Muffler Acoustics - Application to Automotive Exhaust Noise Control

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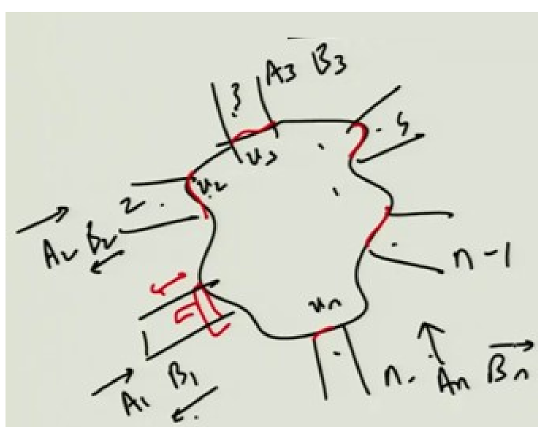
Week-11
Lecture - 51
Transmission Loss in Term of Scattering and Impedance Matrix Parameters

Welcome to this NPTEL course on Muffler Acoustics. So, we are in the week 11 of this course and for the first time in this course although quite late we will be introducing the 3 dimensional analytical solution of muffler systems.

So, we will begin with you know your rectangular chambers, but and in different ways how to characterize such a chamber. You know the different ways like solving a homogeneous set of equations subjected to in homogeneous boundary conditions or the other way around.

Where we characterize the system by solving in homogeneous system equations subjected to homogeneous boundary conditions by means of Greens function and then you know using it further to derive the uniform piston driven model approximation. So, there are number of things, number of ways in which we will analyze these things, but before we do that the way we are going to characterize you know this system is using your impedance matrix things which is basically nothing, but the transfer function.



So, let us say you know let us say you have a you know some arbitrary muffler and you have say the first port 1, 2, 3, 4, 5 and say 6 ok. Let us say we have 6 ports it could be actually let instead of calling it like 5 or 6 you know I would say that n - 1 nth code. So, how do you characterize such a system?

So, if we recall our discussions that we had in the I guess week 9 later parts of week 10. I guess the initial parts you will see that you know an impedance matrix is the best way to characterize such a system. So, I am dropping the tilde excuse me for that. So, here you have this kind of a thing you know so we get this is such a matrix.

So, what is that matrix called where we relate pressure to the acoustic mass velocities at the different ports you know such a matrix is called an impedance matrix. So, how do you write this is basically,

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ \vdots \\ p_n \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ Z_{31} & Z_{31} & \cdots & Z_{3n} \\ \vdots & & & \\ \vdots & & & \\ Z_{n1} & \cdots & \cdots & Z_{nn} \end{bmatrix}_{n \times n} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ \vdots \\ V_n \end{Bmatrix}$$

So, this is the n cross n system. So, how do you get the different parameters for that? You know let us go back to our schematic of the line diagram.

So, what we do let us say we excite the system you know by means of some kind of a piston excitation somewhere here you know to and fro of motion. We generate ways we block all the ports here or you could also block it you know elsewhere you know. So, that the effect of sudden expansions that these ports might also come into effect, but for argument sake you know let us say we want the characterization right from the interface. So, we will move this piston somewhere here.

And excited right at the port chamber interface ok. So, when we excite with the uniform velocity or certain velocity and block all the ports, so what do we get? the mass velocities,

$$V_2 = V_3 = \cdots V_n = 0$$

Under such a thing what will all these things become you know p_1 is equal to Z_{11} times v_1 Z_{12} times v_2 and so on. So, let me expand this guy out

$$p_1 = Z_{11}\, V_1 + Z_{12} + V_2 + Z_{13}\, V_3 \, \cdots \cdots Z_n V_n$$

But then all these things are 0 why? Because you have blocked all the ports you have you know covered with the rigid surface. So, then acoustic velocity normal to the surface is necessarily 0 you know.

So, you get,

$$\frac{p_1}{V_n} = Z_{11}$$

So, similarly now if you were to have a mount a sensor here now put the sensor here p_2. So, p_2 / v_1 will get you Z_{21} p_3 / v_1 will get you Z_{31} because all the other components we can write down this entire equation like this all these things are 0.

So, p_n / v_1 is equal to Z_{n1}. So, you know like this if we block all the other ports you know all the other ports are blocked and only the first excite excitation only at the first port is there, then this entire thing you will get the first column. Similarly, if you block the second port I am sorry excite at the second port and block the ports 1 2 1 and 3 to n you know you will get basically the second row second column I am sorry like this you could traverse the entire thing and characterize the system using n cross n thing.

You know its very similar you know p_1 / v_1 or p_2 / v_1 these are very similar to your transfer functions acoustic transfer function which lot of authors prefer to call them. In essence impedance matrix parameters or acoustic transfer functions are one and the same only they have been used interchangeably in many papers, but they are one and the same and using these transfer functions or the impedance matrix parameters one can also obtain the transfer matrix parameters or the 4 pole parameters we will see soon.

But what we will do in this lecture you know because we are going to use the impedance matrix formulation to characterize you know a rectangular chamber and a circular chamber in this course and I might just talk about little bit about elliptical mufflers using analytical functions may well just a glimpse of that, but perhaps you know an impedance matrix a kind of a thing would be very much suited for analysis of such reactive chamber mufflers.

And before we do that let me sort of more formally establish this theory which I am in the process of course, and also talk about how do you obtain, finally we need to obtain the transmission loss. So, how do you obtain that, and you know how do you know get different conditions in which transmission loss goes to show the peak or show the trough much like your transfer matrix t matrix parameter. So, we have basically,

$$\{p\} = \{Z\}_{n\times n}\{U\}$$

Similarly, A_2 is here, B_2 is here. A_n goes here b_n goes here.

$$\begin{Bmatrix} A_1 + B_1 \\ A_2 + \\ A_3 + \\ \vdots \\ \vdots \\ p_n \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & \cdots & Z_{1n} \\ \vdots & & & & \\ \vdots & & & & \\ \vdots & & & & \\ \vdots & & & & \\ Z_{n1} & \cdots & \cdots & \cdots & Z_{nn} \end{bmatrix} \begin{Bmatrix} \dfrac{A_1 - B_1}{Y_1} \\ \dfrac{A_2 - B_2}{Y_2} \\ \vdots \\ \vdots \\ \vdots \\ \dfrac{A_n + B_n}{Y_n} \end{Bmatrix}$$

So, basically what we can do you know we can write the entire this column. We can do that and this is your Z_{11} Z_{12} and Z_{1n} and this will become your Z_{n1} and Z_{nn} and you know you will get your and mass velocity is mind you know going back to the definition are the characteristic impedance of the ports. So, you know you get this sort of a thing.

So, now what we do is basically,

$$\begin{Bmatrix} A_2 + B_2 \\ A_2 + B_2 \\ \vdots \\ \vdots \\ \vdots \\ A_n + B_n \end{Bmatrix} = [Z]_{n\times n} \left[\frac{1}{Y}\right]_{n\times n} \begin{Bmatrix} A_2 - B_2 \\ A_2 - B_2 \\ \vdots \\ \vdots \\ \vdots \\ A_n - B_n \end{Bmatrix}$$

So, your Y is the diagonal matrix you know I would write this is the diagonal n cross n and you know.

$$\begin{Bmatrix} A_2 + B_2 \\ A_2 + B_2 \\ \vdots \\ \vdots \\ \vdots \\ A_n + B_n \end{Bmatrix} = [Z]_{n\times n}\, diag\, [Y]_{n\times n} \begin{Bmatrix} A_2 - B_2 \\ A_2 - B_2 \\ \vdots \\ \vdots \\ \vdots \\ A_n - B_n \end{Bmatrix}$$

So, what will happen here because, you remember so each of these terms will be multiplied you can consider this Z_{11} multiplied by Y_{11} Y_1 i am sorry Z_{12} multiplied by Y_2 and so on.

So, basically what will happen is that this will become Z_{13} and Z_{21} will be multiplied by Y_1. So, here,

$$\begin{Bmatrix} A_1 + B_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ A_n + B_n \end{Bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Y_1} & \frac{Z_{12}}{Y_2} & \frac{Z_{13}}{Y_3} & \cdots & \frac{Z_{1n}}{Y_n} \\ \frac{Z_{21}}{Y_1} & \frac{Z_{22}}{Y_2} & \frac{Z_{23}}{Y_3} & \cdots & \frac{Z_{24}}{Y_n} \\ \vdots & & & & \\ \vdots & & & & \\ \vdots & & & & \\ \frac{Z_{n1}}{Y_1} & \frac{Z_{n2}}{Y_1} & \frac{Z_{n3}}{Y_1} & \cdots & \frac{Z_{nn}}{Y_1} \end{bmatrix} \begin{Bmatrix} A_1 - B_1 \\ A_2 - B_2 \\ \vdots \\ \vdots \\ \vdots \\ A_n - B_n \end{Bmatrix}$$

So, this will become A_1 minus B_1 A_2 minus B_2 and this will become A_n - B_n ok.

So, we are in the process of obtaining what is known as a scattering matrix parameter. So, basically scattering matrix eventually you might be wondering what exactly we are trying to do here we are trying to relate the reflected wave amplitudes B is that is B_1 B_2 B_3 B_n in terms of the incident wave amplitudes A_1 A_2 A_3 and so on An. So, you know this is now it becomes rather simple what we now basically do is that rearrange a few terms. So, you know.

So, what we can do from here is that arrange terms

$$\begin{Bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ \vdots \\ B_n \end{Bmatrix} = [C^{-1}D] \begin{Bmatrix} A_1 \\ A_2 \\ \vdots \\ \vdots \\ \vdots \\ A_n \end{Bmatrix}$$

So, what exactly is C and D matrix? So, C matrix is nothing it does involve a little bit of algebra. So, I would request all of you guys to please you know do the longhand derivation and practice it by yourself. So, this is nothing,

$$[C] = \begin{bmatrix} \frac{Z_{11}}{Y_1}+1 & \frac{Z_{12}}{Y_2} & \cdots & \cdots & \frac{Z_{1n}}{Y_n} \\ \frac{Z_{21}}{Y_1} & \frac{Z_{22}}{Y_2}+1 & \cdots & \cdots & \frac{Z_{24}}{Y_n} \\ \frac{Z(n-1)}{Y_1} & \cdots & \ddots & & \\ & & & \ddots & \\ \frac{Z_{n1}}{Y_1} & \cdots & \cdots & \cdots & \frac{Z_{nn}}{Y_n}+1 \end{bmatrix}_{n\times n}$$

So, what about the other terms?

So, you know like this one can populate the entire thing you to get the C matrix. So, last term will be Zn - 1 and like this and your D matrix is nothing, but you will pretty much get the same thing, but instead of a instead of a minus sign instead of a plus sign you will have a minus sign.

$$[D] = \begin{bmatrix} \frac{Z_{11}}{Y_1}-1 & & & \frac{Z_{1n}}{Y_n} \\ \vdots & \frac{Z_{22}}{Y_2}-1 & & \frac{Z_{24}}{Y_n} \\ \vdots & & \ddots & \\ \frac{Z_{n1}}{Y_1} & & & \frac{Z_{nn}}{Y_n}+1 \end{bmatrix}$$

So, everything else will be the same and here we will have Z_{22} Y_2 -1 like this and this will be your Y_n and this will be your Z_n + 1 Y_1. So, C and D so basically we get C inverse D and this is C D.

So, you know this can be written in a more much more compact manner where you know your B vector I would say B vector or the vector of the reflected wave amplitude. So, let me sort of write it down in the full blown out form,

$$\begin{Bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ \vdots \\ B_n \end{Bmatrix} = \left[[Z][Y]^{-1} + I\right]^{-1}$$

$$\left[[Z][Y]^{-1} - I\right] \begin{Bmatrix} A_1 \\ A_2 \\ \vdots \\ \vdots \\ \vdots \\ A_n \end{Bmatrix}$$

which you should remember this is the diagonal matrix. So, Y inverse will be you know y is a diagonal matrix.

So, its Y_1 Y_2 Y_3 Yn all the diagonals everything else is 0. So, Y inverse if you do you just have to take the inverse of the diagonal elements. So, this will be + I. So, this is the much more compact way of writing stuff and

So, like this we can keep doing and you know there is yet another well perhaps a little bit more compact way of doing this thing is that you know if you realize this that you know instead of - I if you write + I then you have to subtract - 2I ok. So, once you multiply this guy with this ZY inverse + I whole inverse with this particular thing you will get I matrix and this will be -2 times this thing. So, basically this entire thing is called the scattering matrix.

$$\begin{Bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ \vdots \\ B_n \end{Bmatrix}_{n+n} = \underbrace{\left[[Z][Y]^{-1} + I\right]^{-1} \left[\left[[Z][Y]^{-1} + I\right] - 2I\right]}_{[S]} \begin{Bmatrix} A_1 \\ A_2 \\ \vdots \\ \vdots \\ \vdots \\ A_n \end{Bmatrix}$$

So, scattering matrix one compact way of writing scattering matrix is also given for n cross n system will be given by

$$[S]_{n\times n} = \left[[Z][Y]^{-1} + I\right] \left[[Z][Y]^{-1} - I\right]$$

$$[S] = I - 2\left[[Z][Y]^{-1} + I\right]$$

So, you will get you know this sort of a thing. So, this is your S matrix. Basically, scattering matrix relates the waves that are reflected from the ports to the incident wave.

So, now we might be asking why are we doing this. So, we can you know we can have any sort of this is a very powerful representation you would realize the importance of this thing by actually considering n plus m system where you can have n inputs and m output. So, n can be any number it can you can actually instead of n you know instead of n you can put it n + m. So, you can restructure the matrix by putting the first n rows can be your n inlet us.

And m can be outlet us of course, in you know in conventionally you would analyze the single inlet and single outlet system, but typically you know the photographs of lot of commercial vehicles if you see on the internet or if you go around and have a look at the muffler photographs you see double outlet mufflers as well.

So, you know basically what happens is that the analysis of such a system,

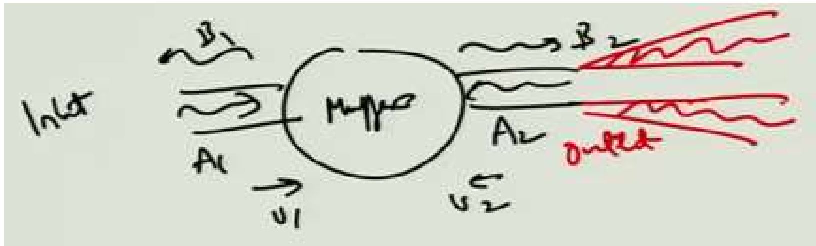
$$\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}$$

Scattering matrix for a single inlet and a double outlet system.

You know when you talk about you know characterizing a system you know you can have actually an n inlet an n outlet. So, the first chunk will become inlet ports reflected waves at the inlet ports and the other chunk will become you know reflected waves at the outlet ports which you know which you intend to find out.

Now let us first you know analyze the single inlet and single outlet system in terms of scattering matrix impedance matrix we can we will you know we characterizing a single inlet and multiple outlet a single inlet and single outlet rectangular chambers or circular chambers in terms of circular functions or Bessel functions whatever the case may be.

But, we will also analyze a muffler with three ports. So, one of the ports can be an inlet port the other 2 can be outlet ports and we can optimally you know choose the sort of location of such ports to make a things you know quite interesting for us. So, in terms of how do you obtain the broadband transmission loss.



$$\begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix}$$

Now, if you set now if you set your let us say you have a system like this ok. Incident wave A_1 and so here yeah one thing that I kind of forgot to mention I should have mentioned apologies for that velocities the direction of velocity is considered positive looking into the system. So, what it means for us it has a great significance is that you know when you consider a system like this A_1 is the incident wave here.

And A_2 is the incident wave here. And the one that goes this is B_1 this is B_2 and you know direction of mass velocity is considered positive into the system v_1 v_2.

$$B_1 = S_{11} + A_1 + S_{12} \cancel{A_2}$$

So, if we set the incident wave to v 0 at the inlet. So, we get A_2 if we set A_2 0 basically, no incident disturbance is there on the boundary. So, we get B_1 is equal to S_{11} times A_1 and $B_2 = S_{21}$ times A_1 because A_2 0.

What it means is that here you know we are basically by putting by exciting the system let us say the muffler proper single this is the inlet port this is the outlet port. So, you know we are connecting the inlet to the we are connecting the inlet to the to the engine to the engine exhaust and this is the tailpipe.

So, there are no reflections if we have an anechoic termination ok. If we have an anechoic termination what we will get is that A_2 will be 0, there will be no incident wave or very negligible for the frequencies of interest and there will be only B_2 waves that are reflected downstream ok.

So, B_2 which we will be of great interest to us because that will determine the amount of acoustic power carried into the anechoic termination and A_2 is 0 because of anechoic termination and here you will have nonzero B_1 of course. So, exactly these are exactly what you would do when you obtain the transfer matrix characterize the system in terms of transfer matrix parameters and find out A_1 B_1 and all that, but here we are moving towards a very general kind of system.

So, B_1 so I will get rid of this part and we will just represent B_1 reflected wave at the inlet port and reflected wave outlet port in terms of this these S_{11} S_{21}. So, what is the transmission loss then? You know transmission loss is defined as,

$$\left. \begin{aligned} B_L &= S_{11}\, A_1 \\ B_L &= S_{21}\, A_1 \end{aligned} \right| TL = 10log_{10}\left|\frac{W_{inc}}{W_{trans}}\right|$$

The acoustic power incident to the acoustic power transmitted in the in a anechoic termination. So, here one thing critic one thing that is very important I am seeing this thing repeating this thing again.

And again is that A_2 must be 0, then only this thing will work and that means the that implies the implication that implies the presence of an anechoic termination. So, what is the acoustic power in terms of this these things let me go to another slide.

So, B_1 mod $|B_1|^2$ divided by 2 ρ_0 Y_1 is W incident ok and mod B_2 I am sorry this is the wave I am sorry my mistake this is the acoustic power that is being carried away back into the this direction. So, this is of less importance the acoustic power that is incident that is given by this thing ok.

$$\frac{|B_1|^2}{2\rho_0 Y_1} \quad \frac{|A_1|^2}{2\rho_0 Y_1}$$

$$\frac{|B_2|^2}{2\rho_0 Y_2}$$

The one that is carried away in the anechoic termination is this thing Y_1 and Y_2 they can be different in general.

$$10\ log_{10}\ \frac{\frac{|A_1|^2}{\cancel{2\rho_0}Y_1}}{\frac{|B_2|^2}{\cancel{2\rho_0}Y_2}}$$

$$10log_{10}\left|\frac{|A_1|^2Y_2}{|S_{21}|^2|A_1|^2Y_1}\right|$$

So, let us do some cancellation and you know A_1 is obviously we know B_2 we could write it in terms of S_{21} ok. And of course, we will have Y_2 on the top and Y_1 on the denominator. So, transmission loss T L

$$TL = 10log_{10}\ \left|\frac{Y_2}{Y_1}\frac{1}{|S_{21}|^2}\right|$$

We get that. So, entire thing remember we got the same thing in terms of your t matrix parameter apologies I am getting something wrong that is probably something

$$= 10log_{10}\ \left|T_{11} + \frac{T_{12}}{Y_1} + T_{21}Y_2 + T_{22}\frac{Y_2}{Y_1}\right|$$

So, this is the counterpart and when you establish when you represent S_{21} you know once you use you know I leave this as an exercise you must do that and we will discuss all the implications of this is that when you sort of figure out that this is the scattering matrix for the 2 port system it looks like this.

And then once you find out what S_{21} is in terms of what S_{21} is in terms of the impedance matrix parameter. You will realize that the transmission loss can be very nicely you know expressed in the following form,

$$TL = 10log_{10}\left\{\frac{1}{4Y_1Y_2}\left|\frac{(Z_1 + Y_1)(Z_{22} + Y_2) - Z_{12}\,Z_{21}}{Z_{21}}\right|^2\right\}$$

But Z_{11} times Y_1 Z_{22} times Y_2 Z_{12} Z_{21} by Z_{21} mod square. So, you know you will get this. So, let us also you know quickly analyze what does it mean. So, clearly you know different conditions under which we will get attenuation peaks.

Conditions under which we get attenuation peaks or trough.

$$Z_{11} \rightarrow \infty, \quad Z_{22}, \quad Z_{21}, \quad Z_{12}$$

$$Z_{22} \rightarrow \infty, \quad Z_{21}, \quad Z_{12}, \quad Z_{21}$$

$$Z_{11} \ \& \ Z_{22} \rightarrow \infty, \quad Z_{21} \ \& \ Z_{12}$$

$$Z_{21} \rightarrow 0$$

So, when Z_{11} is going to infinity Z_{11} is tending to infinity and Z_{22} Z_{21} and Z_{12} generally it was seen that Z_{12} is equal to Z_{21} why because of acoustic reciprocity. Remember our discussions is long time back in I think in week 10. So, you know if you excite the system at 1 port and measure the response at other port keep the things aside and do the reverse you will see the same thing.

So, this follows from acoustic reciprocity that which basically says that if the source and receiver positions are changed then the response does not change well this is true when you do not have a mean flow in general ok, but otherwise we will have things Z_{22} Z_{21} Z_{12} alright. Now all these things are finite and Z_{11} is going to infinity then you its easy to see you will get an at you will you see that the log of the argument is going to infinity. So, you must be getting an attenuation peak ok.

Other condition of course Z_{22} is going to infinity and Z_{11} Z_{12} Z_{21} all are finite ok. And what is the other condition that you must have that where you have your attenuation peak. So, the other condition is that when Z_{11} and Z_{22} Z_{11} and Z_{22} both are tending to infinity, but Z_{21} and Z_{12} are finite such a case is a little rare, but it does happen this can actually happen very frequently by just you know optimally locating a certain port.

So, that you know certain mode is excited by one port, but other port does not excite that particular mode. For example, in a concentric circular duct if you have a port like this you know one port center, one port offset. So, here and due to this port as we can see the only the radial modal radial modes will be excited apart from the plane wave circumferential modes will not be excited and due to this port 2 you will have your all the modes azimuthal modes I mean basically or circumferential modes.

And radial modes all will be excited. So, basically at the cut on frequencies of the 10 20 and n0 mode you know Z_{22} will go tend to infinity Z_{11} will be finite, but for radial mode special case if we offset it at certain distance. So, it at in this is on the nodal circle of the 01 mode radial mode.

Then add 01 mode resonance frequency Z_{22} will be finite and Z_{11} will tend to Z_{11} will tend to infinity. So, you can get a relation p even at the radial at the resonance frequency to 0 1 mode by centering the port toward the at the nodal circle we will all we will have a look at all these beautiful results when we talk about the 3 dimensional analysis. So, that is one thing and that is one thing now Z_{11} and Z_{22} more tending to infinity Z_{21} and Z_{12} remaining finite that will also give you a peak.

Another peak that is obviously evident when Z21 tends to 0 and Z12 will also tend to 0 because, Hence, but Z_{11} and Z_{12} they are all they can be finite or they can be infinite regardless of the magnitude a Z $_{22}$ tends to infinity sorry tends to Z_{21} tends to 0 had a particular frequency you will still get attenuation p.

The one condition for getting a trough is that when all the parameters you know Z_{11} you get troughs number of times Z_{11} Z_{22} and Z_{12} Z_{21} they are all tending to infinity and they are tending to infinity at the same rate.

$$log_{10}\left|\frac{1}{4Y_1Y_2}\left|\frac{Z_{11}Y_2 + Z_{22}\,Y_1}{Z_{21}}\right|^2\right|$$

$$\simeq 10log_{10}\left|\frac{(Y_2 + Y_1)}{4Y_1Y_2}\right| = 0$$

Then what happens? That the numerator of this thing can be very nicely written as you know Z_{11} you know let me go to this thing Z_{11} Z_{22} ok and Y_2 Y_1 into Z_{22}. So, these can be neglected because probably not at this stage. So, use to write it down here

$$Z_{11} \quad Z_{22} + \quad Z_{11}\, Y_2 + Z_{22}\, Y_1$$

$$+Y_1\, Y_2$$

$$-Z_{21}\, Z_{12}$$

So, under the condition when you have your this term tending to infinity you can sort of clearly ignore the Y1 Y2 which will be much smaller ok.

Then is the these things they go to infinity at the same rate each of the parameters then Z 11 Z 22 they are all you know for a reciprocal and a conservative system they are all purely imaginary. So, when they are all tending to infinity at the same rate then what will happen is basically you know this guy can be sort of ignored or probably they cancel out when you get,

$$10\, log_{10} \frac{1}{4\, Y_1\, Y_2} \left| \frac{Z_{11} \;\; Y_2 + \; Z_{22}\, Y_1}{Z_{21}} \right|$$

So, so basically when Y1 and Y2 they are all same you know when Y1 and Y2 they are all same they are equal you can sort of take this out and you can probably since they are all tending to infinity what will happen is that you know this can basically be approximately written as

$$\simeq 10\, log_{10} \left| \frac{(\, Y_2 + \; Y_1)^2}{4\, Y_1\, Y_2} \right|$$

And 4 this is square of course so 4 $(\, Y_2 + \; Y_1)^2$. So, then you know when Y_1 and Y_2 are the same, then you can clearly show the show that this is equal to 0. So, basically these are the conditions under which you will get trough in the attenuation graph.

Now there is another thing that I want to talk about is that you know system when we have double outlet, you have your you know B_2 the transmission loss is given for such a system as single inlet and double outlet as,

$$TL = log_{10}\left|\frac{\frac{|A_1|^2}{2\rho_0 Y_1}}{\frac{(B_2)^2}{2\rho_0 Y_2}+\frac{(B_3)^2}{2\rho_0 Y_3}}\right|$$

And then you have B$_2$ you can write this in terms

$$TL = log_{10}\left|\frac{\frac{1}{Y_1}}{\frac{(S_{21})^2}{Y_2}+\frac{(S_{31})^2}{Y_3}}\right|$$

So, you know then you can basically this is the transmission loss TL 1 I say because one is the inlet. So, you know by doing these manipulations you can get a transmission loss for making the port 2 as the inlet.

And ports 1 and 3 as the are the outlet ports and so on. So, in general you can find out the transmission loss for a single inlet and multiple outlet system you know by these scattering matrix expression and trust me these S matrix parameters get very nasty once you increase the number of port. So, I would just evaluate them numerically rather than finding out conditions under which you are getting peaks or attenuation troughs and all that.

So, you know with this discussion I will you know close of the lecture 1 of week 11 when we meet tomorrow for week I mean sorry lecture 2 and possibly combine lectures 2 and 3 of week 1, we will analyze a rectangular chamber using different methodologies like you know homogeneous system equations subjected to do in homogeneous boundary conditions or in homogeneous equation subjected to homogeneous boundary condition that is your Green's function approach.

Or uniform piston driven approach one of them we will follow. And try to figure out some transmission loss parameters of a certain configurations. Lectures 4 and 5 will be on circular chambers and let us see how far we can progress with though these sort of things there are quite a few things to cover. So, with this I will end the lecture for today and I will see you tomorrow.

Thanks.