Muffler Acoustics - Application to Automotive Exhaust Noise Control
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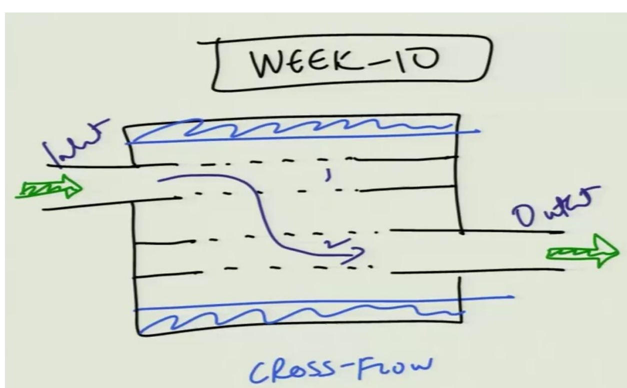
Week-10
Lecture - 46
Multiply-Connected Mufflers: HQ Tubes

Welcome to week 10 of this NPTEL course on Muffler Acoustics. So, we are heading towards the you know towards the end of this course. Well, but there quite a few interesting topics still remaining, that we will cover you know I try to cover as much as possible in the next 3 weeks. And, we will probably next week we will touch up on the 3 dimension analysis probably for the first time for simple you know muffler configurations.

But, in this week the idea of the basic contents for this week is to analyze you know multiply connected muffler systems. Basically, those muffler systems in which the way propagation path can be non unique. So, let; let me just draw a few muffler configurations.

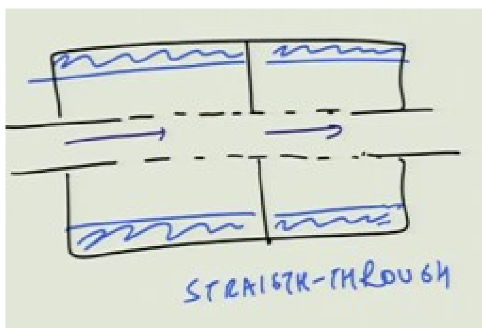
And try to you know prove to the point that what do I mean, what do we mean by multiply connected mufflers and or just cascade systems what is the difference. So, all these while you know some of the commercially available muffler systems, I mean the ones that I used in sort of a commercial mufflers.



They definitely have to operate as I have been sort of insisting. So, they sort of appear something like this you know. So, let us let I am sort of drawing here, I am drawing here, I am drawing here a cross loop perforated element which is open ended something like this.

So, here the waves have really no choice, but to you know go through this perforates and pass on through these two perforate pipes 1 and 2, and then leave this outlet pipe. So, this is the inlet one inlet and this is your outlet portion so, waves have to go like this. Now, another thing is your if you way to do it something like this ok, if you kind of close it. So, it becomes you know closed ended or cross flow element still, but it is no longer open ended that in the sense that the waves no longer discharge here.

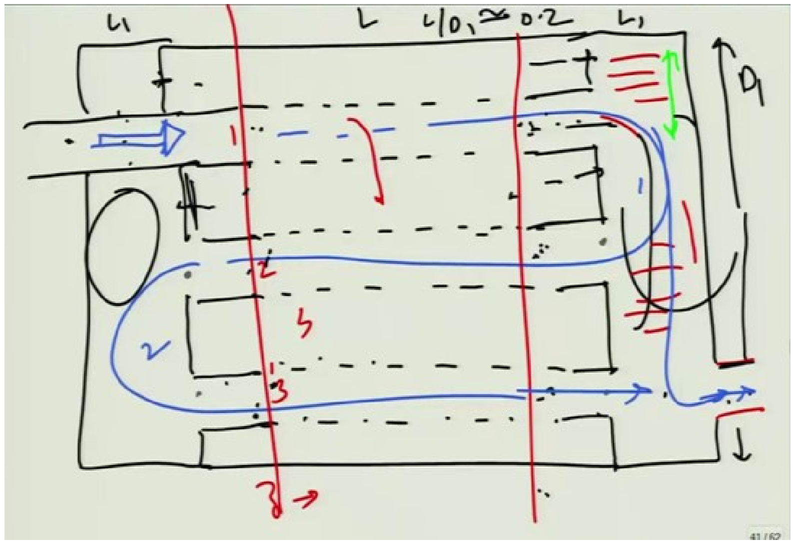
But, the still the waves have to go through these elements. We can keep adding more and more complexities to it, we can have a kind of deceptive lining. So, you know this is just one such configuration in which the waves have to kind of go through one path. or else you can have another configuration.



One configuration that we have been kind of analyzing so far is; obviously, your straight through configuration which; obviously, has only one way in which the waves can go through the system. So, waves can only go through like this, you can have you can keep adding complexities.

For example, you can one can always you know put a kind of plug here as we analyzed, or perhaps what; what also can be done is something like, well we can have this. And, you can have a dual kind of a configuration ok. And, then with extension or with partially perforated thing.

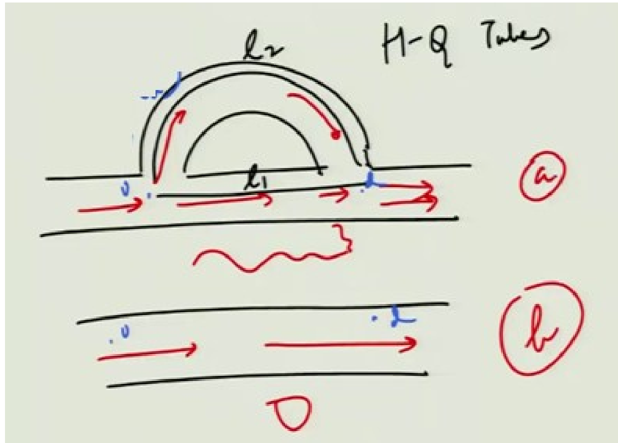
And then you can keep adding complexities like putting a deceptive lining ok, and then the flow goes. So, at the central thing to be noted in this configuration let us say you know, well this is cross flow anyways cross; cross flow configuration, and this is straight through; straight through. So, the idea really is that the flow can take only one; one path ok. But, then there can be many other configurations in which multiple paths do exists. For example, let me go to the slides of the you know week 9 lecture.



So, you know in this I briefly alluded to the fact that, waves you know apart they; they have to you know they the flow enters here and it leaves sort of here. So, the in the process the waves can either you know take this path and if this past tube variant there ok. So, then the waves can directly come and they can go like this also and they can actually come and straightaway go like this. So, basically the idea is that the waves have multiple paths through which they can go.

And, what we probably can do is analyze such, I mean this is just one motivation that why do we study multiply connected muffler. They we can synthesize a number of configuration in which the wave propagation path is non unique. So, let us analyze that using simple network analysis approach.

And first characterize them using you know impedance matrix approach which will probably introduce it for the first time in this course. And, let us first analyze what do you mean by a non, you know simple muffler configuration which makes it very sort of crystal clear that, what would a multiple a wave propagation path look like.



So, you know let us consider a simple duct like this you know this is a pipe and flow goes like this. So, here you have your let us say the wave come here and they are spread into this part and this part. And, then they go like this and the flow again attaches here and then there is a downstream thing.

So, now, compare this with the configuration a simple pipe in which the waves have to go only along this path ok. So, what is the difference between say, a and b? Here the waves can go have alternate path to reach the downstream, this is the upstream point, this is the downstream point, same upstream, downstream.

So, the waves can have multiple paths through which they can go ok. So, you know one needs to do something different to analyze or to relate the state variables between

$$\begin{Bmatrix} p_0 \\ V_1 \end{Bmatrix} = [T] \begin{Bmatrix} p_Q \\ U_d \end{Bmatrix}$$

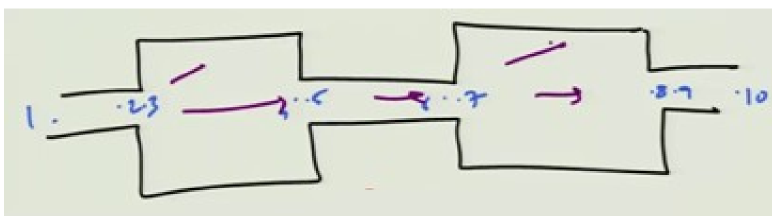
One needs to do something different to analyze such a configuration. So, we are going to study all that is going to happen now. Actually, these configuration that typically called, such pipes are called Herschel Quincke tubes H-Q tubes H-Q tubes.

So, what happens before I discuss something else, so what does you know what does this additional part do? So, this; pipe you know let us say the length is l_2 from this point and from this point it is l_1 ok. So, because of the difference in the, because of the path difference they will they at certain frequencies they will be a constructive interferences happening between the waves. And, at other frequencies there will be destructive interferences, what it means?

In that the waves that come here at certain frequencies they add up to basically increase the amplitude, or basically result in significant increase in amplitude. As a result the transmission loss of such a system is basically becomes almost acoustically transparent system. But, at other frequencies were destructive interference happens between these pipes you know at such frequencies you know almost negligible acoustic power is transmitted downstream.

And, and because of this resiliences they, because of at certain resilience frequencies where destructive interference happens, you know these H-Q tubes perform very well. You get a spike in the transmission loss curve what they are called attenuation peak. If we compare this with the transmission loss for a simple pipe with no expansion contraction; obviously, the transmission loss is 0 throughout the frequency domain. But, just introducing this path difference would lead to multiple resiliences.

You know we will do a proper derivation of that.



And, you know another system that is there you know is basically for example, if you have muffler system like this. As we have been discussing in the beginning of this course, a very simple looking concentrate muffler configurations 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. You can what one can do is that to relate things from

$$\begin{Bmatrix} p_1 \\ V_1 \end{Bmatrix} = [T_{12}][T_{34}][T_{910}] \begin{Bmatrix} p_{10} \\ V_{10} \end{Bmatrix}$$

What we could do is basically, where T_{12} and they will and between 2 and 3, 2 and 3 for a stationary medium it is raided by a identity matrix if we recall.

So, it will be basically 2, 3 is identity matrix. So, one is to lies between the transfer matrix between 3 and 4 ok. Like this if we keep multiplying we will get the things between T 9 and 10. So, effectively what will happen is that

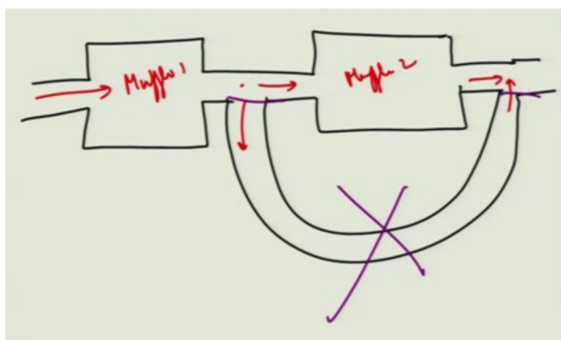
$$[T_{0verall}] = [T_{12}]\,[T_{34}][T_{56}][T_{78}][T_{910}]$$

$$T_{23} = [I]\ \ T_{45} = [I]$$

$$T_{67} = [I]\ \ T_{89} = [I]$$

What exactly am I trying to say? What is the massage that I am trying to convey? For a you know a typical commercial muffler you typically have 2 muffler shells attached to each other, one is upstream and other is downstream. And, internally you can have you know a very complicated structure here and very complicated structure here.

But, now still the wave propagation path is unique from upstream to downstream. But, now let me sort of this rub all these things and introduce another; another path to the wave propagation. So, let us see you know I what I am going to do is and I just also get rid of these numbers.



And you know introduce this thing something like this together ok. So, what happens now the waves acoustic waves they have more than one choice or propagating. So, they can at the moment they come here at this junction they can either propagate downstream here or they can go here.

And, then when, they when the waves come here they again have a choice of constructive interference and destructive interference. So, this can be your muffler proper muffler 1, muffler 2 and what we are doing here is providing additional path through which the waves can go.

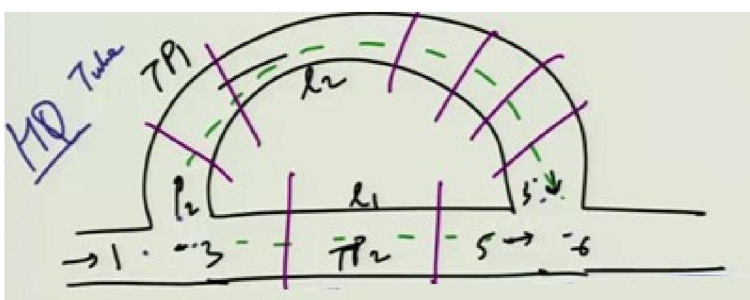
So, such things are called multiply connected mufflers or things with non unique wave propagation path. So, what is the difference between this system and one that I had drawn earlier, or yeah or just between when you have just 2 mufflers connected without you know this tube where and there.

You know if this was a solid pipe solid something like this, what is the difference? Cascading that was used previously now cannot be done. Because, you know essentially the idea of transfer matrix multiplication of cascading is really that the wave propagation happens in a one direction.

And, you can just multiply of the matrixes to get relate things from upstream to downstream. But, because of the very complicated, integrated nature of muffler system in a commercial vehicles like the one that have shown previously multiply corrected perforated mufflers.

Or even the systems that are used in you know heating, ventilation, and air conditioning system where you have multiple side branches or you know complicated integrated networks in gas pipelines. So, there you know non unique wave propagation path does exists.

And, so that basically this quiets a bit of a motivation for us to; to deal with such a topic here introduce it. And, off late there is been this quiet a bit of a research and this field will come to it. And, so let us kind of analyze perhaps the most simplest of a multiply connected muffler systems which is this one. Now, before I begin the analysis of say the system shown here a I would also like to sort of mention is that you know this element, this one.



This is your is the two port element, let us call this as 1, this element 2, this is 3, 4, 5 and 6. We assume that 5 and 6 have collapsed and 1 and 3 also come together. So, this is essentially very small you know one point really I am just drawing it separately just for better conceptualization. But, what I am trying to say now is that this element this guy is a 2 port element, because they have only inlet port or outlet port, or space not to talk in terms of inlet and outlet just 2 ports.

Where either you can relate the pressure and velocity is here to the pressure and mass velocity is here. Or else we relate the pressure, acoustic pressure here and here with the mass, with the respective mass velocity, were impedance matrix. And, as I briefly talked about in the last few lectures of week 9, this impedance matrix thing is the beautiful concept because. Now, we do not have a compelling, we do not have a kind of a constraint that the system might should have only even number of ports.

Its not necessary that you have a muffler system with only 2 ports, we can have 3 ports also. In such a case impedance matrix is very useful for a characterizing the system. And, then we can do some network analysis which anyways we are going to do now. So, for a simple 2 port system and than we will consider another text case.

So, here for such a pipe here another 2 port element is your this guy. So, I would write this as TP_1, port TP_2 between 3 and 5 and then junction laws will be apply. So, exactly you have your Kirchhoff law for current. So, you know basically, current is basically redistributed number of paths, number of sub circuits which comprises an electrical network. So, here also we can redistribute write mass velocities as being split into number of paths, which we will talk about now.

And, at the same point a voltage is the same, so what we will do is that we will equate the pressures. You know at; at this point 1, 2, 3 where they have come together is a very small point, $\tilde{p}_1 = \tilde{p}_2 = \tilde{p}_3$ they are all the same.

So, these are junction laws, so junctions are really points where two or more elements are connected together. So, those elements themselves can be a 2 port element or a multi port element we will soon sort of figure out, but depending on the network involved.

But, for analyzing such a system what we will do now is basically, let us write down the junction laws. So, here we

$$\begin{aligned} \tilde{p}_1 = \tilde{p}_2 = \tilde{p}_3 \\ \tilde{V}_1 = \tilde{V}_2 = \tilde{V}_3 \end{aligned} \qquad (1)$$

And, your velocity gets split into 2 parts, v_1 that comes here gets. So, it gets split into 2 parts and another thing that you have is a, let us first get done with a junction laws.

$$\tilde{p}_4 = \tilde{p}_5 = \tilde{p}_6$$
$$\tilde{V}_4 = \tilde{V}_5 = \tilde{V}_6 \qquad (2)$$

And, velocity that, well acoustic waves that come here they all contribute.

So, v_4, note that v_4 all these vs are mass velocities, or you can also have volume velocity if you divided by density, I choose to work in mass velocities. So, that is what is constant, so v_4 plus v_6 is equal to this thing. Now, note that the wave propagation direction is such that velocity at the inlet of the muffler.

Or the at the assumed inlet the velocity is taken into the system positive. And, at the outlet the waves that the velocity is taken positive in the direction facing outside the system, that is towards the side. But, we can use another sign convention also later perhaps.

But, for now we will probably stick to this convention, because the reason being is that we already know the transfer matrix from 2 to 4, relating thing from

$$\begin{Bmatrix} \tilde{p}_2 \\ \tilde{V}_2 \end{Bmatrix} = \begin{bmatrix} C_2 & jY_2S_2 \\ \frac{jS_2}{Y_2} & C_2 \end{bmatrix} \begin{Bmatrix} \tilde{p}_5 \\ \tilde{V}_5 \end{Bmatrix} \qquad (3)$$

I am just writing c let me call this length as l 2 and this as l_1. So, this is what it is this is a transfer matrix here. I will call this equation as say 1, 2 and 3 ok. Now, the transfer matrix that relates 3 and 5 would be something like something like this.

$$\begin{Bmatrix} \tilde{p}_3 \\ \tilde{V}_3 \end{Bmatrix} = \begin{bmatrix} C_1 & jY_1S_1 \\ \frac{jS_1}{Y_1} & C_1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_5 \\ \tilde{V}_5 \end{Bmatrix} \qquad (4)$$

So, we have now the relevant number of equations let me call this as 4. Now, how many variables we have? That is given by the number of nodes we have, for such a system that is the most general analysis that I am trying to present.

So, 6, 1, 2, 3, 4, 5, 6, so 6 into 2 12, 12 variables we have including both pressure and velocity, because at each port there is 2 variables ok. And, how many equations do we have? You know 1 and 2, because this one and this one they are together form 2 equations, and plus this one so 3 ok.

So, 3 plus 3 6, plus 2 is 8 and 8 plus 2 is 10. So, we have 10 equations; we have 10 equations alright, 12 variables and 10 equations. So, what does it mean? That any 10 variables you can express in the terms of the remaining 2 variables by theory of general linear algebraic equations.

Ultimately what we want like I said cascading cannot be applied here you know not in the straight forward manner.

So, what we can do is that you know, adopt the following practice, we can rearrange the system in a certain manner in a network analysis. So, now, we are properly entering into the network analysis. So, what we do we choose to represent p 1 v 1 in terms of p 6 v 6, alternatively we can also represent p6 p1 and p6 in terms of v 1 and v 6.

But, let me just move ahead with the transfer matrix formation nothing different what we have discussed in the preceding lectures, they are large number of permutations one can do. We can just like we have put here a transfer matrix relation in equation (3) and (4), we can also relate the port 3 and 5 or 1.

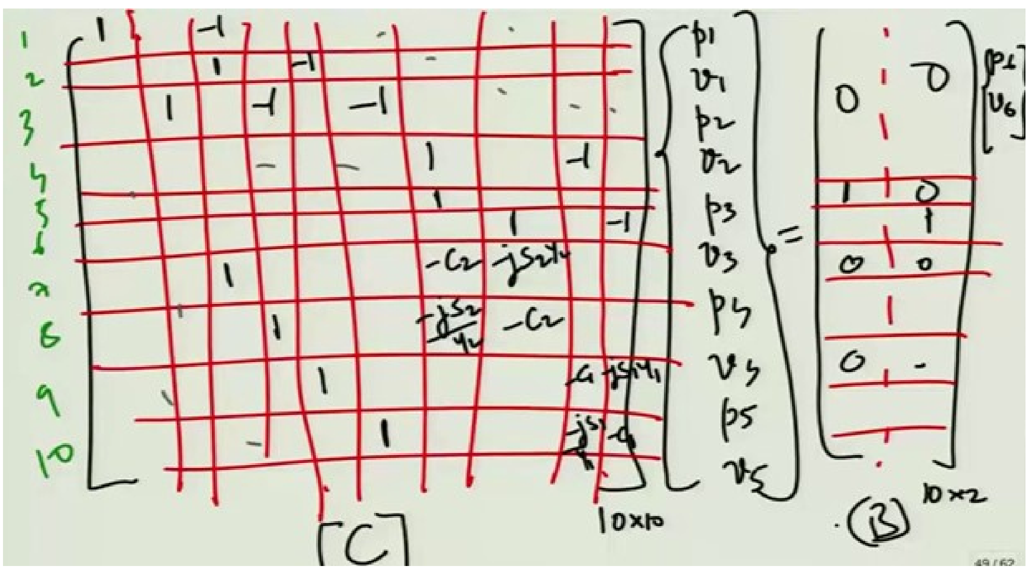
And I guess well, 2 and 4 using an impedance matrix. We will slowly cover that and see what are the pros and cons of that. But, for now let me just write down your p 1 matrix p, I am just dropping the tilde signs for simplicity. So, p_1 v_1 p_2 v_2 and eventually at the end of the day we want p; p_6 v_6 is not it. This would be your 10 cross 10 matrix and this would be your also rectangular matrix 10 cross 2 matrix, and here we have your p 6 and v_6 ok.

And, let me just sort of you know start taking things like rows and columns 3, 4, 5 like this you know 1, 2, 3, 4, 5, 6, 7, 8. And, then we can probably add one more 9, and possibly one more here 10, so, and then 1, 2, 3, 4, 5, 6, 7. So, we can add possibly one more thing here 8, so 1, 2, 3, 4, 5, 6,7, 8, 9.

So, we can sort of put one thing here as well. So, we have eventually 10 equations. So, sorry for the its becoming a little untidy, there lot of ground to cover, so we have typically written thing like this. So, the first equation what is the first equation look like, the first equation is just $p_1 = p_2$.

So, we can write this guy as $p_1 - p_2 = 0$ ok. So, it is not related to this so we can just in place of this thing we write it 1 and in place of p_2 we will, by default all the entries are 0. So, I am not writing it wherever 0s are there. So, I am just writing the non 0 entries. So, this will be minus 0 ok, this will be minus 1 the first; first row ok. And, now p_2 is equal to p3, what is it mean? It means that this will become 1 and this will become - 1 ok.

So, these are the 3 equations for the first junction laws. We can write the balance of the junction laws, also that we get done with the junction laws first. So, the fourth equation is your p_4.



So, for the first time we will probably we relating this. So, p4 we can for the next equation p4 is equal to the same kind of row what we will do is that we will relate p4 thing with the p6 thing. So, this will p_4 is equal to p_6. So, here it will be not be minus because its from this side, and everything else is 0 here, 0 here. And, then v_4 is equal to, v 4 is basically your this entry. So, this is equal to this and this will become minus 1 here ok, and here it will be 1 alright.

So, we are done with the junction laws here, now comes the crucial point of putting the transfer matrix in the proper form. So, what I am going to do is that just simply put this thing where these things are there. So, in so this is the 7th equation, so the 7th row. So, p_2 you know I for; for simplicity of for your better understanding I can sort of just number the rows. Now, p7 is where your p_2 that is the thing row, so p_2 - c_2, so minus c_2 times p_4.

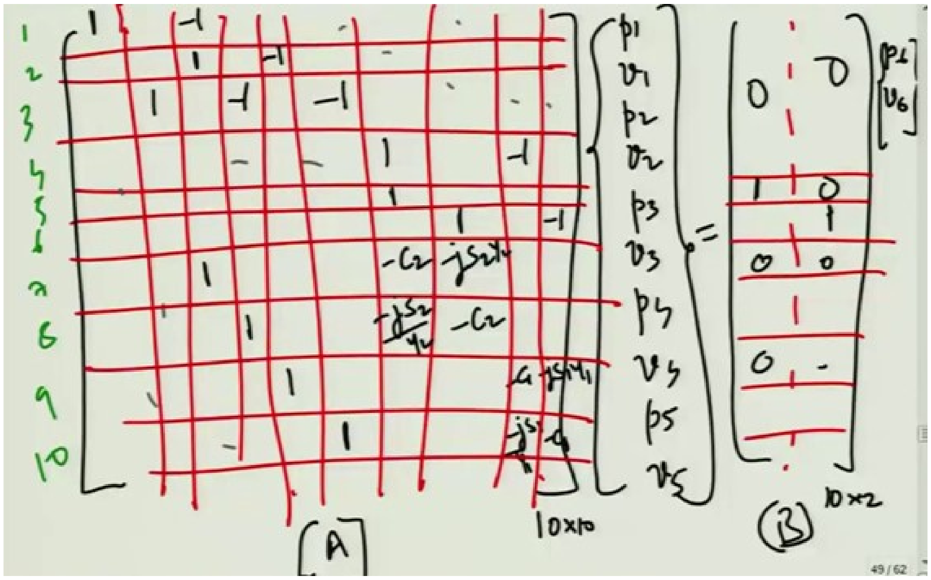
So, here in place of this entry will become 1 and then your p 4 entry, so this will be your -c2 is not it. And, then there will be minus jy 2 sign thing sitting here. So, here there will be minus js 2 y_2. And, what is the other equation that we have? We have v_2 ok.

V 2 is your this thing and minus j sign y 2, you have you know such a system ok. So, we have this equation 7 and 8 and now comes the final bit, p3 v3 relate to p_5 v_5 that is your this thing ok. So, where do we go from here? p 3 and v 3, so this requires a lot of patience i must honestly tell you. So, this will be 1, because once you make an error in the book keeping entry entire thing will be wrong. So, you will get a some extra transmission loss, so you will get to immediately know it.

So, that is a good part about it that you get to know where you going wrong. So, you can correct it for yourself, so p3 and v3 they are 1 1 alright. And, then you have your you have your c_1 relate things relate to 5th kind of a port. So, here will be pretty much the same thing - c1-js 1 /y1 and this will be minus c1 ok. So, we have covered the 10 equations, you know all the places like where I am marking you know all these this is, where I am not written anything in black colour they are all 0s.

So, this connectivity matrix you can put this in the following form. C matrix which is the, or the matrix that rather than calling it a connectivity matrix I would call that as a something like matrix that contains all the information pertaining to the connections between the port how other ports relate.

So, this is a simple tubular connections so as this. We will in this week's lecture we will come across different methods how to arrange these equations. This is just one way of manually doing the stuff this is the first stuff I am doing for you.



So, then all these things you know will be 0s. So, now, what and let me call this matrix as B matrix and actually instead of calling it C let me call it A matrix ok, A would be better.

So, eventually what we will get is that the big vector standing p1 v1 all the way until p 5 v_5 will be A inverse B and p_6 v_6. So, this is your C matrix, what we will get is that this thing p_1 v_1 and this kind of a A inverse B.

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{V}_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \tilde{p}_5 \\ \tilde{V}_5 \end{Bmatrix} = \underbrace{[A]^{-1}[B]}_{[C]} \begin{Bmatrix} p_6 \\ V_6 \end{Bmatrix}$$

So, this is finally, what we will obtain p_1 v_1 is related to the things that report here through the C matrix where C is A inverse B as analyzed in the following manner. And, of course, one can sort of do a lot of algebra right.

$$\begin{Bmatrix} p_1 \\ V_1 \end{Bmatrix} = [C] \begin{Bmatrix} p_6 \\ V_6 \end{Bmatrix}$$

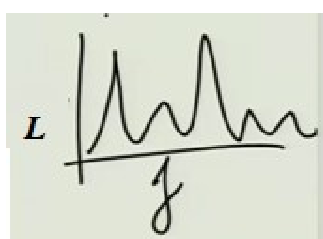
At least for this configuration one can straightaway start you know eliminating some variables, and derive a nice clean analytical formula to get the stuff. But, this is just one way of doing network analysis, because for more; more complicates situations, more complicate muffler analysis we have to resort to this.

And, also things like integrated transfer matrix approach which is nothing, but pretty much the same network way of synthesizing things. So, that is why I suggest that we follow this approach of at least manually arranging things in a connective in a A matrix. And, then you know inverting a that matrix and all that.

So, what we are going to do in next lecture that is lecture 2 of week 10 is that derives the transmission laws for such a tube which is, what is it called? This is called a HQ tube, Herschel Quincke tube. It has minimum 2 tubes, you can have multiple such tube and because of the difference in path. Path difference we have constructive or destructive interference going on between waves and leading to some non 0 transmission laws. So, note that these all these things in the section.

We have a uniform cross section you know across this thing. So, in spite of having uniform cross section you are having non 0 transmission laws in general because of this destructive interference, now at certain frequencies. Now, what we are going to do in next lecture that is lecture 2 of a week 10 go to MATLAB write down all these equations in a show, basically do a MATLAB demonstration of all these equations work. And, we know eventually how we get to this form.

And find out the transmission laws which is probably going to look something like, there is a frequency this is TL we will have multiple such peaks something like this.



So, its going to have a lot of spikes depending on the length and at certain frequencies. So, we are going to get that and possibly compared the result with some other previously reported stuff in literature. And, you know lets also analyze some one of the analytical formulas, and discuss some discrete relationships, frequency relationships.

So, we will do that and then we will move to more realistic muffler configurations So, till that time good bye and I will see you in the next class.

Thanks.