Robot Motion Planning
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Lecture – 13
Potential Field Method – Part I

Hello and welcome to lecture number 13 of the course robot motion planning. In the last class, we looked at sampling based methods. Today we will move on to the next method which is potential field based methods, very quickly revising what we were doing in the last class.

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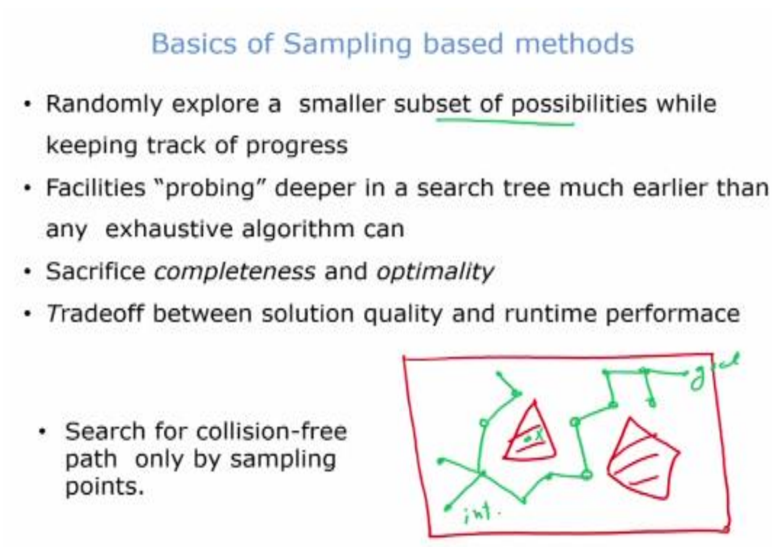

So, in the last class, we looked at the sampling based methods in which we do not search in the complete workspace of the robot and try and focus our search such that we get with the path faster than the other methods like some like cell decomposition methods, etcetera. So, very basis, very quick revision of sampling based methods. So, in sampling based methods, basically, we have the workspace and we have obstacles. So, this is my workspace and there are obstacles here, so, these are obstacles.

And, what we do is we have the robot which is a point robot here and we have my initial point here, initial and that is my goal point. So, in this method, what we have to do is, we randomly explore a smaller subset of possibilities while keeping track of progress, because that means I start looking at various nodes. So, I from my initial point, I take my node here. And basically this node is whether it is possible to go to that point or not possible to go that point whether the robot can go to that point or not.

So, essentially, what we do is we randomly explore a smaller subset of possibilities for example, if this is free space, it can go here, this is not free space inside the obstacle, so it cannot go there. Then what we do is we make connections between these nodes by means of edges and we make a connected graph. So, we have a set of points and then we make connect the points with edges and see whether can connect the initial point to the goal point. So, if you do this, then what happens is that we do not explored the complete workspace, but we can get a path which is much faster, you know, using an algorithm which can work in a much faster or lesser time.

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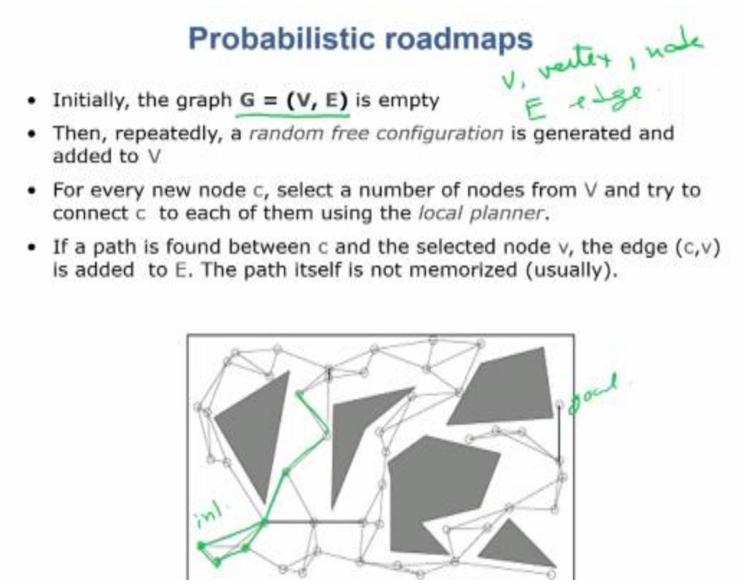

Now, so in the sampling based methods, one of the ways one of the methods is probabilistic roadmap, in which we generate a graph which is given here G and where V is a vertex, so V is vertex, this is also called a node, and E is an edge. So, what is the vertex? Vertex is points where the robot can go. So, these are points where the robot can go. And we connect these points by means of edges. And by connecting this by means of edges, we make what is a connected graph.

Then what we do is basically, we try after we have made this set of connected graphs, we try and find if there is a path between the initial point and the goal point suppose my goal point is here.

Now, this is basically the probabilistic roadmap method by which it is a sampling based method and in which we make a connected graph.

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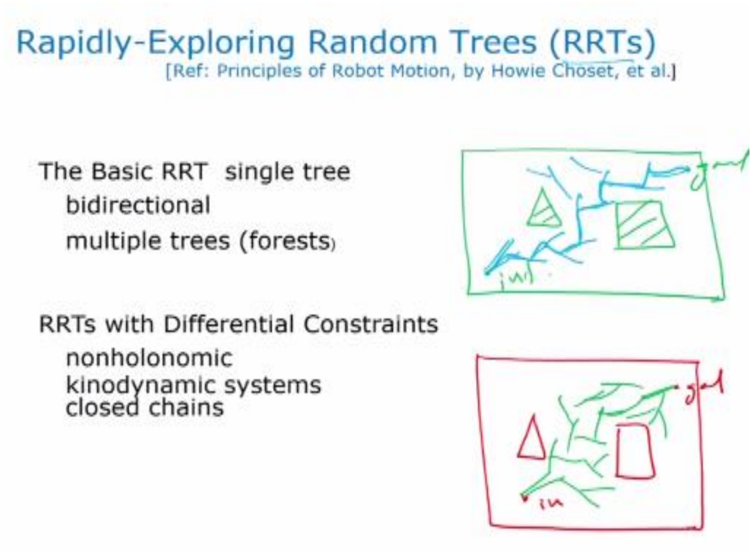

Now, the other way is randomly exploring random trees rapidly exploring random trees in which what we have to do is basically we have again a workspace and we have a obstacles. So, these are my obstacles. And the robot, let us say is here the starting point is here and the goal point is here. So, this is my initial point and this is my goal point. Now, in this case, what we do is it is something similar to growing a tree. So, this is the base of the tree, the roots of the tree and the tree is growing branches like this.

So, it is almost same as the previous one, but the only difference being that this is a connected graph in the form of a tree, and then we see where they can connect the goal. And once the branch connects the goal, then we know that there is a path which is taking it from here, here, here, here like this to the goal, this is basically what is called RRT. Now, you can also grow the trees in 2 directions. For example, if you have a workspace in which we have an obstacle here, and this my initial point and that is my goal point.

So, in this particular case, what we are doing is we are growing the tree in 2 different directions. So, there is one tree which is going from here, like this, like this, like this, like this, these are branches of the tree and the tree is going out and from the other direction another tree is coming.

And then they are meeting somewhere in between, then they know that then we know that this is a meeting in between, so both the trees are meeting in between. And what we can do is? we can search for a path which will take me from the initial point to the goal point, they can be one part they can be more than one path.

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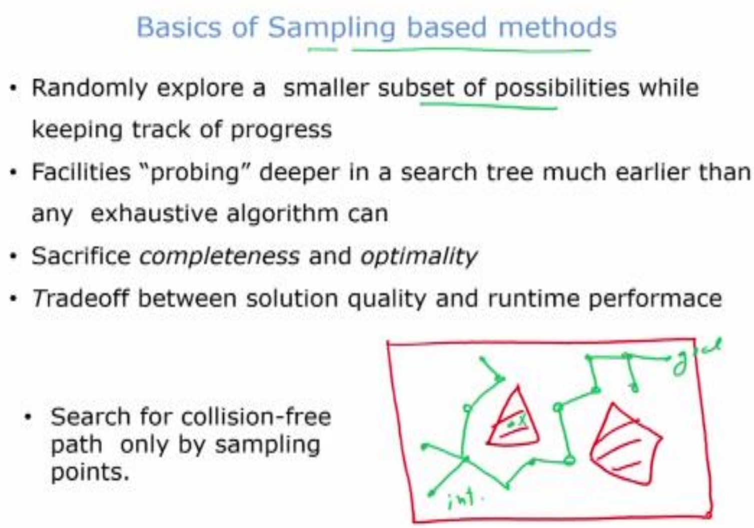

So, what we see is in this sampling based methods, we do not search the whole workspace, but we basically look for a smaller subset of possibilities which makes the search faster, but then we are sacrificing optimality and you may not be able to find a path even when a path can exist. So, this is something we discussed in the last class. Now, we will move on today to the next method.

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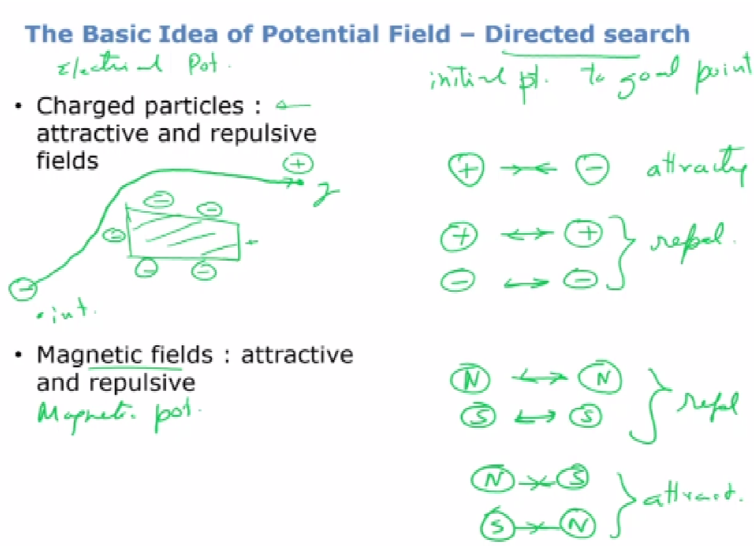

So, the next method is potential field based method. Now, this is a directed search now, what we mean by that is in the previous case, if you look what we are doing here, we were generating this nodes randomly very often we were generating the nodes randomly. So, I have a node here I generate the next node which is here or here or here. So, all these nodes are possible, but we are generating the nodes randomly.

Now, there are of course, directed ways of searching for example, if you are going near an obstacle, then you should sample near optical more than new sample in the free space. Now, in the directed graph methods, what we are doing is we are trying to direct the search in a direct the search from where to where. So, we are basically directing the search from the initial point to the goal point just like we were trying to guide the robot from the initial point to the goal point. Now, how do we do that is basically we use the idea of a potential function?

Now, potential function you would understand very easily you have dealt with charges for example. So, we have a positive charge so, you have a plus charge and you have a minus charge, so, if you have a plus charge and a minus charge what would happen is these 2 charges will attract each other. So, this will be attracting and if you have a plus and a plus or we have a minus and then minus, minus and minus what will happen? This will repel. So, these 2 charges, these 2 are going to repel each other.

So, now, if I can assign potential or charges to the goal and the robot and the obstacle, then this attraction and repulsion can be made use of in directing the robot to go from the initial point to the goal point, for example, if I have a robot which is standing here, so, there is a robot, this is an obstacle and there is a goal that is my goal and this is my initial point. So, what I can do is? I can give the robot a negative charge; I can give this also a negative charge. So, what will happen it will repel the robot and I can give a positive charge here?

So, what is going to happen the robot will get attracted towards the positive charge and it will go like this towards the positive charge get repelled by the negative charges and then it will go towards the goal. So, we are directing the goal and directing the robot towards the goal. Now, so, one example is charged particles this is an electrical potential. Now, we can also have magnetic fields and have magnetic potential, so, we can have magnetic potential.

Now, in case of North Pole and North Pole, north and north, they are going to repel or south and south are going to repel. So, these 2 are going to repel each other, this we know this will repel now, we can also have a north and a south, south and north. Now, this is going to attract each other, they are going to attract. So, basically we can place the North Pole the South Pole on the robot or on the obstacle or on the goal accordingly and ensure that the robot is attracted towards the goal and it is repelled by the by the similar charge on the obstacles. So, this is the idea of using magnetic energy or magnetic potential.

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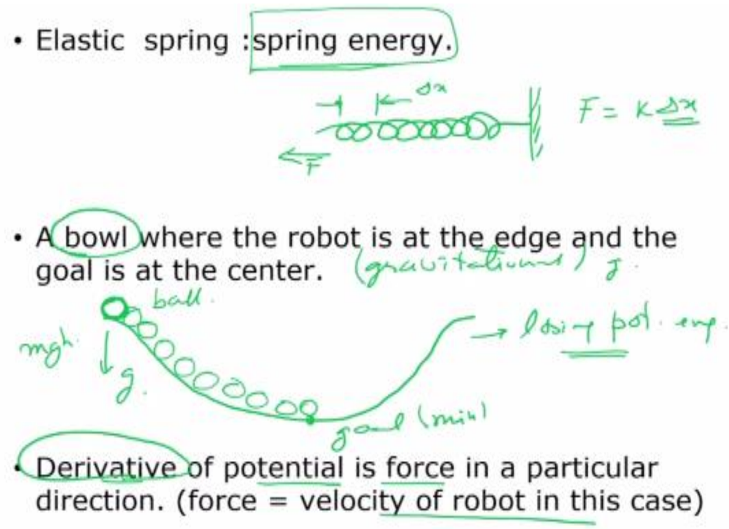

Now, you can think about an elastic spring or rubber band and this elastic spring has spring energy. So, basically if I have a spring like this one is fixed and I pull it. And or I compress it, then F = k Δx small deflection in deflex a little bit and this my Δx. Now what it is storing? It is storing the spring energy. So, the elastic spring or this tension spring, when you pull it or you compress it, it is storing a potential energy. So, when I leave the force, what will happen the spring will come back to its original shape?

So, it is gaining the energy and it is giving back the energy in that way. Now, you can also consider this to be gravity there are different kinds of energy as you know, so, we can also consider this to be all this principle to work on magnetic energy, sorry not magnetic, we have already seen magnetic we can look at gravitational energy or g so, if you consider a bowl, for example, a bowl and the robot is sitting here, the robot is there and this is a bowl and gravity is pulling it in their direction.

Now the robot is at the edge and the goal is at the center, let us say now, what would happen to the robot gravitational energy is gravity is going to attract the robot and the robot will start rolling down. So, it is like a ball. So, this is like a ball which is sitting there. Now, what is happening is? It was having a potential energy mgh. So, as it is coming downwards what is happening it is losing potential energy, now, the robot is coming down because gravity is pulling it down and it is coming in where will it rest? It will rest of the goal.

So, in this configuration, the minimum energy is at the goal. Now, so, what is the direction which it is moving? It is moving in a direction in which it is losing potential energy. So, if we talk in terms of the gradient, then we can say that it is following the negative gradient of the energy there. So, the derivative of potential is force potential energy we can talk about in terms of electrical potential, we can talk in terms of magnetic potential, we can talk in terms of gravitational potential.

Now, the derivative of potential is force in a particular direction. So, this is something to remember here that the derivative of potential is a force in a particular direction and force we can take the force as velocity of the robot in this case, so, the force is pulling it, the force is pulling the robot towards the goal. And the obstacle is repelling the robot away from the obstacle. And this can be taken as the velocity of the robot in this particular case, as we go along, it will become clearer.

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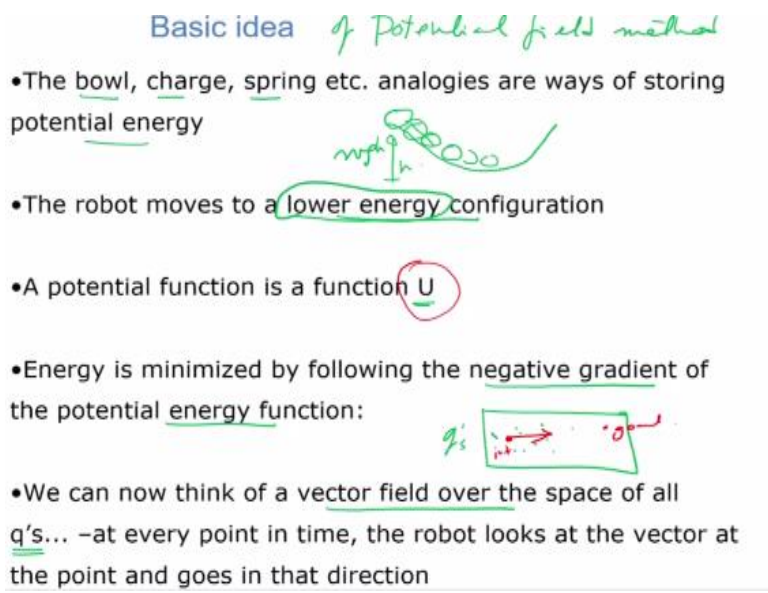

Now, the basic idea of a potential field method is that the bowl, the charge, spring these are analogies and these are ways of storing potential energy. So, these are ways of storing potential energy. And they can be used to guide motion in a particular direction. So, the robot moves to a lower energy configuration. As we saw in the case of the bowl there, the robot was actually coming down. So, it was mgh here. So, this was h. So, the ball was rolling down and it was losing potential energy.

So, the robot moves to a lower energy configuration. So, in this method, what we have to do is first is that we define a potential function U, so in this case, we are defining a potential function U and energy is minimized by following the negative gradient of the potential energy function. So first, I need to write a potential energy function which is U and then I take the negative gradient of that the robot follows the negative gradient such that as it is moving, it is losing energy and moving towards the goal.

Now we can think of a vector field. Now over the space of all q's, and q's are the configurations. So, we can think of a vector field. Let us look at it this way. So, these are various points which are the q's, the configurations in the workspace. And this is a vector field over the space of all the q's, not every point in time. Let me talk, let me marked here so at every point here, the robot looks at the vector at that point and goes in that direction. So, at this point here, suppose this is my robot is here initial point. That is my goal point.

Now, if the robot is being attracted by the goal, then the vector would be in this direction. Because it has been attracted towards the goal that is an attractive potential there. So, at every point, the robot looks at the vector at that point and goes in that direction. So that is how basically how the potential field method works, first we define a potential function. And we take the negative gradient of the potential energy function, and the robot looks at every point it looks at the potential there and goes in the negative direction. So that it basically is reducing the potential energy. Now in terms of force, as we go along we will see more about this.

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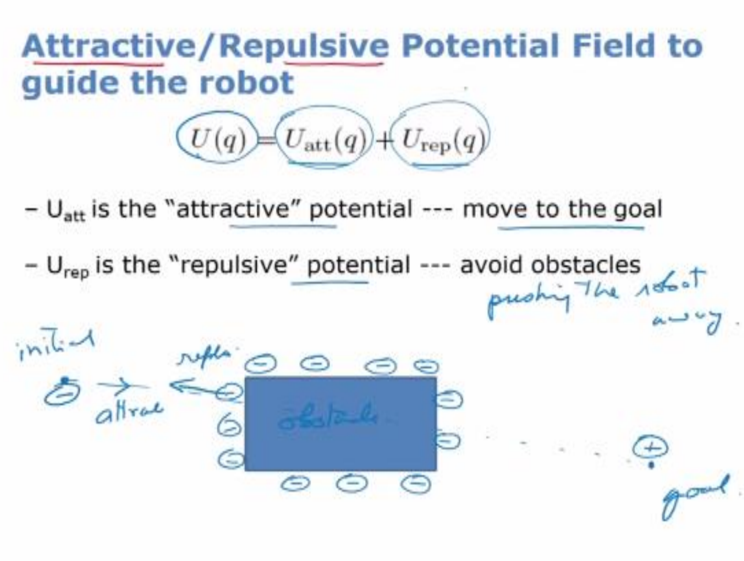

Now let us see what are the various potential functions that we use and how it basically works? So, we have an attractive potential, and we have a repulsive potential field to guide the robot. Now, the attractive potential will move the robot towards the goal and the repulsive potential will push the robot away from the obstacles it can move to the goal and this will avoid obstacles by pushing the robot away, now, so the total potential at any point is the sum of the attractive potential plus the repulsive potential.

Let us see this is my initial point of the robot, this is my goal point these 2 points initial point, goal point, this is my obstacle. Now, let me assign charges to this let me assign a negative charges. So, these are all negative charges, I am assigning to my obstacle and the robot has to be repelled by the obstacle. So, the robot also was sending a negative charge, but the robot has to be attracted towards the goal. So, I have to assign a positive charge there.

So, now if you look at every point here, what is going to happen is that the robot is going to be pulled by the goal in what direction it will be a straight line like that it will be attracted in that direction, but at the same time at this point, there is attractive force which is coming from the goal in that direction along the straight line, now, the obstacle is repelling it the robot with a force in that direction. So, if this one is pulling it that is pushing it like that.

So, what is going to happen at this point is there is going to be a net resultant of the attractive force this is my attractive force and this is my repulsive force. So, what is going to happen at that point is the robot will be subjected to the vector sum of these 2 forces and it will go in the resultant direction the repulsive force is more it will push it away, if the attractive force is more it will pull it towards the attractive force.

So, depending on the attractive force and the repulsive force, wherever is the resultant of the 2 forces that is the direction in which the robot is going to go. Now, this is the basic idea of potential field method which is a guided search. So, the robot is being guided away from the obstacles and towards the goal. Now, so the first thing that we need to do is we need to find we need to assign an attractive potential and we have to assign repulsive potential.

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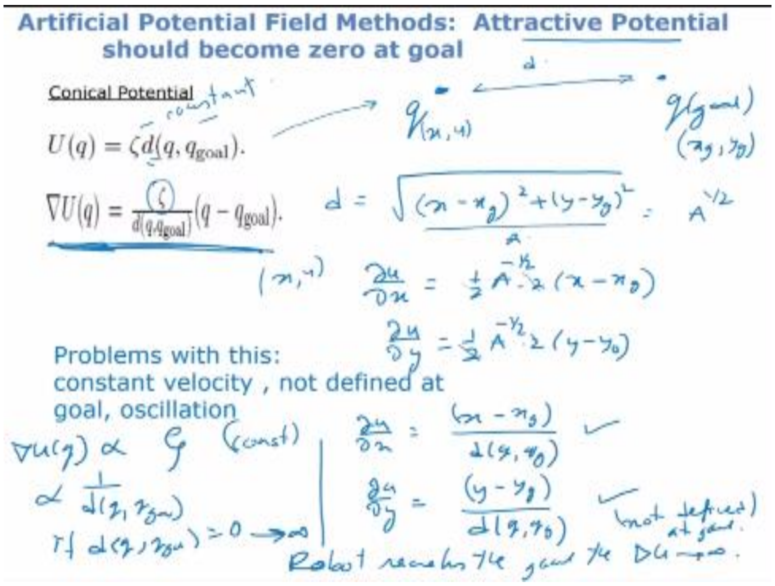

So, attractive potential should so, let us assign an attractive potential, let us define a function. So, U(q) is the attractive potential, and the simplest is to have a function $U(q) = \zeta d\left(q, q_{goal}\right)$, d is distance between the robot and the goal. So, let me put it this way. So, this is my let me just put it here. So, this is my point q, it is in this configuration q and this is q_{goal}. So, what is d? d is the distance between these 2, that is my d. So, this is the $q_{(x,\, y)}$. And let us say this is x_{goal} and y_{goal}. So, d is the distance between the 2 configurations, q and q_{goal}.

So, my simplest potential function that I am putting here is a conical potential where we have a constant ζ was it is a constant and d is the distance which is the distance between these. Now, if this is the conical potential then I need to find the gradient that is given by the grad of U. So, the gradient of this now let me see what is the distance between q and q $_{goal}$. So, $d = \sqrt{\left(x - x_g\right)^2 + \left(y - y_g\right)^2}$, this is the gradient distance. So, I can write this as $A^{1/2}$ where this is equal to A just to understand.

So, now when I am going to take the derivative of this conical potential then there are 2 variables there is x and there is y. So, first I take a partial derivative with respect to x then I take a partial derivative with respect to y. So, if I write it that way, what do I get is $\frac{\partial u}{\partial x}$. So, this is my $\frac{\partial u}{\partial x} = \frac{1}{2} A^{-1/2} \cdot 2\left(x - x_g\right)$. Similarly, I can $\frac{\partial u}{\partial y} = \frac{1}{2} A^{-1/2} \cdot 2\left(y - y_0\right)$. Now, what will happen this 2 will cancel this 2 this 2 will cancel this and this will cancel.

So, what does this come out to be so, $\frac{\partial u}{\partial x} = \frac{\left(x - x_g\right)}{d\left(q, q_{goal}\right)}$ so this is A actually. So, that is my distance A. So, now, $\frac{\partial u}{\partial y} = \frac{\left(y - y_g\right)}{d\left(q, q_{goal}\right)}$. Now, this is what we have seen $\nabla U(q) = \frac{\zeta}{d\left(q, q_{goal}\right)}\left(q - q_{goal}\right)$, this is what we are seeing here, because this and this for x and y. Now, what do you see for this gradient of this conical potential is that number one is that it is proportional to ζ?

So, U(q) ζ, it depends on the constant you put. So, if you put a larger constant the force is going to be more if you put a smaller constant the force is going to be less then it is inversely proportional to $\frac{1}{d(q, q_{goal})}$ which means that if $d(q, q_{goal})$ = 0, this will go towards infinity this is an infinity that means, when the robot reaches the goal the $\nabla u \to \infty$ because the distance will become 0, so, it is not defined at that point not defined at goal.

So, if you use a conical potential, what we see is that it the gradient is proportional to the constant ζ and it is inversely proportional to the distance which means that at the goal what will happen is it will become undefined, because the distance between the configuration and the goal will become 0. So, if the conical potential does not work or there is a problem in the goal, what we do is?

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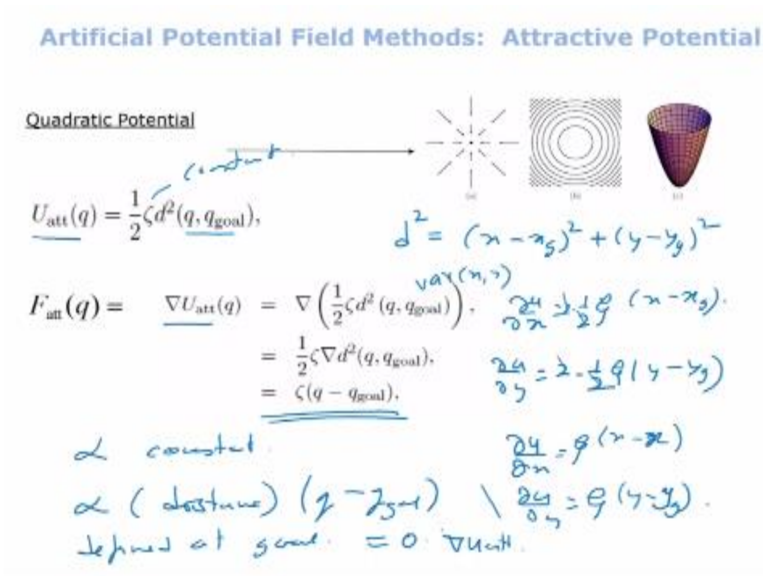

We can use a quadratic potential now, the quadratic potential attractive potential is written by $U_{att}(q) = \frac{1}{2}\zeta d^2(q, q_{goal})$. Now, in this case again ζ is a constant. So, this is a constant and d is the distance between the q and q_{goal} just like in the previous case and the grad of the attractive potential in this case the attractive potential is equal to so, $d^2 = (x - x_g)^2 + (y - y_g)^2$e. Now, again, there are 2 variables x and y. So, variables are x and y. So, we have to differentiate it twice.

So, what would happen is $\frac{\partial u}{\partial x} = 2 \cdot \frac{1}{2}\zeta(x - x_g)$. And $\frac{\partial u}{\partial y} = 2 \cdot \frac{1}{2}\zeta(y - y_g)$l. So, there is 2 and 2 will cancel and what we will be left with is so, in either case, we will be left with $\frac{\partial u}{\partial x} = \zeta(x - x_g)$ and $\frac{\partial u}{\partial y} = \zeta(y - y_g)$. So, what we see here is that in this particular case it is ζ which is a constant so, it is proportional to constant.

So, the larger the constant the more the force and it is also directly proportional to the distance (q- q_{goal}). So, what we are seeing here is that in the case of the quadratic potential it is defined at goal. So, at the goal what will happen this will become equal to 0, $\nabla U_{att} = 0$ at the goal, unlike in the previous case where this was not defined at the goal. So, there is a problem suddenly it goes to infinity or if it comes very near the goal this will start going towards infinity now.

Whereas if you use a quadratic potential, then what happens is that it is directly proportional to the constant that we are assigning plus it is directly proportional to the distance between the goal and it is defined that the goal unlike the conical potential, so, one good idea would be to use both of them together.

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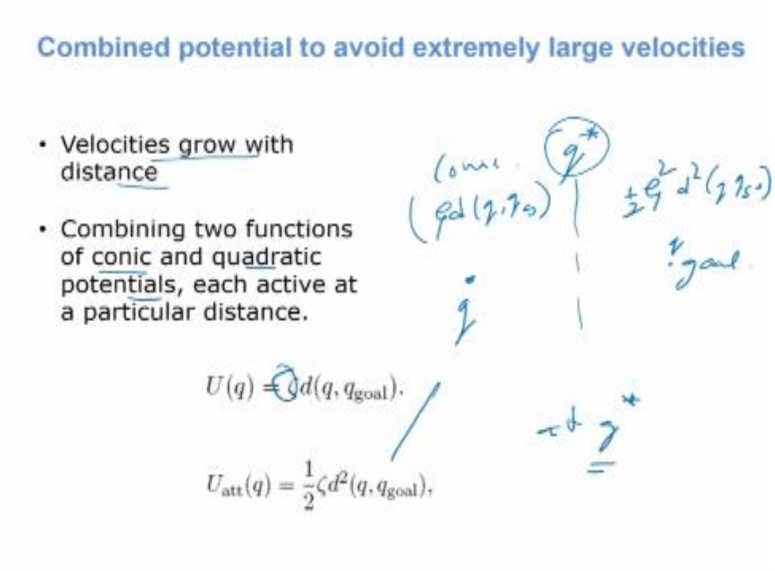

So, combining potentials to over extremely large velocities for example, the velocities grow with distance there is something we have seen now combining 2 functions of conic and quadratic potentials each active at a particular distance. So, what we can do is we can assign a distance now says that if I have my q here and I have my q_{goal} here I can try and define a distance which is called q*. Now, here I can use ζ into d into q and q goal here so, I can use the conical flow here.

Now, the chronic flow as it comes closer and closer to the goal what will happen is there is a chance that the velocities will become very large again it will become undefined at the goal. So, what we can do after the distance I say $\zeta^2 d^2 \left(q - q_g\right)$. So, at this distance what will happen is q * so, we have a particular distance at which they will be switching over from the quadratic sorry from the conic to the quadratic. So, there will be a switch here that q * and we can define this function.

So, what you can see here is that there is a constant here q which you have to define, second is you can define this distance q when you want the switchover to take place. Now, so much about the attractive potential where we can have 2 potentials and they can be active at a particular distance.

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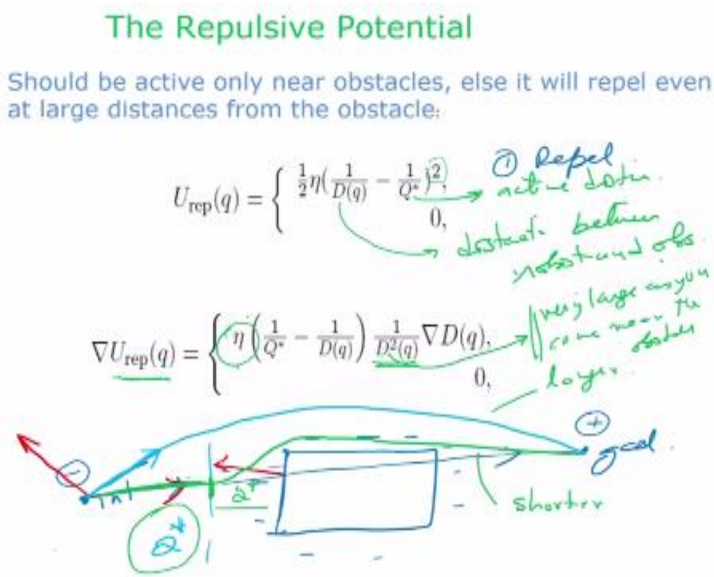

Now, the repulsive potential now, we the repulsive potential to repel the robot, which is clear, now, it should repel, but first of all that it should repel which is fine. But if it starts repelling from very far distance, then what will happen the distance on the path will become longer. Let me take an example here. So, this is my goal and it is my initial point. And there is an obstacle here let us say there is no let me draw the goal here and there is an obstacle here and there is an obstacle here.

Now, suppose I say it is attractive it is plus and this is minus and this is minus, minus, minus now, what is going to happen is that the attractive flow will have a force in which direction in this direction it will connect straight to the sorry it will connect straight by a straight line that is my attractive potential to attractive the side and the repulsive potential will tend to repel. So, what will happen is the 2 forces will have a resultant at this point depending on their magnitudes of course.

So, what will happen is the robot will not move in a straight line but will tend to get pushed off like this why because there is a force and let me draw it in some of the colour there is one force in this direction and there is another force which is normal to this in that direction. So, what is going to happen is the robot is going to move in it result in direction. So, if I draw it like this, then depending on the magnitude the resultant of these 2 forces or in which direction it is in this direction depending on the magnitude of course.

So, now, the robot is moving like this, like this, like this, like this, like this something like this it is reaching the goal yes, but it has taken a longer path. So, what we can do is we can ensure that the repulsive force does not repel the object at longer distances, but only when the robot comes very near the obstacle. So, again we put a threshold distance which is Q * and say that the repulsive force will act only at Q * otherwise; there is no point to tend to push the robot away.

So, if this happens that the repulsive force is active only here, then the robot will follow a straight line like this till here, then it will go like this like this like this and then go to the goal. Now, this part is shorter you can see it very clearly that flow is longer. And because of which we use this Q * as a distance again, it can be set by the person is just not there is nothing automatic here, it has to be set like you set a value of a constant you can also set the distance here, that at this distance, let us say this is my distance Q *, the robot will start getting repelled.

Now, what should be the repulsive potential? So, we are saying the repulsive potential $U_{rep}(q) = \frac{1}{2}\eta\left(\frac{1}{D(q)} - \frac{1}{Q*}\right)^2$ and q is the effective distance or active distance. So, when this robot comes within the distance q only then this is active otherwise this force is inactive. So, this is my potential $\left(\frac{1}{D(q)} - \frac{1}{Q*}\right)^2$. So, this is my potential function I am using now, if I take the grad of this what will be the grad of this? $\nabla U_{rep}(q) = \eta\left(\frac{1}{Q*} - \frac{1}{D(q)}\right)\frac{1}{D^2(q)}\nabla D(q)$.

So, what is happening here is that we are having a D* here and this is coming in the denominator. So, it basically means that it will become very large as you come near the obstacle.

So, as you are coming nearer and nearer to the obstacle what is happening this D^2. So, the repulsive forces increasing very, very quickly. So, this is the repulsive potential that we are using again ζ is a constant. So, the constant would mean that you have to assign the value of the constant.

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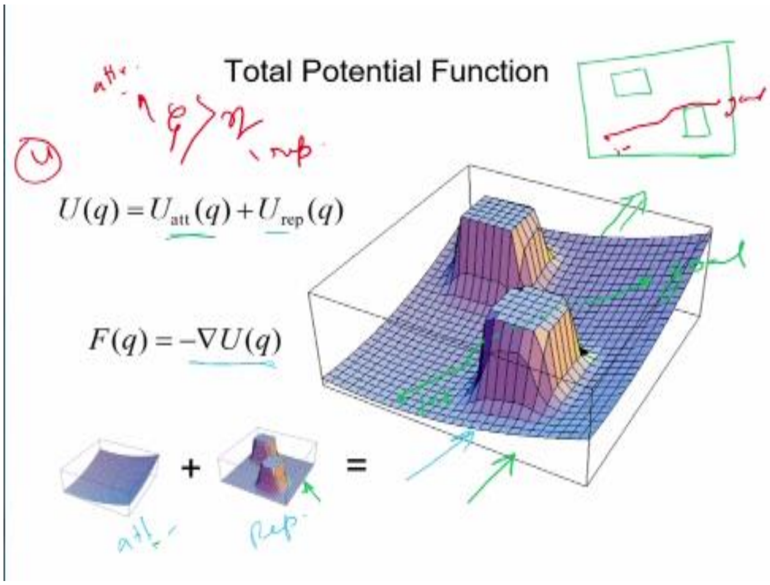

Now, the total potential acting would be the sum of the attractive potential plus the repulsive potential. So, here we are having the attractive potential which is this way so, it is attracting the robot towards the goal. So, it is increasing as we can see that this is the attractive potential. This is the repulsive potential which is repelling the object and what will be the resultant of both of them the attractive and the repulsive potential will be shown here. So, this is the sum of the attractive potential and the reversal potential.

So, suppose I want to go this is my point this is my goal this is my initial point, so, it will be going in this direction like this as soon as it comes near the obstacle to avoid the obstacle and then go towards the goal. So, let me draw it just going on the other side. So, this dashed line in the draw to this is my initial point. So, it starts of towards the goal this my goal point starts off towards the goal then it starts avoiding this obstacle come that side and then goes to the goal that is when behind in between these 2 obstacles actually.

So, here if I draw it in top view, so, this is my top view to there is here and there is here. So, the robot was this easier to understand. So, their goal was here and this my initial point, then the robot was start going like this then what not so sharp avoid like this avoid like this and the good to go. So, it is going like this that was like this like this. So, this is what we are seeing how the attractive potential and the repulsive potential together affect the motion of the robot.

Now, we saw that there was a constant ζ and there was another η, η was for the repulsive potential and ζ was for the attractive potential and the potential functions that we used. Now, these are constants which mean that you have to assign them numerical values so, if you make them very large, now, obviously, the attractive potential should be larger, because it would not attract the repulsive potential will push it away. So, these are some things which are logical that the effective potential should be larger than they should be larger than that one. And whatever the numerical values that depends now on the problem, how fast you want it to go.

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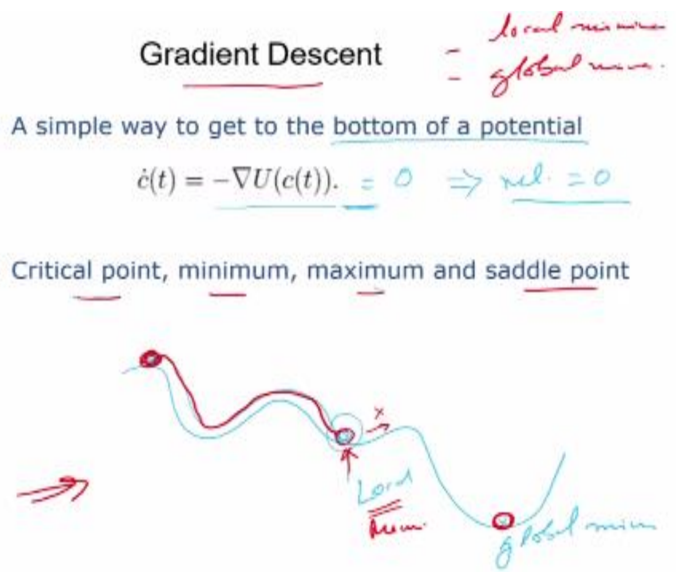

Now, the very simple way by which this method works is that it goes to the bottom of the potential so it basically means it follows a gradient descent algorithm basically, now, if it is following a gradient descent algorithm, there is a question of a local minima or a global minima. So, if you look at this case, here, the robot is standing here let us see this is the profile the robot is well a robot is a ball you can say now it is going down like this decreasing potential energy going like this like this like this, but once it comes it gets stuck.

It does not have enough energy to overcome this one. Now this is not a global minima, this is a local minima here know. Now, whereas the global minima is somewhere here, so this is a problem that comes up with the potential field method. The question of getting caught in a local minima and not reaching the global solution. So, there are critical points maximum minima and saddle points. So, this is a standard problem of potential field method any gradient descent algorithm has this problem that it can get caught in local minima.

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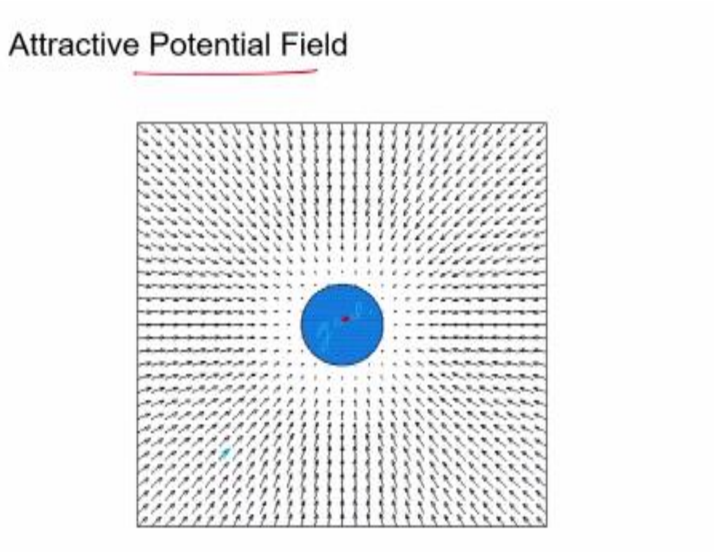

Now, this pictorially shows the attractive potential field you can we can imagine this to be charged direction of charge. Now, this is my goal, this my goal is the central of goal let us say that was the goal and that is my center now, what is the direction in which the lines of force will be moving so, they will be moving normal to the surface like this towards the center right like this. So, we are seeing that this lines of force which are pulling in a particular direction are moving towards the center there you can see that from there so, they are moving in that direction.

So, this is the attractive potential field. So, you can imagine that if there is only a attractive field here it is pulling towards the center of the goal.

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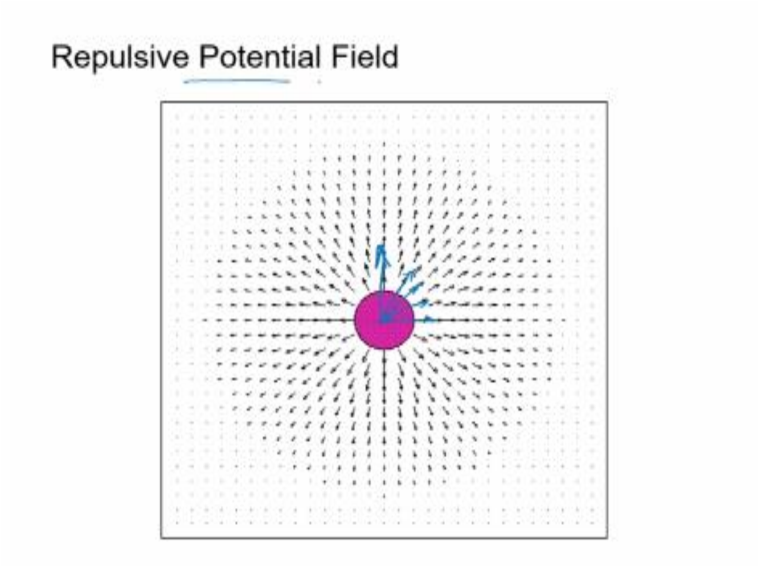

Now, what about the repulsive field now, the repulsive field if you can imagine it to be here, so, this is repulsive we take some other colour this is repulsive it is here it is forcing it in that direction, so, it is going out in the direction. So, it is forcing the robot away in that direction. So, this is a repulsive potential field. So, the sum of the 2 fields this is the attractive potential field this is the repulsive potential field.

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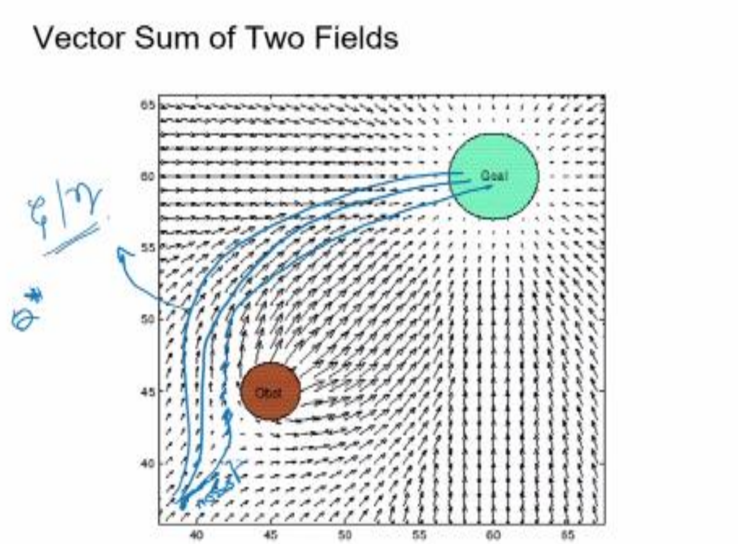

What will be the sum of both the attractive and repulsive potential fields. So, at every point in this vector field there is going to be a vector which will be a resultant of these 2 fields. So, if you are looking at an obstacle and this is the goal, suppose this is my robot so, the robot will be attracted in this direction. And as it is moving towards the goal, what is happening as it is coming here you can see this the vectors of these points are changing direction as it is coming nearer and nearer the obstacle is getting is changing direction.

So, it is coming here like this like this, that is you can see very clearly now, how far will it be like this will be like this will be like this, which one this depends on the values of ζ and η. So, the attractive field is very much larger than the repulsive field, then what will happen is it will go closer and closer to the obstacle and it will go almost like a straight line towards the goal and also on your Q *, which is the distance at which the repulsive field is active.

So, at every point in the workspace, there is a vector which is guiding the robot towards the goal and away from the obstacle. And the direction of the vector would depend on the numerical values of this ζ and the η that we are going to assign.

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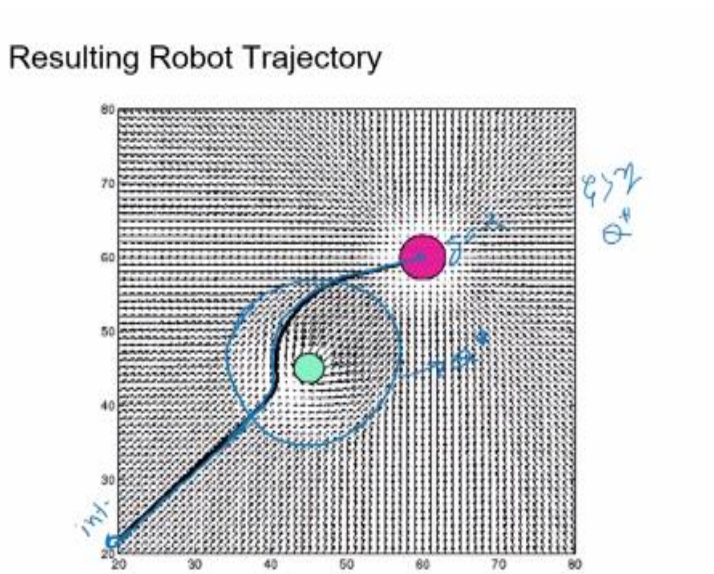

So, this is the basic fundamental of, so you can see here so, if this is my distance, if I say this is my Q *, then the repulsive potential is active only in that Q *. So, the robot starts going I suppose, the robot is here. So, this initial point, that is my goal point, then the robot starts going in a straight line towards the goal, because then that is the direction is the vector is going then the moment it comes inside that effective area, then it starts getting repelled.

It goes like this, like this, and then goes in a straight line again. So, here we see that it depends on ζ it depends on η and also Q *. So, when you are writing the program, basically, you have to numerically tune these parameters, depending on and we will see as we go along, how do you tune these parameters using optimization. So, this explains the potential field method, how it works, how potentials are assigned, and how the negative gradient of the potential is followed such that the robot can reach the goal.

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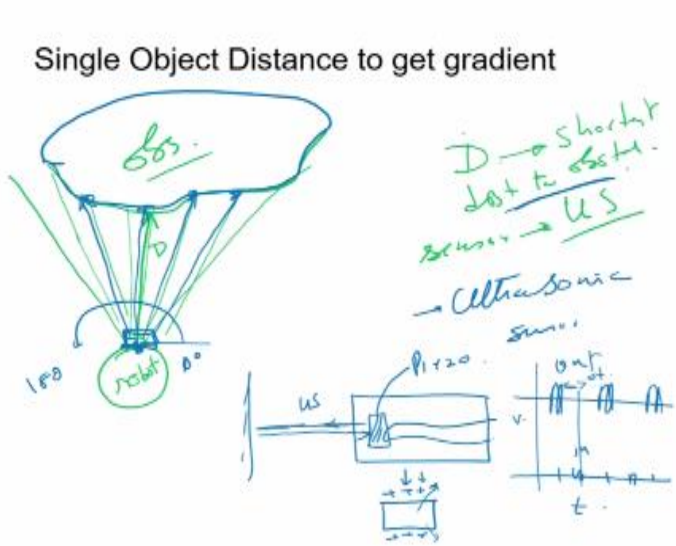

Now, what we talked about is number 1 is local minima. Number 2 is distance and gradient. So, suppose I have an obstacle here like this, this is obstacle. Now a real robot would have sensors, like ultrasonic sensors. So, sensors would mean ultrasonic sensors, so this ultrasonic sensors can find distance and as you would be familiar with ultrasonic sensors, how it basically works is that there is a ceramic transducer here and so this is a piece of crystal.

And the piece of crystal is is actuated in a particular frequency, so, in a particular frequency we give it activation signals. So, this is my piece of crystal, you know that in a piece of crystal, what happens is if you give it charge, it will expand, it will expand a little bit. So, what happens is if you are giving it this my time axis and this my voltage axis or charge. So, what would happen here is if I give it a charge, it will expand and then I give this in a particular frequency. So, this my frequency.

And every time it expands an ultrasonic wave, it is expanding and contracting, so, this wave will go out. So, this my ultrasonic wave, this wave goes out, hits an object and then it comes back again. So, it goes out, now, it comes back means it applies a pressure on the crystal now and if it applies the pressure on the crystal, it produces charge because it is a piece of crystal. So, the moment the wave hits the piece of crystal a small wave is produced here. So, it has gone out this is going out this it has come in.

And if I see the distance between here to here, I see the time distance ∂t I know the velocity of sound in air, I know exactly how far the obstacle is. Now, how this works basically is it is basically rotated in a particular direction. So, if you can imagine that this sensor is mounted and it can rotate maybe 360^{0} or 180^{0}, 0^{0} to 180^{0}, it gives out this waves and wherever the wave is hitting it is basically in short, it is giving the feedback that the obstacle is there.

So, you are being able to profile of the object also. So, ultrasonic sensors can are used also to get the profile of the object as well as to get the distance between the robot and the object. So, this distance is the shortest distance that we are talking about. So, this distance is the distance here that we talked about in the potential field function. So, where is the d, so this d. So, this d is found by an ultrasonic sensor, how far the robot is from the obstacle.

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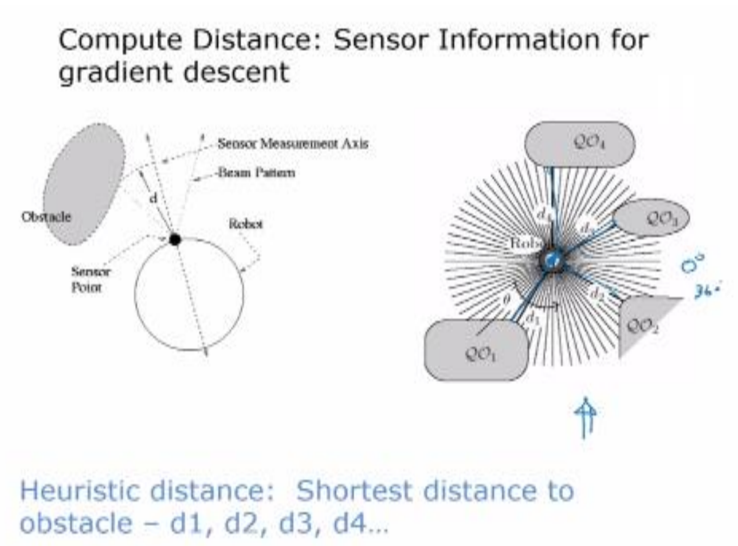

Now so, the right side basically gives us a distance gives us the example. So, we have a robot q here and there are obstacles Q_1, Q_2, Q_3, Q_4, so the ultrasonic sensor can scan let us say from 0^0 to 360^0 and it gives this distance metric. So, finding this distance metric, the robot can actually compute at that point, what is the force the repulsive force is subjected to from the various obstacles?

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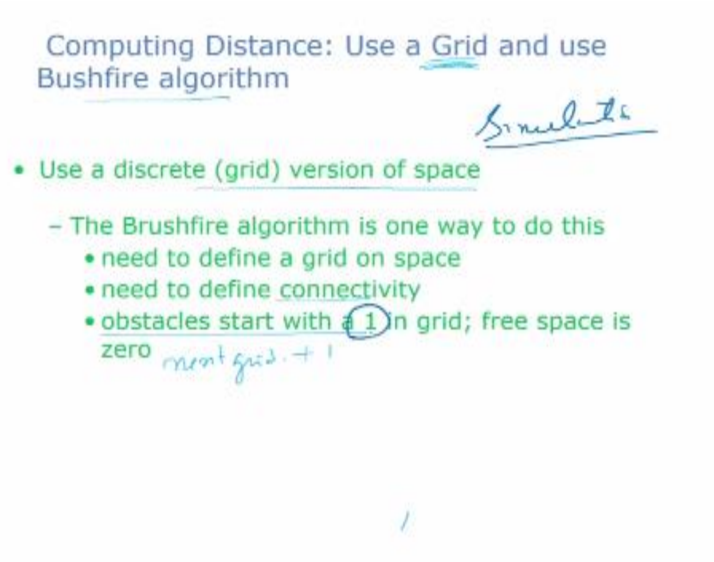

Now, computing the distance, if you have an ultrasonic sensor is in the case of a real robot. Now, suppose you are doing a simulation, we are having a simulation in which you do not have a real robot, but I am simulating the robot obstacle and goal point. So, what we can do is we can use the book a brushfire algorithm. So, we had looked at the brushfire algorithm earlier in which we use a grid. And the brushfire algorithm actually is one way to find out the distance between the robot and the obstacles. How this basically works is the obstacle start with a 1 in the grid and free space is 0.

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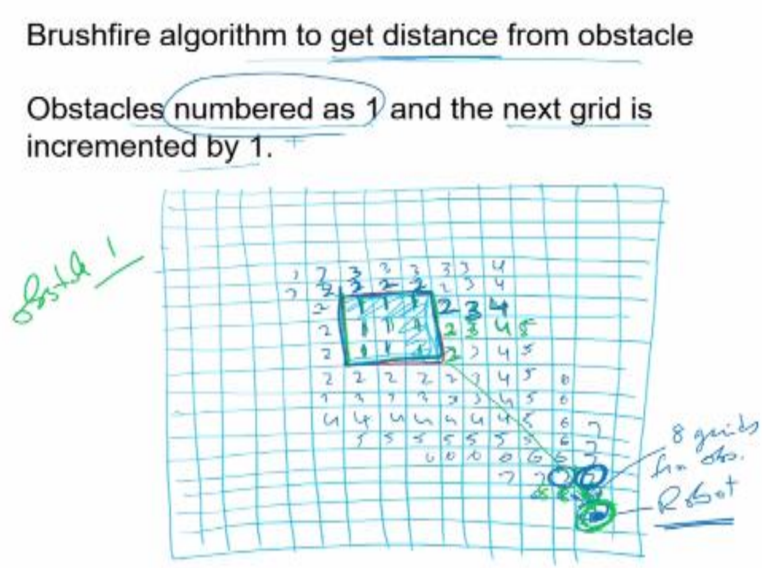

So, let us look at this. So, this is my obstacle. So, the obstacle is assigned 1 and we increment the next grid from this grid sorry the next square rather by 1 unit. So, this is 1 then that is 2 and this is 2 this is 3, 4, like that we keep increasing. So, these are all 1. So, the next is going to be 2 2 2 next to the 2s will be 3 3 3 like that. Now, if we cover the whole workspace like this, basically I can find out if my robot is here, then what is the distance between the robot and the obstacle is very easy to just look at the number 7.

That means that this is the metric of the distance between the robot and that obstacle. So, this is a very easy way of finding out what is the distance between the obstacle and the robot that is my D in the form for grid.

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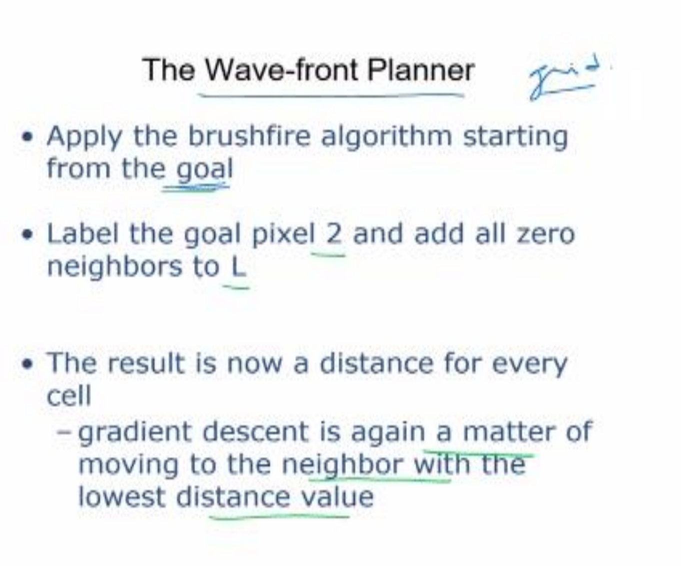

Now, in the case of a wave front planner, this is also a grid kind of planner, in which we apply the brushfire algorithm starting from the goal this is a little different way in the earlier case we were applying the number 1 to the obstacle and then we are incrementing every grid by 1. Here we are saying that we are starting the brushfire algorithm from the goal. And we are applying the algorithm from the goal. Let us look at it how it goes.

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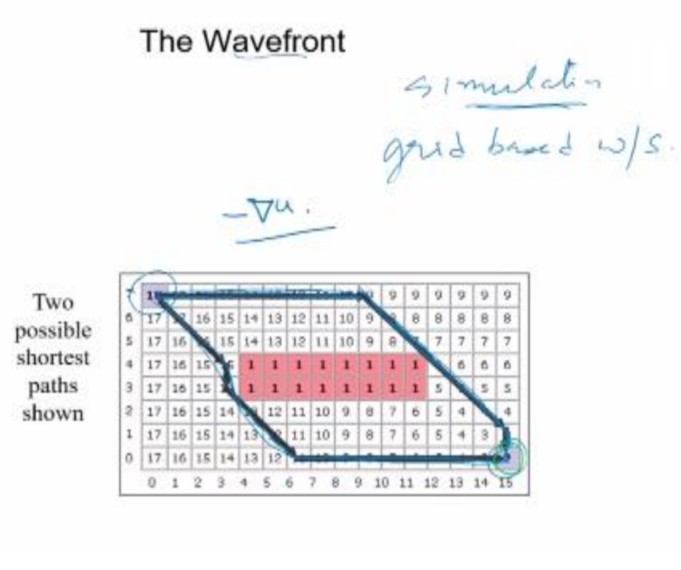

So, here the obstacle is 1 and the goal point is marked as 2, in the way from planner almost similar. Now from 2, we might the next one is 3, so we increment from goal the previous case we were incrementing from the obstacle. So, if this is 2, the next one is going to be 3. So that is 3, then the next is going to be 4. So, we are incrementing it this way, and what you see is that we are covering all the full workspace. So, this is my goal, that is my robot which is 0 there, an obstacle is marked 1. So, robot is there in this obstacle.

So now, we have incremented the whole workspace and what we are seeing is that we are incrementing it this way. And we have finished the complete covering all the grids. Now, it is called a wave front planner, because it moves like waves on water, the way if you drop a stone on a surface of a pond or a lake, then what would happen the disturbance will generate waves, the waves are going to move out outwards. So, it is similar to that which we call a wave front planet it moves like waves.

Now, if you look at this, now, you can find out using gradient descent again. So, what this robot has to do basically it is using gradient descent that was just choosing the lesson number. So, if it is 18 it has to choose the next number which is smaller than 18 so, it is choosing 17. So, it is let us go back here. So, it is choosing this 1, 10 9. So, from 9 it will come to 8 it will come to 7, 7 to 6 that way. So, basically what it is doing is it is choosing numbers it is using gradient descent.

And it is going in choosing numbers which is smaller than itself and this is one part that finds now there is another part that you can find can be like this, then the next question is which path it will take, which 1 is faster. So, this was basically finding using gradient, some gradient negative gradient. So, this is one way of finding out the distance in case of a simulation, if you do not have a real robot where cannot be where you are actually computing the distances.

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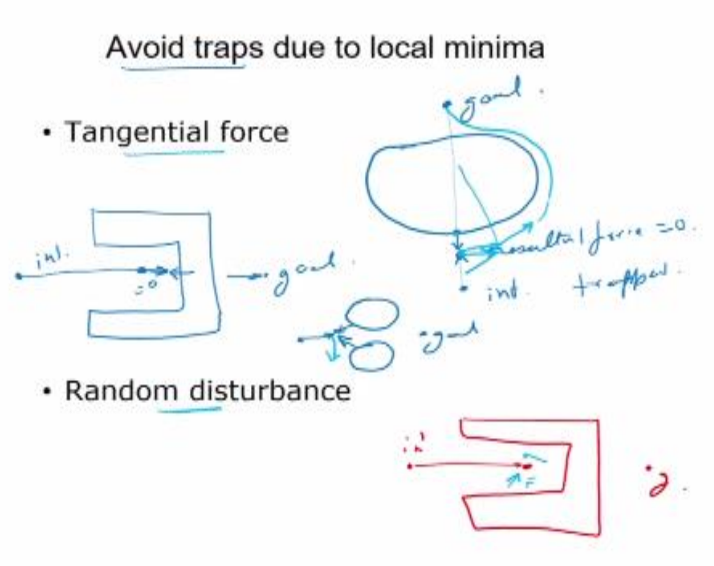

Now, we talked about the potential field method which is susceptible to this problem of local minima, because it is a gradient based method. So, what can happen is it can get caught in a local minima and it appears that it is the goal but it is not really the goal. And these are in terms of path planning, these are called traps. So, how to avoid traps due to local minima number 1 is suppose we have local minima this way, let us look at our case here I have my goal point here.

So, this is my goal and this initial point now, the attractive force from the goal will be in this direction. So, it will move like this. Now, what is going like this what would happen is it will get caught here, because at this point, what is happening, the repulsive and the attractive, maybe at this point is becoming same in the same direction. So, they will cancel each other. So, what will happen is the resultant force is equal to 0. Once the resultant force is equal to 0 means the robot is not moving anymore, it is not subjected to any force.

So, it is caught here or trapped. Now, let us look at another example here. Let us look at this example. So, this is very common in path planning problems. The U shaped kind of an obstacle robot is your initial point goal point, the robot moves in a straight line and at some location what is happening the attractive force which is pulling it and the repulsive force just coming from here are going to become 0 and the net force is 0. Now, the robot is caught there, because the net force is 0, so, it does not move in any direction.

Now, this can also happen if you have a set of obstacles say one obstacle here, another obstacle here and this is my goal, same thing happens. So, it is pulling in this direction, it is forcing it in that direction. So, at some point it will become result in becomes 0. So, in such case what can be done is we can give a tangential force. At this point, we subject it to a force which is tangent to the surface here. So, I give a tangential force here, and this tangential force moves the robot in that direction.

Again, it tries to go here again it is forced in that direction by tangential force and finally it goes like this and goes to the goal. So, we subject the robot to tangential force once it gets caught in a trap. Similarly here we can give it a small force like this in terms they say now, the other way could be to give a random disturbance suppose I have let me take this example again. So, the robot has got caught here this is my goal point this is my initial point the robot has got caught there.

So, what we can do we can give it a disturbance force a small random disturbance. So, I get the robotic force. So, from here the robot goes there. Now, again it goes then out tries again it may not be able to get out but this is one way of disturbing it and trying so, that it can try again.

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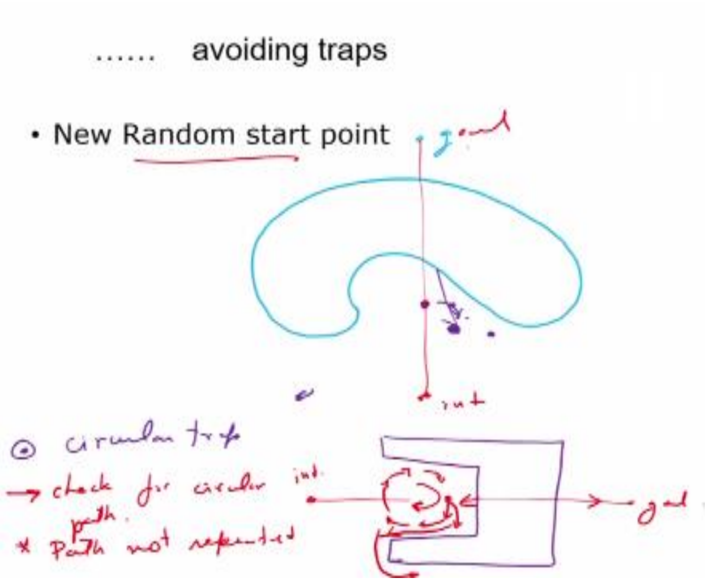

The other way of avoiding traps is a new random start point for example, I have an obstacle which is of this shape and this is my goal point and this is my initial point. So, what is happening here is just coming till here and is getting caught here. So, the moment it gets caught I can give it a new random start point, this is almost similar to giving you a disturbance. So, from here I start the algorithm again and from here I push it here it comes there. So, this is the random start point I push it from there to here now, it starts again.

It may be able to overcome interview to overcome now, if it gets caught here I give it another point it goes there. So, it could be random so, it could be here also. Now, you may be able to get out you may not be able to get out. Now, the other way of is by looking at you should also be able to circular traps so, this circular trap is very common again in provincial fill method where we have an obstacle which is of this nature and the robot is moving from here the initial point to the goal point and this is my straight line.

So, it comes here it gets stuck, let us see, it comes here it gets stuck. Now, this is my repulsive and this metric it gets caught there. So, I move it is following the tangential force it is going the side, then it goes like this, then it goes like this and they will tend to go back ground and it will tend to circle now, this is another kind of trap, but it is caught it keeps circling instead it keeps moving in that circle thinking that it has come out of the trap, but in actual fact it has not come out of the trap, because of the shape of the obstacle.

So, in this particular case, you should be able to check for circular motion, circular path. So, path should not be repeated. So, the way it is done here is path not repeated. So, if it is repeating this path, the robot finds it repeating the path it should not take the same path it should change the port immediately. So, for example, it should immediately change it to some other path and try this one then probably will come out from here. So, new random start points circular trap, then we have tangential force, random force, these are all combined together.

Depending on which one again, the question which will work best? That is a difficult question. It is very much what planning is completely geometry based. So, depending on the kind of obstacle that is there, a particular method will work.

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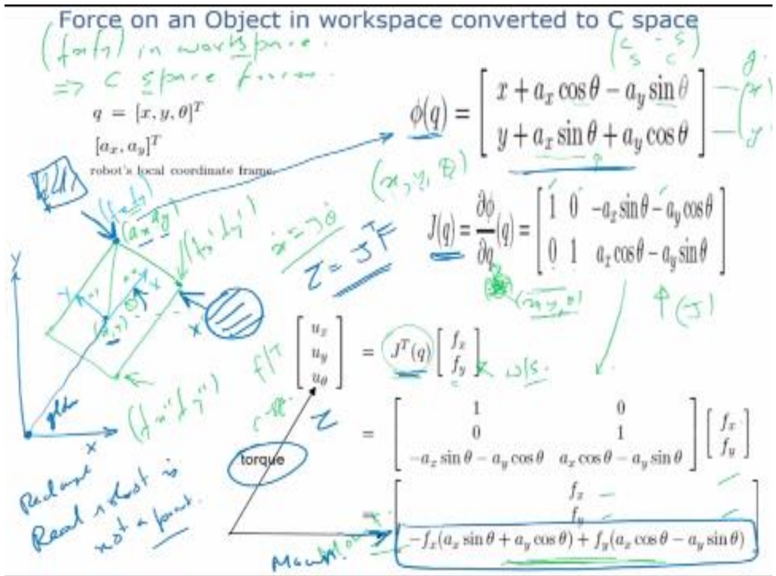

Now, so far what we have been seeing, we talked about a robot that is essentially a point robot, but a real robot is not a point robot. So, real robot is not a point robot. that means it has some specific dimensions. So, in that case, if I see a real robot like this, let us say this is my rectangular robot, real robot rectangular. So, there is my x axis, this is my y axis. Now, if I take this then I have my center point x y coordinates x y.

Now, suppose I say as per potential field method, there is a force acting at that point, which is f_x f_y and the coordinate of that point there is a_x a_y. Then with reference to the base to the global frame, there is my global frame, what is the coordinate of the point? So, Φ is given by and the rotation of this robot is a θ the x axis of the robot versus the global x is θ now. So, the coordinate of that point is given by $\phi(\text{q}) = \begin{bmatrix} x + a_x \cos\theta - a_y \sin\theta \\ y + a_x \sin\theta + a_y \cos\theta \end{bmatrix}$.

So, we are seeing that the force which is acting at that point a x and a y can be converted to the global frame in terms of movement by using this Jacobin we know that, let me write $\tau = J^T F$ those of you who have not studied introduction to robotics statics, but please refer to the textbook regarding this relation that the joint top $\tau = J^T F$ and from here what we do is I take the derivative of Φ and I get J and there are 3 variables x y and θ.

So, first I take the derivative with respect to x then with y then with respect to θ and what I get J is like this now, I get this my torque which is $\begin{bmatrix} u_x \\ u_y \\ u_\theta \end{bmatrix} = J^T(q) \begin{bmatrix} f_x \\ f_y \end{bmatrix}$ and the final relation we get is this. So, this is the moment which is acting, so, this moment which is acting at the origin of the global frame. Now, by doing this why do I need to do this because in the case of a real robot, you would understand the forces would be acting at different points. So, real robot is not a point.

So, suppose I have a rectangular robot and there is an obstacle here this fellow is pushing at this point and there is another obstacle there which is pushing at that point now. So, now, you see that the robot is subjected to forces at 2 different points and the forces could be different. So, we can convert it to a moment and then add them up and see what is the net result in force which is acting on the robot. So, this is the method used for converting the force acting at different points on a real robot, either it is a serial arm or on a mobile robot to convert it.

So, that we can convert to the moment acting at the origin of the global frame and then add at the moments. So, today we will stop here, what we discussed today is the potential field method in which we talked about assigning potentials to so, we were assigning potentials to the object, the robot and the obstacle and then the robot was subjected at every point in the workspace the robot is subjected to a force in a particular direction and then it effectively moves in that direction.

And in this method, there are constants which is ζ and η which we have to define manually, which will determine what is the force acting on the robot, so we will stop here today and continue in the next class on potential field method.