attenuation even in a very low frequency, as much as 35 degree 40 degree flow really guide so there is no flow separation noise that would have otherwise occurred you know.

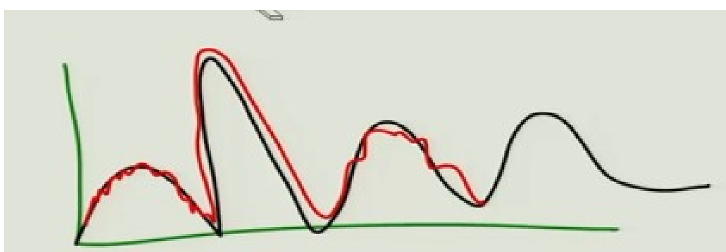
And, these kind of things are there. So, it is all good. So, you know and this is incidentally the same kind of a behavior that we see published in the literature when we have a you know plug muffler analyzed you know, these kind of domes and troughs and then the peaks based on the length of the resonator that will also be sort of formed.

So, we stand with what we sort of expected ok. Now, there is one thing that I wanted to sort of tell here. A general comment about the analyzing the perforated sections. So, basically what I am trying to say is, that you have your EXPM command that you are using in Matlab, like 15 20 years back when computational power of such functions were not that readily available, you know you still have to solve this X dash is equal to ax matrix.

So, it eventually to find out EXPM exponential of a matrix you need to find out the Eigen values, Eigen vectors. So, ξ into diagonal matrix of the Eigen values into ξ transpose, where ξ is the Eigen vector matrix, so we need to write those routines or use some linear algebra package and fortran or stuff like that. So, those are the standard techniques that you use and Matlab has quite good inbuilt solvers.

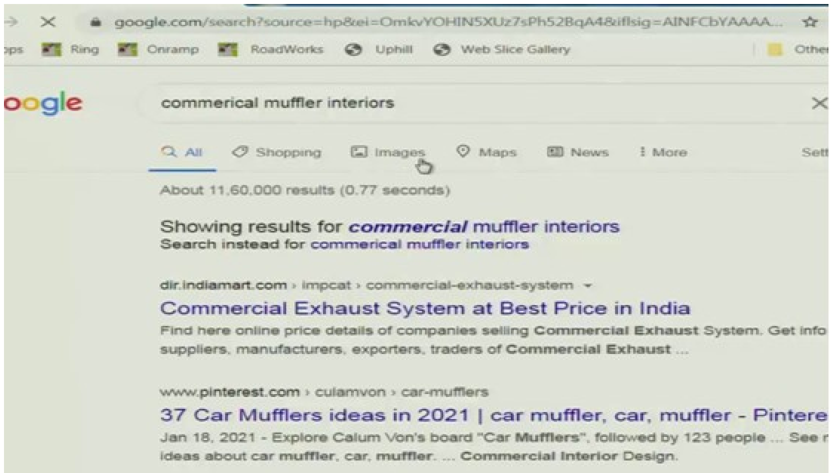
But the fact of the matter is that we still have the problems when you use EXPM command for certain parameters. You know you get these sort of a wavy kind let me go to the presentation you know, here and you know talk about what I mean is that you know typically you get this kind of a value then you know something like this. Now, for certain parameters just starts to do this and you know then it kind of stabilizes and all that.

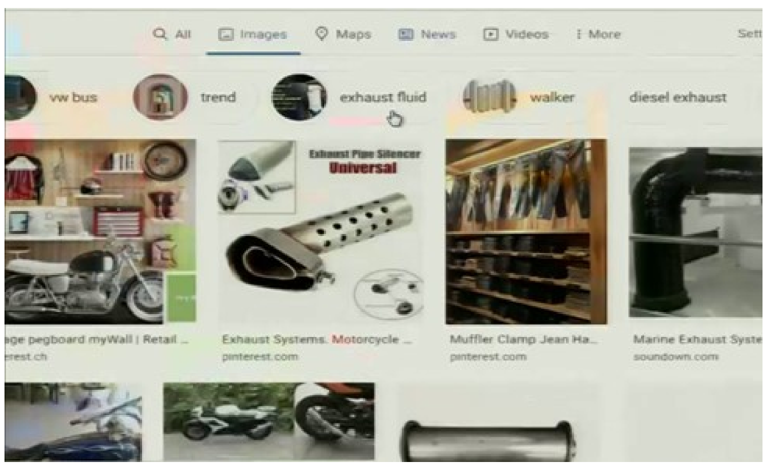
And suddenly you see for some parameters increases is doing completely garbage it is giving and its this is probably not the thing, but what I am saying is that you get a rapid oscillation something like this you know this massive instability numerical negative transmission loss and all that. So, you know these are really your numerical artifacts the EXPM command also is not able to probably you know kind out for the length considered, because you know eventually you are doing EXPM minus a into l.

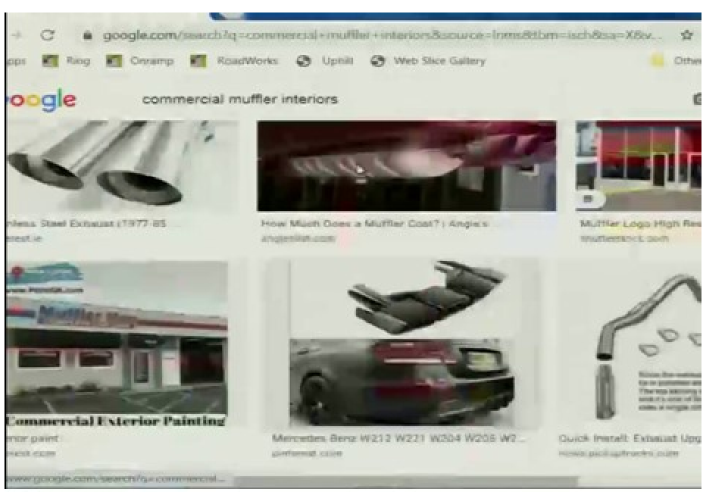


So, basically for a much larger length you know you have problems. So, there is one there is a decent enough scope of some additional investigations to be carried out in future in this thing. So, what probably could be done is that you could instead of directly using EXPM command, there are things like boundary condition transfer algorithm published in the paper by and Munjal in Jasa, acoustic society of America several years back.

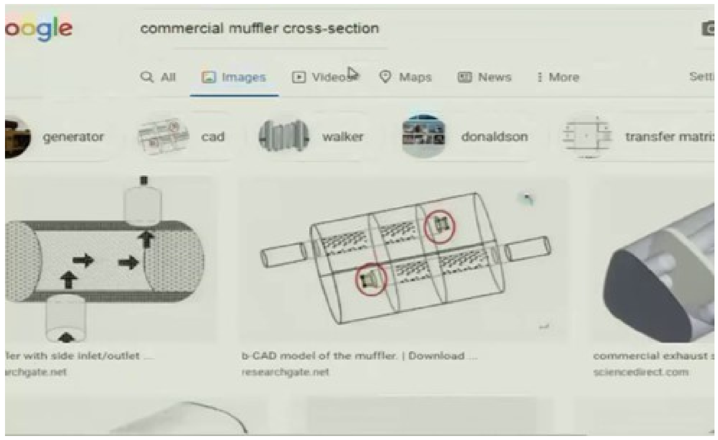
So, there are techniques where you could do that sort of a thing or possibly consider a better alternative to using EXPM command by using some numerical sort of techniques. These are eventually you know at the end of the day these are all constant coefficient ordinary differential equations coupled one of course. So, you know you might actually consider you know using some sort of a one dimensional finite element scheme or maybe one dimensional finite difference schemes with good frequency resolution, wavelength resolution capacity. So, those kind of things can be done, or some special novel techniques can be done. So this, the reason that I am telling you this is because you know all mufflers all commercial mufflers.

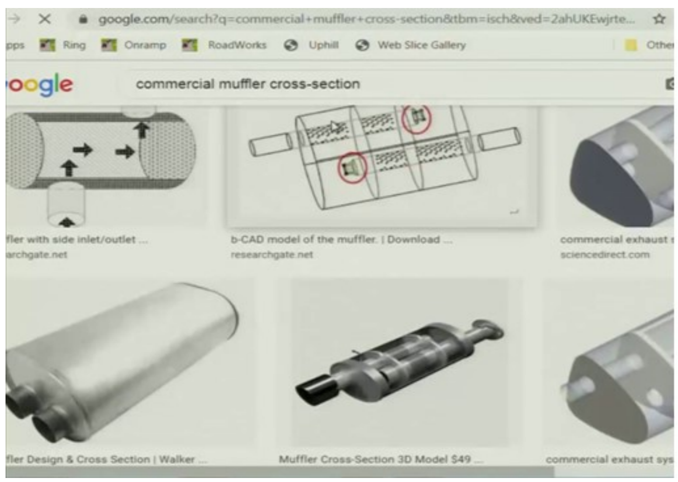


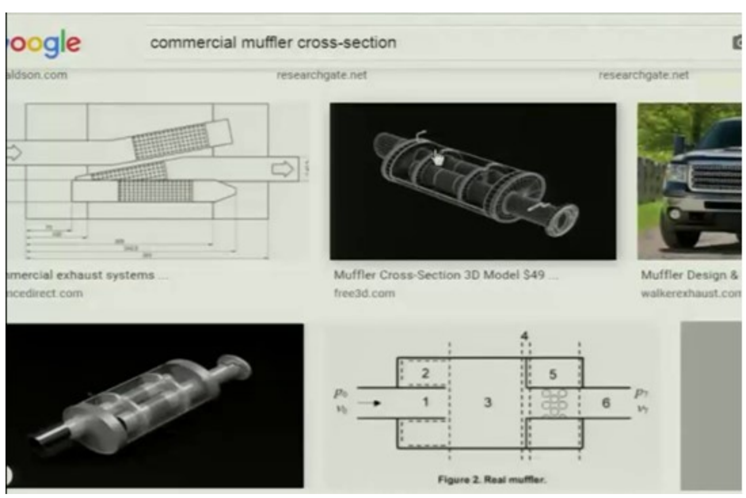




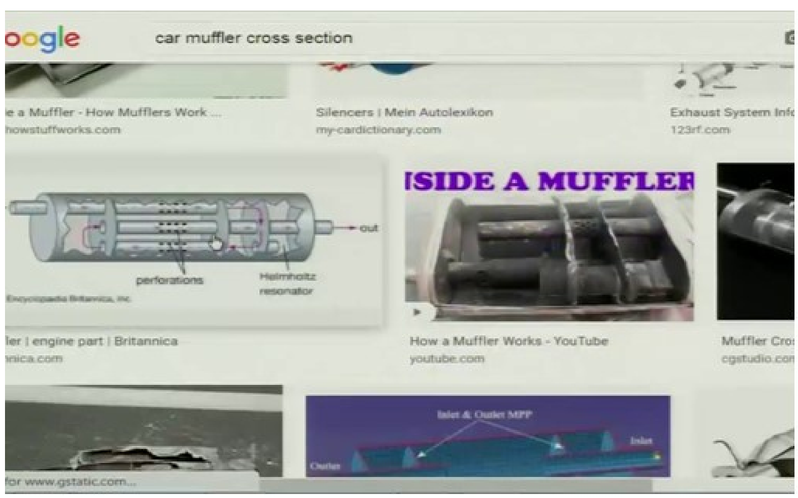
Let us go to Google, what we need to do is that just have a look at some of the commercially available muffler, interior commercial muffler interiors and you will see invariably the presence of perforated pipes in all the mufflers.

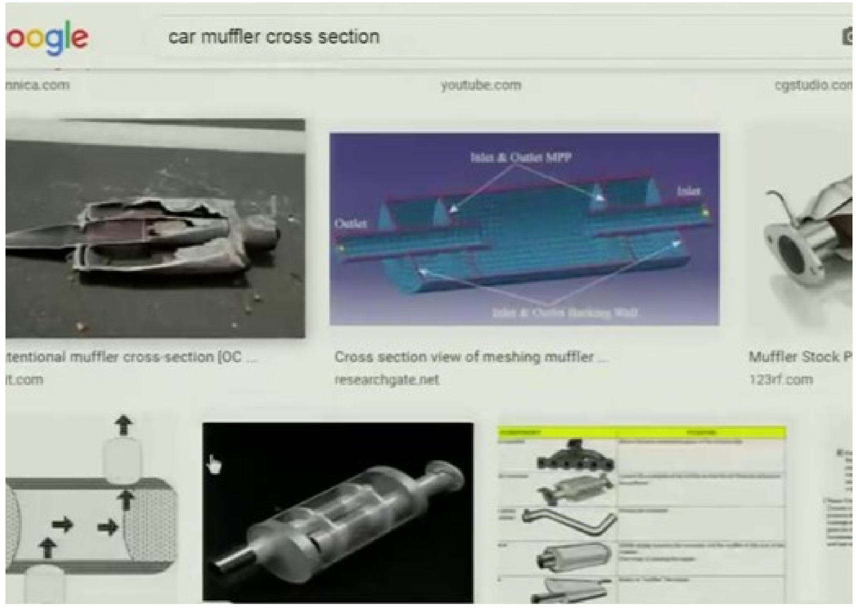


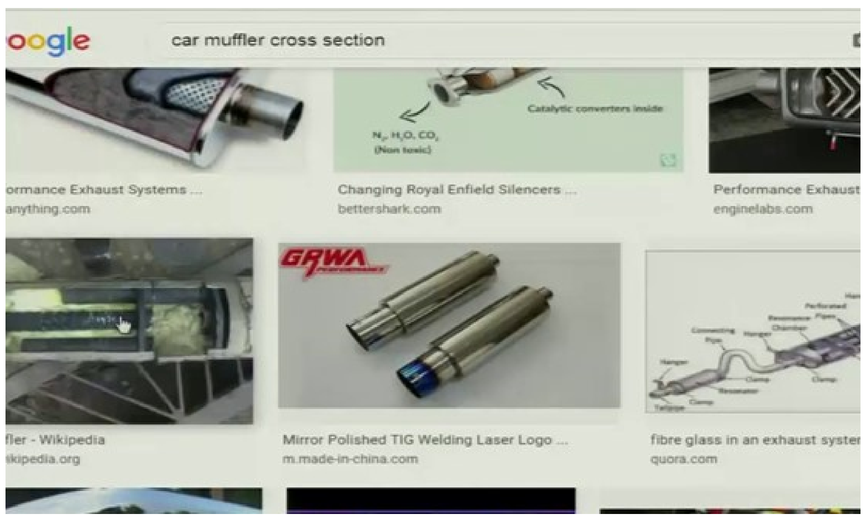




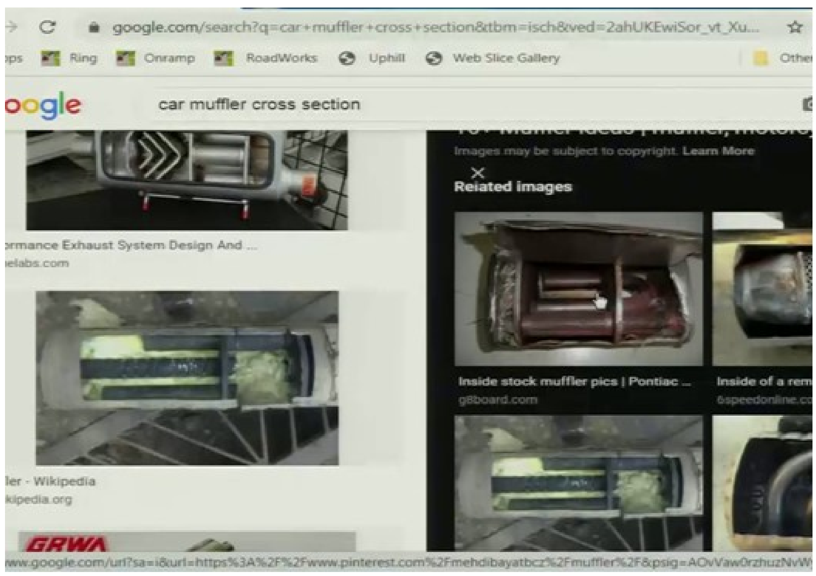
So, well these are some of the perforated things, but let us say muffler cross-section. These are simple google images that when I am trying to show you just right now. So, these are some other schematics, but what I really wanted to point out is that maybe some nice cool photographs you can have a look and sort of appreciate for yourself, what is it like.





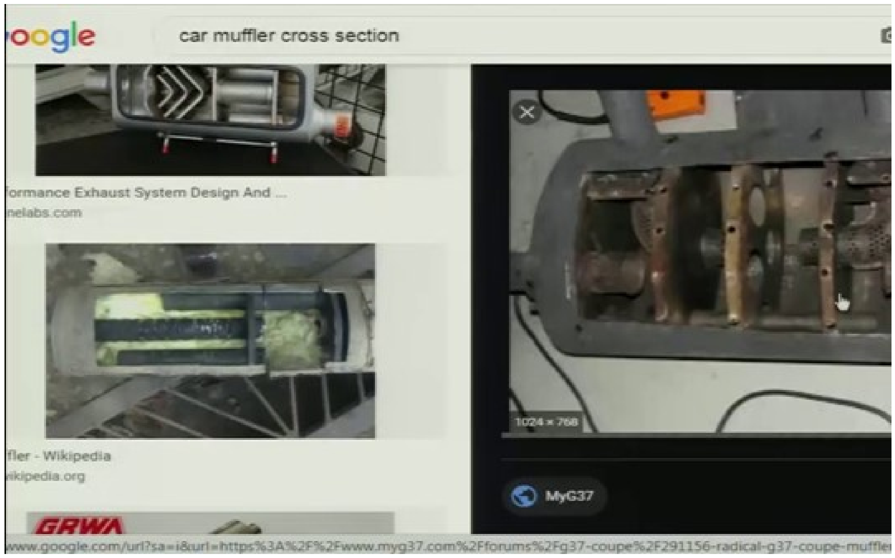


So, well you know let us say this one this is like a old and worn out muffler, but you know this is a you tube video it seems, and so the point is, that there is so many of them well all of them have perforated thing, let us have a look at this thing perhaps.



So, so these are you know on a different kind of a perforated holes basically in in a central pipe through which the waves are allowed to interact with annular cavity which is filled with an absorbent material.

So, this is something we have not really seen so far. Maybe this is a nice cool photograph in P interest you know the is in here and interact with angular cavity there are two interacting ducts that you sort of see here. Then they are you know this is yet another one, so look cavity there is a YouTube there is a u bend here, things like this and then there are.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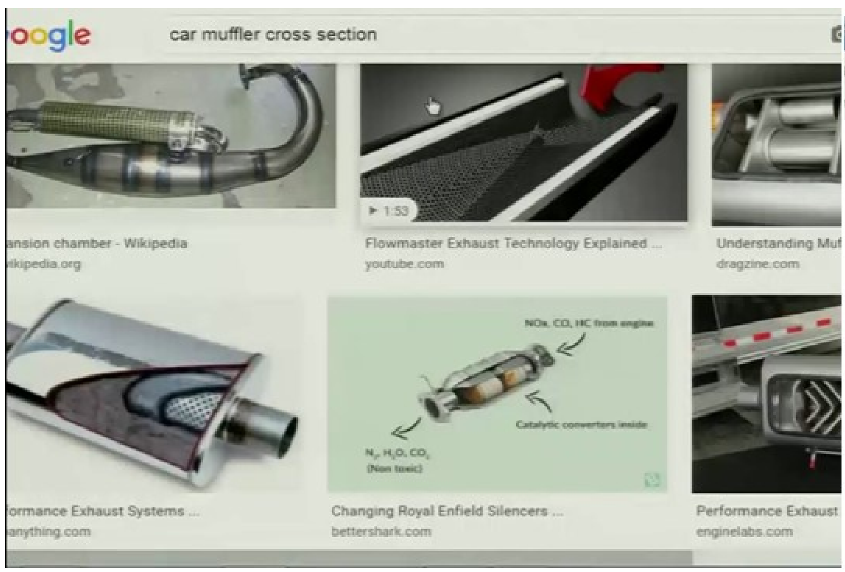
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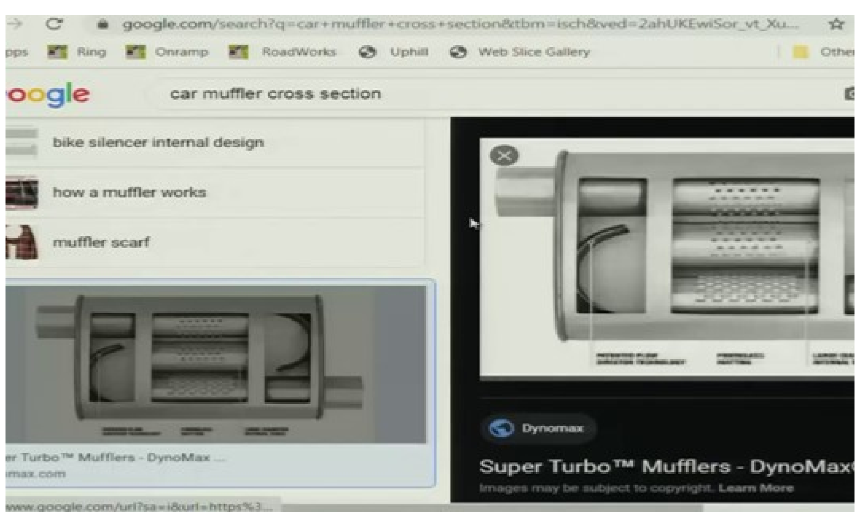
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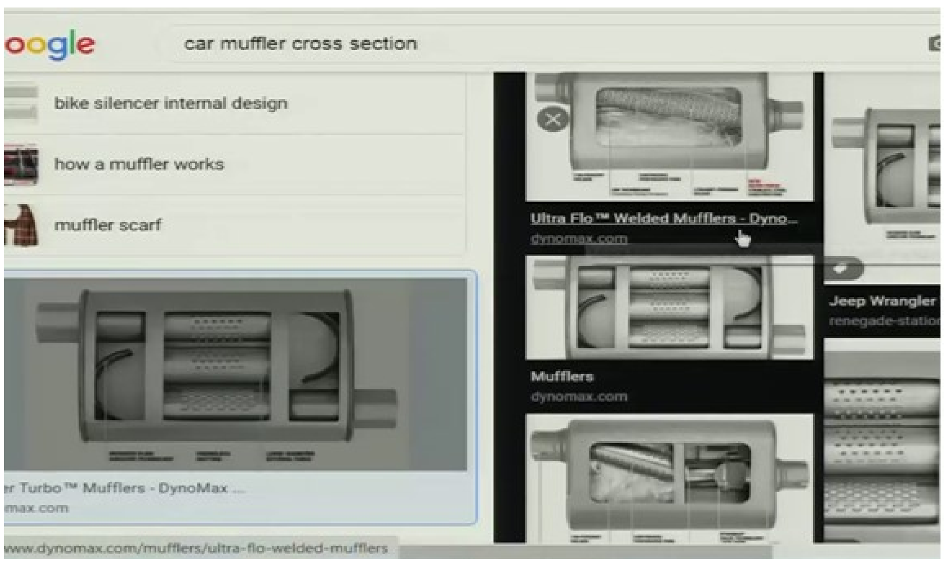
So, many heaps of them all of them have perforate pipes.

So, what exactly am I trying to say here you know, invariably they all always have you know perforated, muffler perforated elements like you can as you are seeing here and you know in these this is a nice photograph you know from some commercial company. So, this is the one that three pass muffler configuration flush tube what we would be analyzing just to give you a glimpse of how it works, and also have a look at some of the photographs there are unequal perforated patterns.

You can do a lot of things, lot of parametric studies and completely thoroughly analyze it, and these are n chambers which kind of facilitate flow reversal and waves propagate typically along the major axis, not along the axial length for such short upcoming book that is almost published now.

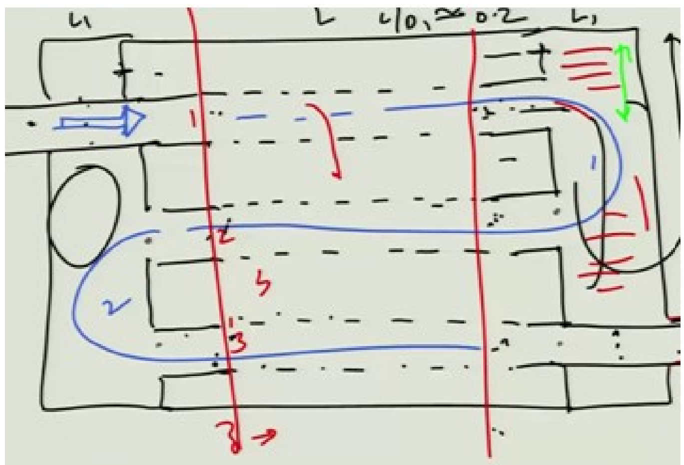


And so, the point I am trying to make, well you can keep looking at these photos on the internet at your leisure. The point I am trying to make is that there are so many muffler components and so many configurations which have. It becomes quite important for us to analyze or pay great attention to the accuracy of our transmission loss predictions for commercial mufflers, given their you know popularity.

So, either we need to develop codes which can you know resolve this in numerical artifacts or numerical instability issues that typically happen as we are seeing if we just change the parameters beyond a certain range.

Using some numerical scheme like you know finite elements or perhaps a finite difference as I was mentioning about you know, and maybe kind of instead of just using the EXPM command in a very brute force kind of an approach one could do a lot of other things as well, you know maybe divide the sections and then, maybe do some kind of a cascading and you know properly analyzing the round of errors that are happening.

So, this is sufficient room for you know computational accuracy of such systems. So, one really needs to think about analyze analysis of such a system. Now, we probably would like to go to three pass perforated duct mufflers. So, these are some of the photographs that you had seen just here. So this is a three duct perforated muffler which is flush tube. So let us let us first also have a look at some of the nice papers that have been published in this thing this is a very practical commercial muffler.



Let me draw such a configuration for you know, draw this three interacting ducts thing. So here we are and there are three ducts, so these are all perforated you know something like this. So, what really happened is this is called a pass tube you know.

So this region, this can be elliptical in cross section or possibly circular in cross section does not really matter as far as plane wave analysis is concerned. And, then you know you have your this kind of a thing. So, I will rub this guy and draw this thing here again. So, the waves really flow really comes here and you know the waves are it is kind of going the flow is allowed to its forced to take multiple things.

So, this is called a double reversal, the first reversal, second reversal and this is called a three pass thing.

So, this can be perforated throughout its length or you know it can be partially perforated forming cavities here and here resonator cavity is much like your extended ECTR element and here this length is pretty typically quite short L_1 this is much more shorter than the cross dimension, let us call it D_1 assuming this to be elliptical in section. So, $\frac{L}{D_1} \simeq 0.2$ for commercial mufflers.

So, because the space is really limited, most of the space is occupied by this entire thing ok. So, waves typically go along the major axis rather than going like this ok. So they do not travel like this. They travel in this manner, but this is you know this is really the least of our concern now, right now our concern is basically to figure out this three duct or actually four duct system and eventually you would end up with a matrix which resembles something like this.

So, this is going to be your pretty heavy duty matrix. So, if you, if you number, if you choose to number this as duct 1, 2, 3 and 4 is the annular thing, being the length measured in this direction and

$$\frac{d}{dz}\begin{Bmatrix} p_1 \\ \rho C_0 U_1 \\ p_2 \\ \rho_0 C_0 U_2 \\ p_3 \\ \rho_0 C_0 U_3 \\ p_4 \\ \rho_0 C_0 U_4 \end{Bmatrix} = \underset{8\times 8}{[A]} \begin{Bmatrix} p_1 \\ \rho C_0 U_1 \\ p_2 \\ \rho_0 C_0 U_2 \\ \vdots \\ p_4 \\ \rho_0 C_0 U_4 \end{Bmatrix}$$

So, you know you really have 8 cross 8 matrix. So, and then you know this we are gradually moving towards what is called a network analysis. So, that will be the part of the next weeks week 10s content, but before you know let me just tell you something that such systems you know become algebraically very tedious. So, instead of you know we definitely do have boundary conditions somewhere you know across this section and across this section.

Eventually what we would want to do is that you know get a transfer matrix relation between the points just here, this one, this guy, this guy and relating you know kind of sort of you know all the pressures and velocities here with those possibly at this point this point and this point is not it, and assuming that these ends are region you have some sort of a resonator cavity formed in here.

And, then once you from here to here is a simple tubular element transfer matrix, same thing from here to here and eventually we would we would also want a transfer matrix that is relating things somewhere here and here and the one that is related here and here. And these tubes are flush flushed with this end chamber they need not be, the length can be actually more also depending if the space is available typically it is not. If the more space is available you can have things like this here or you know you can typically have a whole cavity here.

So, then you will have multiple multiply connected muffler. Right now, you know let us analyze this thing a little more carefully, and you know what we typically do that the waves are allowed when the waves are allowed to interact with the annular duct through these holes, but they are also allowed to go through this thing. So, the flow however can take only one route, you know it has to bend through these things, typically it bends you one must still do a CFD analysis.

But what I am trying to say is that even in this, there is an element of multiple connections you know there you definitely have multiple connections. Suppose, you know this guy was not there, suppose this pass tube was not there ok you would just have this kind of a thing, then definitely the waves this is a this is for sure cannot be analyzed using a simple transfer matrix cascading approach as the one that we are going to talk about now.

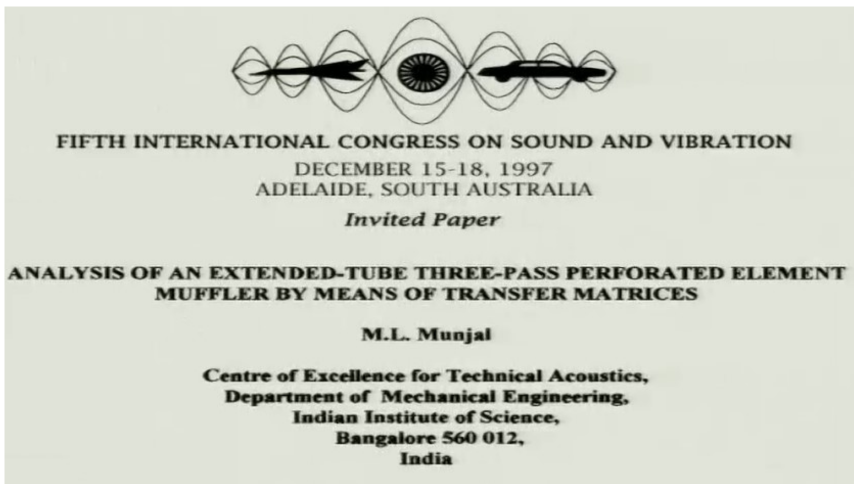
But, you know this is a multiple connection thing you know if the pass tube were not there. But, suppose this pass tube is here you know then definitely this is a non multiply connected muffler, but with certain amount of interactions are happening parallelly between the waves that are here and this cavity and the waves that go here.

Eventually, the since transfer matrix cascading can be used here, the point I am going to make is that at the end of the day is going to be pretty tedious. For the first things first we need to analyze or relate basically pressures $p_1 p_2 p_3 \rho_0 C_0 U_1$ three here is the ones at this section at z = l one including incorporating the boundary conditions that is here and here, resonator conditions, get the transfer matrix between this point all the way until this point and similarly the transfer matrix between this point and this point.

So, we have transfer matrix between this point and this point this point and this point in the entire transform matrix between relating 1, 2, 3 at z = 0 with variables at z = l, and then this is a simple tubular element and the transfer matrix between this and this really under really incorporates all that is happening inside this cavity and this cavity.

So, all that simple algebra, but it can still be you can do some sort of a cascading approach and get the transfer matrix between this point and go about you know finding out the transmission loss of such a muffler system.

So, like to go to a paper, this was your analysis of an extended tube three pass perforated element muffler the means of transfer matrices.



FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
DECEMBER 15-18, 1997
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Invited Paper

ANALYSIS OF AN EXTENDED-TUBE THREE-PASS PERFORATED ELEMENT MUFFLER BY MEANS OF TRANSFER MATRICES

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India

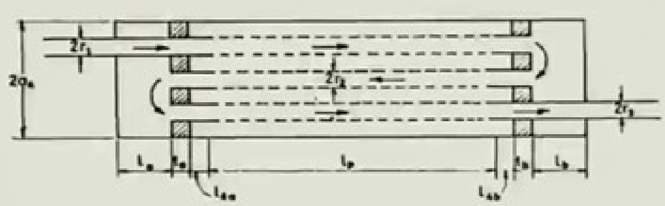


Fig.1. A functional sketch of a flush-tube three-pass perforated element chamber

So, you know the configuration that was analyzed was pretty much what I am talking about, except that the thickness effects of the end plate I mean the plates that demarcate or separate the perforated region between the end chambers have the thickness of such a plate has been considered while computing the transfer matrices.

So now, what is happening really is that you know this is a multiple interaction duct, so I am just going to rely on this paper and subsequent journal article was also published, but this is making use of the generalized decoupling approach well at that time it was a generalized decoupling approach, but what I am saying is that this entire thing once we get it in this form, for us it is a matter of using this command and relate upstream and downstream variables, may be like I was just hinting at developing clever numerical or computational strategies to analyze such things within the context of a plane wave analysis also.

So, plane wave is not all that bad, once we get S_1 S_2 S_3 and S_1 S_2 S_3 at 0 and l_p where S_1 is your you know pressure and volume velocity $\rho_0 C_0 U_0$ you know velocity.

$$\begin{bmatrix} S_2(0) \\ S_3(0) \end{bmatrix} = \begin{bmatrix} G & H & K \\ P & Q & R \end{bmatrix} \begin{bmatrix} S_2(lp) \\ S_3(lp) \end{bmatrix}$$

Where

$$\{S_1\} = [p_1 \; v_1]^T$$

And D,E,F,G,H,K,P,Q, and R are 2x2 sub-matrices. Explicit expressions for elements of all these sub-matrices have been derived in ref. 3.

$$\{S_1(l_p)\} = [A]\{S_2(l_p)\}$$

Where [A] is product of the matrices

So, once we get such a representation you know between 0 and l and then we have our transfer matrices. So, if you have a look at this thing you need to have a transfer matrix between this point and this point and this point and this point.

$$\{S_1(l_p)\} = [A]\{S_2(l_p)\}$$

$$\delta_1 = ect(0.6r_1)(1 - 125r_1/r_b) \qquad (3)$$

$$\delta_2 = ect(0.6r_2)(1 - 125r_2/r_b) \qquad (4)$$

So, that is so I am just trying to walk you through the derivation here. So, I may not go through each of them in a great detail, but just skim doing the derivation the state variables that z p in duct 1 may be related to those in duct 2 the right end cavity that in figure 3. The right hand cavity means what this one. So, between this point and this point incorporating the tubular elements, and there are some end corrections also in the length of the pipe which is ok.

$$r_b = (r_4^2 - r_3^2)^{1/2}, \text{ and ecf} = 1$$

$$r_{1,}r_{2,}r_{3,}r_4 = (d_{1,}d_{2,}d_{3,}d_4)/2$$

$$\{S_2(0)\} = [B]\{S_3(0)\} \qquad (5)$$

Matrix equation (1), (2) and (5) may now be combined to obtain the transfer matrix relation for the entire chamber (see Fig. 3):

$$\{S,(0)\} = \big[[D][A][W] + [E][W] + [F]\big]\{S,(l_p)\}$$

$$= [C]\{S,(l_p)\} \ (Say) \qquad (7)$$

$$[W] = \big[[G][A] + [H] - [B][P][A] - [B][Q]^{-1}\big]\big[[B][R] - [K]\big] \qquad (8)$$

And

$$[C] = [D][A][W] + E[W] + [F] \qquad (9)$$

So, you get this relation and then you get relation between the left hand cavity like this with certain end correction lengths all those things are there. And finally, S_1 0 is related to S_3 l_p you know eventually what we want, let me go back to this figure.

We want relation between pressure and velocity just you know somewhere here to those somewhere here, and then it is a tubular element, so it does not really matter what happens no matter the length whatever the length is.

So, that is what the author has done by subs by you know cleverly doing some clever algebraic manipulations, he has found out that $S_1$0 is related to S_3 l_p using a [C] matrix [C] matrix is a big [D][A][W] and all these matrix are there where your this matrix is your sub matrices really. So, it is written in a certain fashion, so I will probably write I mean I will show some Matlab coding results and [W] is your this matrix [C] is this matrix.

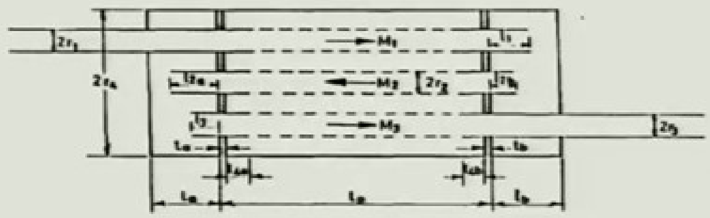


Fig.3. A functional sketch of the extended-tube three-pass perforated element chamber

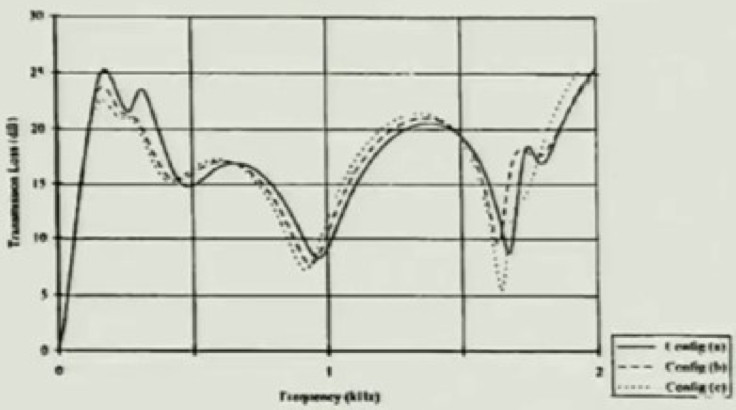


Fig. 5. Effect of the extended outlet tubes, with the inlet tubes being flush with the three-pass chamber.

So, I am sort of not going into the algebra because of want of time, but there are some parametric studies, the result of which I would sort of show. So, I am assuming mach number 0 only he is considered point 1 and there are some nice cool results that the author has published here, and then you have a longer chamber with tube protruding inside and then a transmission loss which.

The good thing about such configuration such perforated thing is that, you know the reason why do we use such a muffler? Because in automotive mufflers we do not have very much of a space and the flow within the constraints of a limited space and with the understanding that you can tolerate a much higher level of back pressure, you are trying to bend the pipes multiply so that the flow allowing the flow to go through number of perforated holes and turning and taking a number of turns so incurring losses.

So back pressure will be more, but it is not, but that not that much because you have number of interacting ducts. But at the same time because waves are allowed to take a larger path through perforated sections and you know resonator cavities and all that you would expect to have a number of peaks and sort of domes or things like that, eventually with the understanding that you hope to get some sort of a broadband at least some 10 db 15 db constant attenuation at beginning from very low frequency.

So, you know that is what plane wave analysis parametric studies promises you. Just vary the parameter and figure out one of them which gives you a good estimation and so, this is one of the curves that we get, broadband transmission loss and so, these kind of things you know happen here.

So, we do get you know such a, such a transmission loss configuration. So now the thing is that let us let us do some Matlab coding for certain parameters perforated length l a and l b are the length of the 70 mm, 100 mm at the length of the left and right cavities.

Dimension of the configuration of fig. 3 are:

$$l_a = 0.07m,\ l_p = 0.2m,\ l_b = 0.Im, \qquad l_{4a} = l_{4b} = 0.0055m,$$

$$t_a = t_b = 0.002m,\ porosities\ \sigma_1 = \sigma_2 = \sigma_3 = 0.05$$

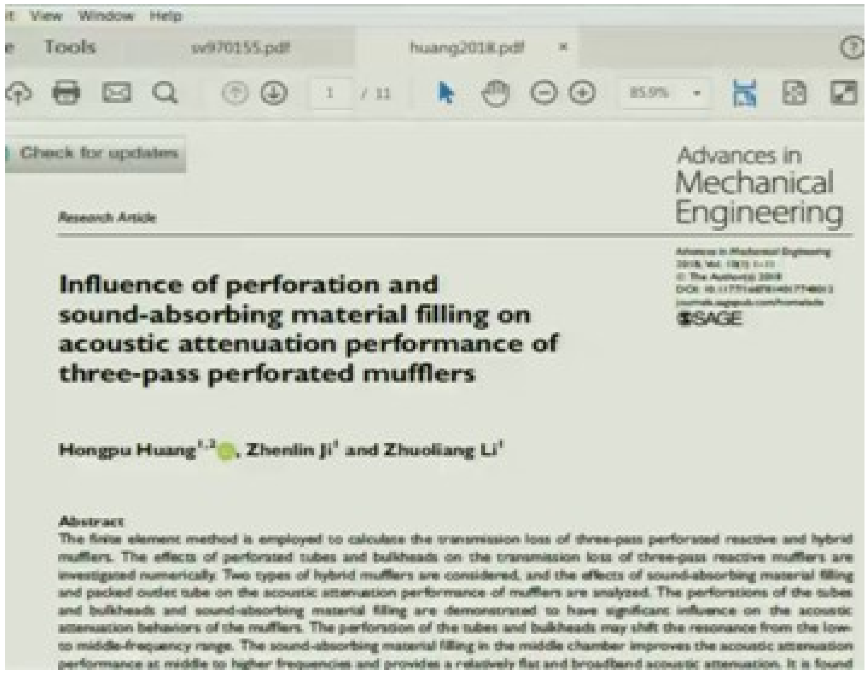
$$radii\ \ r_1 = r_3 = r_3 = 0.0246m, r_4 = 0.0825m,$$

$$wall\ thickness\ th_1 = th_2 = th_3 = 0.0008m,$$

$$hole\ diameters\ dh_1 = dh_2 = dh_3 = 0.0025m,$$

$$and\ the\ mean\ flow\ Mach\ number\ M_1 = M_2 = M_3 = 0.1$$

This is some connecting length about just about 5.5 mm or something like that, thickness of the pipe porosity is about only about 5 percent, the radius of the duct is something like this, thickness of the pipe and the whole diameter.



Advances in Mechanical Engineering

Research Article

Influence of perforation and sound-absorbing material filling on acoustic attenuation performance of three-pass perforated mufflers

Hongpu Huang[1,2], Zhenlin Ji[1] and Zhuoliang Li[1]

Abstract
The finite element method is employed to calculate the transmission loss of three-pass perforated reactive and hybrid mufflers. The effects of perforated tubes and bulkheads on the transmission loss of three-pass reactive mufflers are investigated numerically. Two types of hybrid mufflers are considered, and the effects of sound-absorbing material filling and packed outlet tube on the acoustic attenuation performance of mufflers are analyzed. The perforations of the tubes and bulkheads and sound-absorbing material filling are demonstrated to have significant influence on the acoustic attenuation behaviors of the mufflers. The perforation of the tubes and bulkheads may shift the resonance from the low- to middle-frequency range. The sound-absorbing material filling in the middle chamber improves the acoustic attenuation performance at middle to higher frequencies and provides a relatively flat and broadband acoustic attenuation. It is found

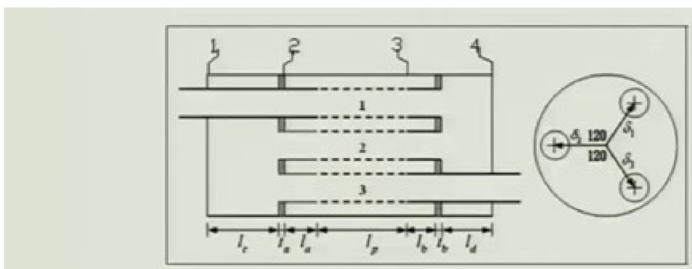


Figure 1. Three-pass perforated tube muffler with circular cross section.

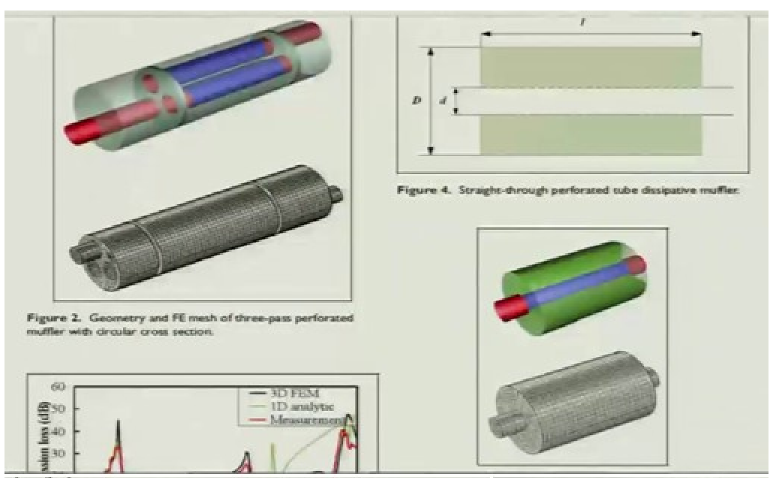
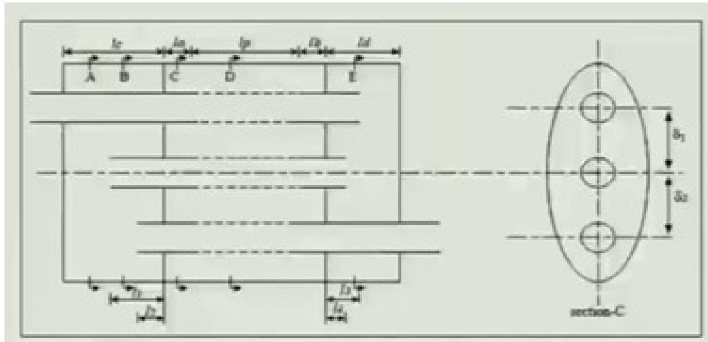


Figure 2. Geometry and FE mesh of three-pass perforated muffler with circular cross section.

Figure 4. Straight-through perforated tube dissipative muffler.



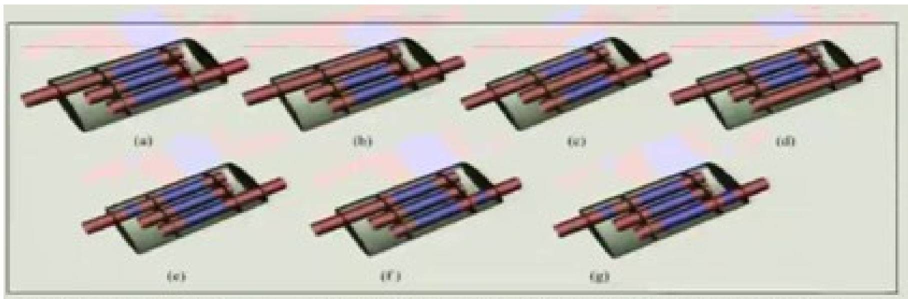
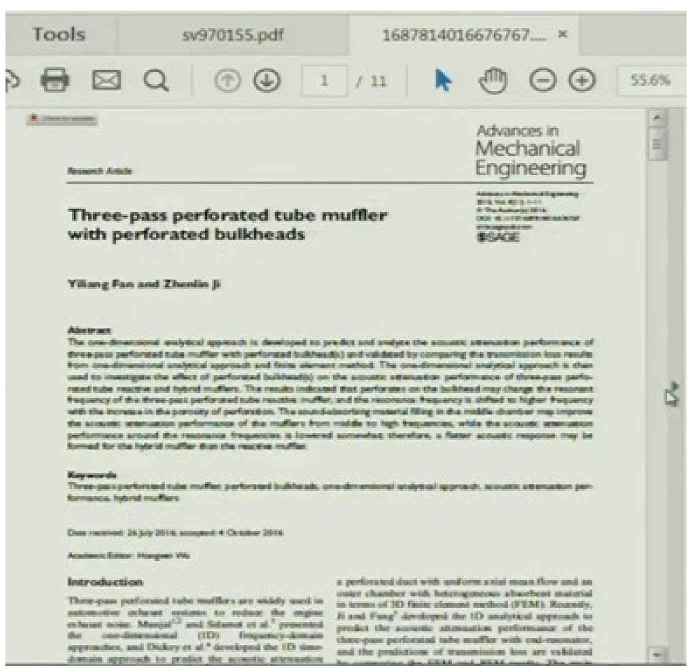


Figure 8. Configurations of three-pass reactive mufflers: (a) baseline, (b) solid inlet tube, (c) solid center tube, (d) solid outlet tube, (e) perforation on inlet tube within the inlet chamber, (f) perforation on outlet tube within the outlet chamber, and (g) perforations on inlet and outlet tubes within the inlet and outlet chambers (blue region: perforated portion; red region: solid portion).

So, those are some values and this like ever since this paper and you know this is probably another such configuration such 3 plus muffler configuration with circular cross section, exactly the one that I am talking about. Analyzed in comsol multi physics and they did some finite element modeling full 3D modeling and figured out some transmission loss curves and all that published in in 2018 really, so yeah.



Research Article

Advances in Mechanical Engineering

SAGE

Three-pass perforated tube muffler with perforated bulkheads

Yiliang Fan and Zhenlin Ji

Abstract

The one-dimensional analytical approach is developed to predict and analyze the acoustic attenuation performance of three-pass perforated tube muffler with perforated bulkhead(s) and validated by comparing the transmission loss results from one-dimensional analytical approach and finite element method. The one-dimensional analytical approach is then used to investigate the effect of perforated bulkhead(s) on the acoustic attenuation performance of three-pass perforated tube reactive and hybrid mufflers. The results indicated that perforates on the bulkhead may change the resonant frequency of the three-pass perforated tube reactive muffler, and the resonance frequency is shifted to higher frequency with the increase in the porosity of perforation. The sound-absorbing material filling in the middle chamber may improve the acoustic attenuation performance of the mufflers from middle to high frequencies, while the acoustic attenuation performance around the resonance frequencies is lowered somewhat; therefore, a flatter acoustic response may be formed for the hybrid muffler than the reactive muffler.

Keywords

Three-pass perforated tube muffler, perforated bulkheads, one-dimensional analytical approach, acoustic attenuation performance, hybrid mufflers

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Introduction

Three-pass perforated tube mufflers are widely used in automotive exhaust systems to reduce the engine exhaust noise. Munjal[1,2] and Selamet et al.[3] presented the one-dimensional (1D) frequency-domain approaches, and Dickey et al.[4] developed the 1D time-domain approach to predict the acoustic attenuation

a perforated duct with uniform axial mean flow and an outer chamber with heterogeneous absorbent material in terms of 3D finite element method (FEM). Recently, Ji and Fang[7] developed the 1D analytical approach to predict the acoustic attenuation performance of the three-pass perforated tube muffler with end-resonator, and the predictions of transmission loss are validated

So, all good, and then you have yet another paper with perforated bulkheads published in here. So, there are number of such sort of advancements that has really happened. So, what we will do is that we will quickly jump on to a Matlab code.

```
j=sqrt(-1);
c0=343.1382; %%%% speed of sound....
S1=pi*((d1/2)^2);  S2=pi*((d2/2)^2); S3=pi*((
Y1=c0/S1; Y2=c0/S2;Y3=c0/S3; Y4=c0/S4;
%%% cross-sectional areas and characteristic
k0=(2*pi*freq)/c0;
alpha=(D4^2)- ( (d1^2) + (d2^2) + (d3^2) );
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[imp] =perforate_impedance(pr1,pr2,pr3,t1,t2,
zeta1=imp(1);
zeta2=imp(2);
zeta3=imp(3);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
stic impedances........

 );
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
1,t2,t3,dh1,dh2,dh3,freq); %%% perforate impe



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
grated for realting state variables upstream
```

```
zeta1=imp(1);
zeta2=imp(2);
zeta3=imp(3);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A=zeros(8,8); %%%% the matrix to be integrat
              %%%% applying BC's......
%%%% p1 rho0*c0*u1
A(1,2)=-j*k0;
A(2,1)=(-j*k0)-(4/(d1*zeta1));  A(2,7)=4/(d1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% p2 rho0*c0*u2
A(3,4)=-j*k0;
A(4,3)=(-j*k0)-(4/(d2*zeta2));  A(4,7)=4/(d2
```

And, and the, let me also open the common perforate you know carefully have a look at these are all standard parameters by now, you should be familiar alpha is some value what I used in the last code.

So, you know net effective annular area and this is the perforate impedance some value to give the perforate impedance.

```
A(5,6)=-j*k0;
A(6,5)=(-j*k0)-(4/(d3*zeta3));   A(6,7)=4/(d3*
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% p4 rho0*c0*u4
A(7,8)=-j*k0;
A(8,1)=(4*d1)/(alpha*zeta1);     A(8,3)=(4*d2)
A(8,7)= -(j*k0) - ( (4/alpha)*( (d1/zeta1) +
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B=expm(-A*Lp);   %%%% calculated the matrix ex
%%%% converting into [p v ] form from [ p rho
%%% first the column operation.......
B(:,2)=B(:,2)*Y1;
B(:,4)=B(:,4)*Y2;
```

```
B(6,:)=B(6,:)/Y3;
B(8,:)=B(8,:)/Y4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% now the integrated matrix B is in [ p v]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Z1=-j*Y4*cot(k0*la); %%% impedance at the lef
Z2=-j*Y4*cot(k0*lb); %%% impedance at the rig
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C=zeros(6,6); %%% matrix after applying rigic
K1= Z1*( (B(8,7)*Z2) + B(8,8) ) + ( (B(7,7)*Z
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i=1:1:6  %%%% for row i...
    for k=1:1:6 %%% for column k...
```

Now, this is an 8 cross 8 matrix ok. Exactly what I was talking about. Momentum equation continuity in the duct 1 in the duct 2 duct 3 and finally, you have duct 4 where you have such a thing. All these effects are there mean flow has been not been considered and finally, you get EXPM times thing and get it in the p V form and possibly with some rearrangement I guess or probably not.

```
    for k=1:1:6 %%% for column k...
    fac=( (B(i,7)*Z2) + B(i,8) );  %%% factor
    C(i,k)= B(i,k) + fac*( (B(8,k)*Z1) + B(7,
    end
    clear fac
end
%%%     C(i,1)= B(i,1) + fac*( (B(8,1)*Z1) +
%%%     C(i,2)= B(i,2) + fac*( (B(8,2)*Z1) +
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% now, the transfer matrix C is  computed a
%%% BC's ...i.e. [p1 v1 p2 v2 p3 v3](z=0) = C
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T_mat = C;
```

And then applying the impedance conditions at the left and right of the thing and then relating and then sort of reducing it to the 6 cross 6 thing and finally, what you get is a 6 cross 6 matrix. So, this code computes the transfer matrix in the state variables forms up from upstream or the downstream to the common perforated section after applying rigid wall boundary conditions for the annular shell.

So here note that we are not applying the you know, if you go back to the presentation and so here when I talk about boundary conditions, I really mean applying the boundary conditions here and here I mean pertaining to the rigid end here. So, at this is at this section this is at this section ok, so that is what I do. So, Matlab coding can be fun provided that you know how to do it and keep manipulating things.

```
function [T_overall]=overall_trans_mat(D4,d1
%%%%%%%%%%%%%%%%%%%%%%%%
[T_mat] = tran_mat_common_perforated_sec(D4,
%%%%%%%%%%%%%%%%%%%%%%%%
[D,E,F,G,H,K,P,Q,R] = sub_transfer_mat(T_mat
%%%%%%%%%%%%%%%%%%%%%%%%
[A,B]=end_cavities(D4,d1,d2,d3,la,lb,ta,tb,L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
W=inv( (G*A) + H - (B*P*A) - (B*Q) )*( (B*R)
C=(D*A*W) + (E*W) + F;
T_overall=C;
```

```
on [T_overall]=overall_trans_mat(D4,d1,d2,d3,
%%%%%%%%%%%%%%%%%%
 = tran_mat_common_perforated_sec(D4,d1,d2,d
%%%%%%%%%%%%%%%%%%
 G,H,K,P,Q,R] = sub_transfer_mat(T_mat);
%%%%%%%%%%%%%%%%%%
end_cavities(D4,d1,d2,d3,la,lb,ta,tb,La_cav,L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 (G*A) + H - (B*P*A) - (B*Q) )*( (B*R) - K );
W) + (E*W) + F;
ll=C;
```

And then once we get that the overall transfer matrix is exactly what following Munjal's approach in his paper, so you can follow other you can. So, this particular configuration allows you some sort of a cascading or simple multiplication of transfer matrices network analysis, when you have multiple wave propagation paths.

We will study about that in the next week like I said by introducing some simple elements to begin with, but for now what I am what I suggest is, let me just run the code it is going to take some time.

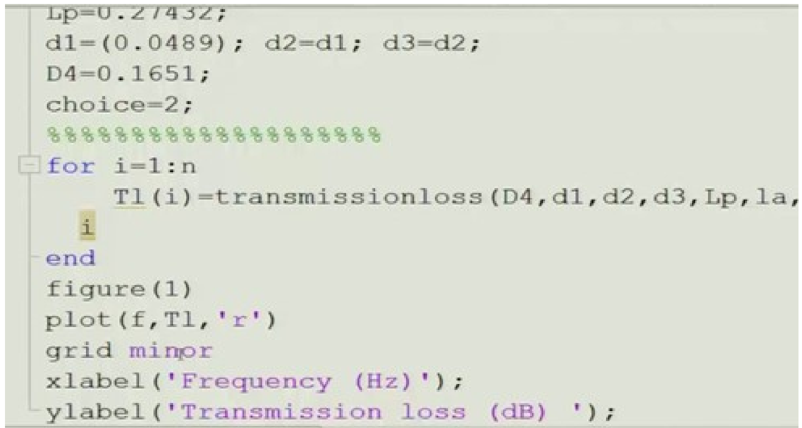
```
function [] =transmission_loss_plot (frange1,
f=frange1:5:frange2;
n1=size(f); n=n1(1,2);
%%% parameters....
pr1=0.045; pr2=pr1; pr3=pr2;
t1=0.0008; t2=t1; t3=t2;
dh1=0.00234; dh2=dh1; dh3=dh2;
ta=0.0127; tb=ta;
La_cav=0.15; Lb_cav=0.102;
la=0.02794; lb=la;
Lp=0.27432;
d1=(0.0489); d2=d1; d3=d2;
D4=0.1651;
```

```
ion [] =transmission_loss_plot (frange1,franc
nge1:5:frange2;
ze(f); n=n1(1,2);
arameters....
.045; pr2=pr1; pr3=pr2;
0008; t2=t1; t3=t2;
.00234; dh2=dh1; dh3=dh2;
0127; tb=ta;
v=0.15; Lb_cav=0.102;
02794; lb=la;
27432;
.0489); d2=d1; d3=d2;
1651;
```

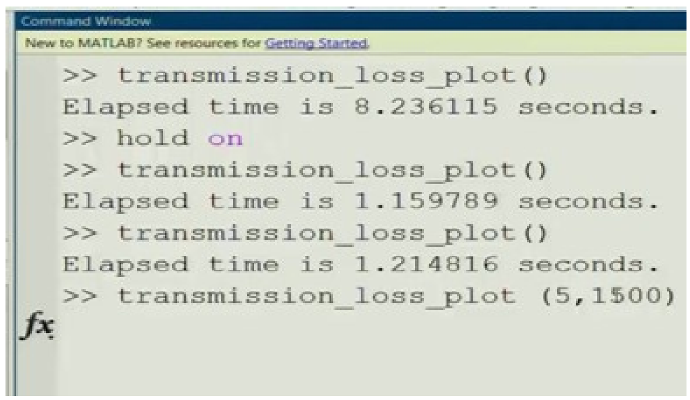
Let me sort of increase the value, let us say 5 hertz because this is going to take some time.

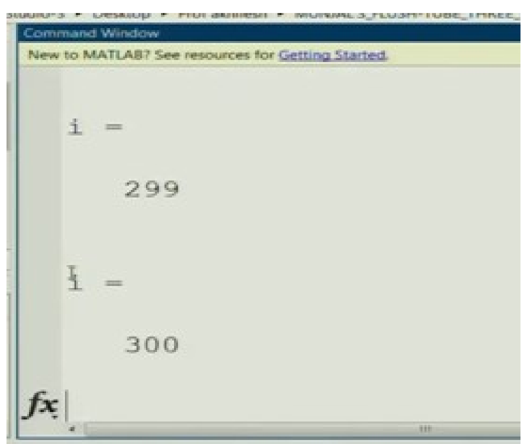
And maybe do only up to 1000 hertz or maybe 1500 hertz. So, what let me also bring out the porosity value 4.5 percent is the porosity really, thickness is 8 mm, diameter of the hole is about 2-3 mm or something like that, thickness is 127 mm and l a cavity l b cavity is about 150 mm and 102 mm, that is basically the end chambers that is at the left and the right given by these lengths and this is the length of the common perforated section I guess so.

No, so this is the length of the common perforated section I my mistake and La and Lb have to see what it is diameters of the duct diameter of the shell.



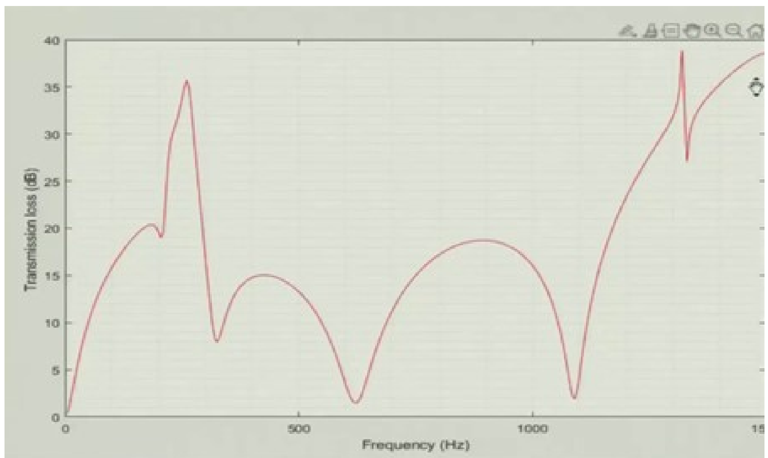
So, I am doing choice 2. So, it might be because of some something some changes in the code I would have done. It is been some time now that I have used this code.





But, let me use let me get some nice curves. So, beginning from 5 hertz up onto 1500 hertz. Let us see what we get. So, this is going to take about 300 runs or 301 runs. So, we should be getting something like you know this kind of a curve that, we sort of got here you know this kind of broadband we are setting of course, mean flow is not considered really in, and we are just considering the perforated impedance by the classical one, not using the one.

One thing is there you know, while the code is executing one piece of advice is that you know whenever you write codes again comment it and you know kind of securely you know you know save it at different location and you never know when you will use certain things. So, like this you can actually, I would encourage you guys to actually, the ones who are interested to build your own library of you know muffler elements, that is what all muffler consultants you know they do.



So, while we were talking a transmission loss curve a beautiful nice t l curve has popped out and you know you see exactly the same stuff. So we are able to sort of reproduce what the author has done in this paper of course, this is with mean flow effect of distance, so this is with mean flow.

We have not really considered mean flow, but we know for sure what is going to happen when mean flow is there, this peak will come down and this trough might be raised a bit especially this one, and this would be further. There will be leveling effect we know by now we should be knowing, but what is to be seen we are still getting about you know roughly, we would easily get about 8 to 10 dB 8 dB transmission loss up to about thousand hertz, because especially when flow is there because this will raise the trough, no doubt about it.

You know these kind of things will happen, so this configuration you know presents nice insight into the actually used commercial mufflers and with this thing what we will do?

We will stop for the day and I will see you in the next week, but on a parting note what I would allude to is that in the next week we are going to talk about the impedance matrix concept for the first time how do we all this while we are characterizing things in terms of pressure and velocity at 1 o 2.

The ones at the downstream port it was really, at the end of the day you would have got even something like 2n cross 2n matrix or you know 2 for a simple case 2 cross 2 matrix, but you know you can have ports with 3 ports single inlet double outlet muffler. So, how do you relate that, how do you characterize such a system? So, by characterization I mean relating some state variables to the other state variables using some sort of a matrix representation, there should be some relation between them.

For a general n port system, n need not be an even number and can be odd number; three port, single inlet double outlet, you know a double inlet triple outlet you know things like that. So, you need to work in impedance matrix which is more general and is used a lot in electrical theory. So there is a like we have been talking about electro acoustic analogy. So, here also again impedance matrix.

So, we will worry about, we will talk about impedance matrix characterization in the next week, and how to analyze you know fairly complicated and you know pretty formidable looking system by means of a network analysis where each of the mufflers you know at least the 2 ports 1 can be characterized using the transfer matrices, but need not be you can use an impedance matrix depending upon what you want to do.

And then you apply junction laws and characterize such a thing, and possibly you know use such an impedance I am sorry network representation talk about different elements which involve multiple acoustic wave propagation path. For example, a Herschel Quincke tube.

Before moving to this multiply connected perforated thing element. So we will analyze such a thing and if time permitting we will also have a look at some of the recently published ideas, like integrated transfer matrix which can be used to analyze very complicated commercial mufflers all within the realm of plane wave we can analyze such complicated multiply connected mufflers and time permitting we probably would also be going to a CCTR configuration, remember?

The things that we probably discussed, I think in week 5 if I can see my notes is, we probably discussed that in not well, I guess in week 6, I am sorry my apologies. So, we did only conical mufflers, but we can probably have a central pipe with a perforated thing and with extensions.

But the outer of the annular thing is no longer uniform in shape it is conical in shape. So, we have a CCTR; conical concentric mufflers which can also be analyzed using a matricent approach. Let us see how we go about doing that time permitting, so this is all going to be the focus of the week 10 and till that time I would strongly encourage you to start writing your own codes and developing your own libraries for the plain wave thing.

It will be a very useful exercise for you guys, if you want to really link learn things from the you know from the coding point of view at least from a designers point of view ok ah. So, this is one aspect that you should have. Good analytical and computational skills and before and then there are of course, people who do experiment, so they are good at that. So till that time, I would stop here and you know I would see you in week 10 with the contents I described below thanks.