Robot Motion Planning
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Lecture – 18
Kinematic Constraints and Multifinger Robot Hands

Hello and welcome to lecture number 18 of the course robot motion planning. In the last class we looked at motion planning in systems which are subjected to kinematic constraints that is non-holonomic constraints like a car which can move sideways because there is a velocity constraint. We also looked at the concept of controllability and we saw that although the car is subjected to a single non-holonomic constraint and it cannot move sideways, but even then it is fully controllable.

That means it can go and park at any position in the workspace. Now, today let us complete the discussion on controllability and then move on to the next topic which is the multifinger robot hands Multifinger robot hands which are used for manipulating objects. For example, you have multifinger hand like this which has maybe 3 fingers, a robot hand with 3 fingers. And when you catch an object and you manipulate this object, say something like this.

I am holding this mouse and I am manipulating it, first is I am grasping the mouse for example I am just grasping it like that, I just grasp. Second is after grasping I am manipulating the object, I am moving it in some direction like this. Now, this fingertip point and this object is also subjected to kinematic constraints that means it can move freely in some direction, it may not be able to move freely in another direction that is something we will look at today after we complete our discussion on controllability.

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So, in the last class we were talking about motion with kinematic constraints. For example, we talked about this single wheel which can rotate about an axis. So, it can rotate about that that means it can roll forward or backward and it can turn about this axis. So, this is my steering.

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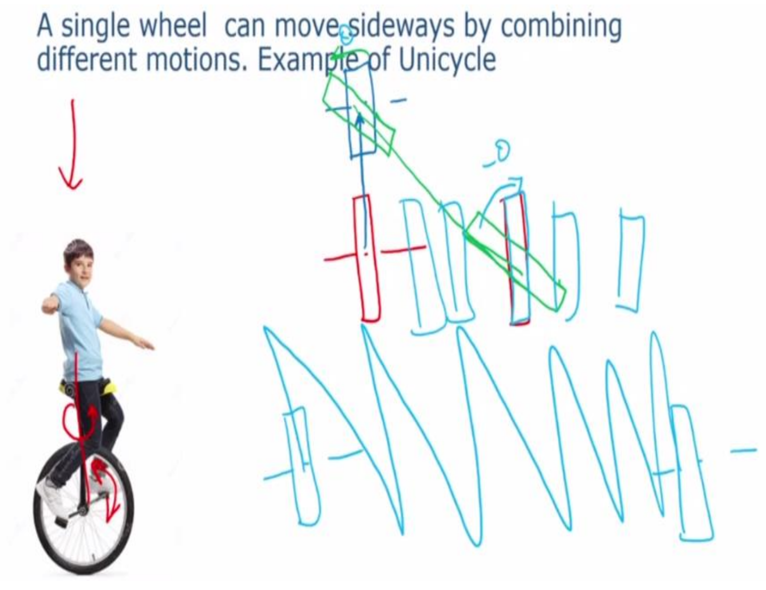

This is very similar to this particular case where which is a unicycle. For example, this boy who is riding the unicycle, when the paddles this moves forward, so this can roll forward in this direction or backward and it can twist about this axis. So, it can basically roll forward backward and twist. In such cases, we see that it cannot move sideways. For example, this wheel that I just drew here, it cannot move sideways because it is subjected to a non-holonomic constraint.

But if I want to make it move sideways, there are some motions which I can perform by which it can move sideways. Now, this wheel can move forward and backward. It can rotate about its axis, so that is my rotation about the axis which is the steering actually. So, if you see a car which is made up of four wheels, each of the wheel, the front wheel can be though about like this and a unicycle of course is the same wheel that is being shown in the video here.

Now, it cannot go sideways. So, I cannot make it go sideways like that because of that single non-holonomic constraint. But if I want it to go and park it sideways, that means I want to park it or place it at some other configuration in the workspace then what I can do is I can make a series of manoeuvres where I can perform certain motions and a collection of motion such that which will enable the wheel to go and park sideways and what are those motions?

First for example, I go straight then I rotate that is my second motion, third is I go back and then rotate again. So by performing these four actions, you can see the car can go or this wheel can go and park itself in any orientation actually and this is what we mean by controllability of the problem that the system is subjected to a non-holonomic constraint, but it can still go and park at different locations because it is fully controllable.

This was the example of the unicycle. Now, if I take the top view of the unicycle, this is my top view and this is my central axis because suppose I am seeing from this direction and I want to make the unicycle go and park here. So, what I can do is I can take it forward in the front direction, I can move it in the front direction that is it rolls forward and goes here up to that location that is my axis.

Then what I can do is I can use the steering angle and steer, it turns like this that is my steering angle like this and then it comes backward like this till here. So now it has come backward like this and just positioned like this. I just come backward in this position like that. And then what I do is my fourth manoeuvre is like this, I turn it on that side. So this is my θ and this my $-\theta$.

Then I turn it this way and this comes and parks here; which means that I can park it anywhere, I can park it here, I can park it here, I can park it here, I can park it here. Now I may be able to park it with just these four manoeuvres. Suppose it is very far from here and I want to park it here now. So I may not be able to park it in just one manoeuvre, but I can do a series of manoeuvres like this and then go and park it there.

So, this is a path planning problem where we are trying to trying to park this unicycle in some other orientation in some other location in the workspace and the system is fully controllable, which means I can actually go and park it anywhere I want.

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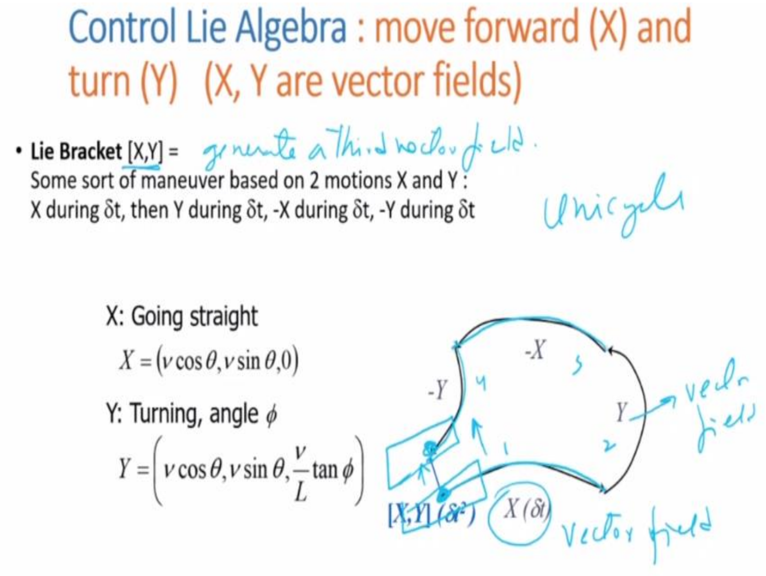

Now, the mathematics involved in this is basically what we call the control Lie algebra. And what the control Lie algebra tells us is that if I want to go from this position here to here, I am making the set of manoeuvres in the case of the unicycle we were going straight which can be taken as a vector field, vector field X okay, then I am going to turn this is another vector field.

Then I am going to go back which is – X that is another vector field and then I am going to go by –Y. So, by making these four manoeuvres 1, 2, 3 and 4, I was able to go from here, this is my unicycle, I was able to go from here to there which is effectively going sideways. Now what the Lie bracket tells us is that if you take the Lie bracket of X and Y when X and Y are two vector fields one is going straight and the other is we are going to generate a third vector field.

So taking the Lie bracket of X and we are generating a third vector field and this vector field is telling us that you can actually go sideways like that, not in one shot but by making different manoeuvres.

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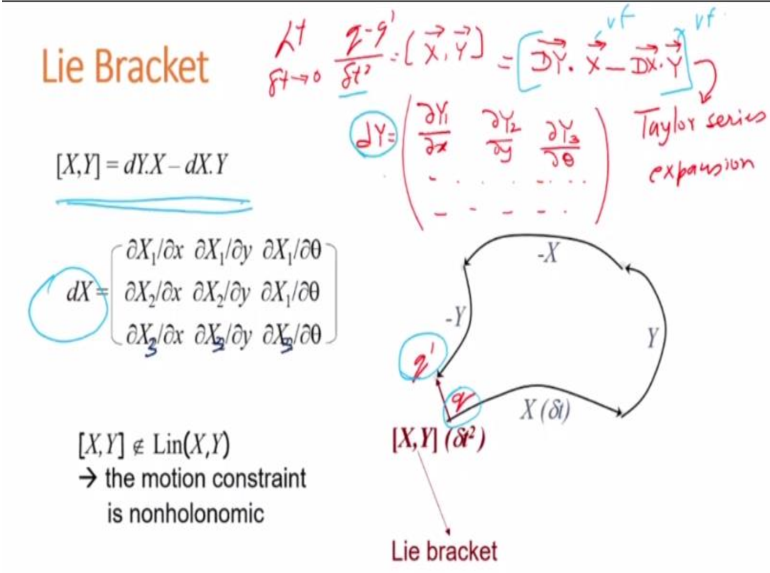

Now in order to do that and where did that come from? That is basically when we are at a location say q and you are going to q' by q', you are taking the Taylor series expansion and only retaining this term, where $\underset{\delta t\to 0}{Lt}\frac{q-q'}{\delta t^2}=\left[X,Y\right]=\overrightarrow{DY}\bullet\vec{X}-\overrightarrow{DX}\bullet\vec{Y}$ and this is one vector field, this is another vector field. Now, dX is equal to this and dY is equal to that.

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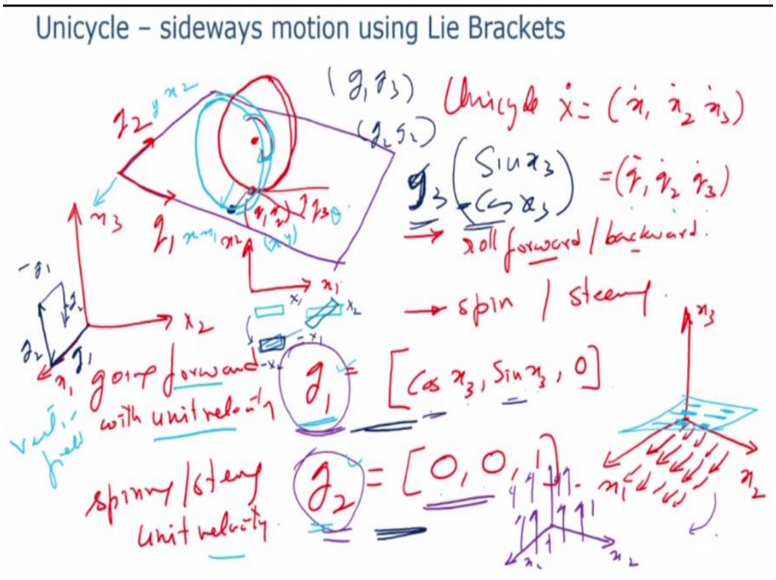

So, if I perform this operation what I would get is a third vector field and that vector field will actually be able to show me if there is any other direction which it can go. So, in the case of the unicycle, something I discussed in the last class, I will go through this very quickly here. So this is a wheel, this is my single wheel, it is making a contact q $_1$, q $_2$, or x, y we can say and it has an angle θ which is q $_3$.

So the unicycle has three a q_1, q_2 and q_3, x and y and θ. Now, if I have two vector fields, the first vector field is the unicycle going straight which is $g_1 = [\cos x_3, \sin x_3, 0]$. This is just going straight or going forward with unit velocity, this is one vector field. So, this is one vector field. The other vector field is spinning or steering with unit velocity so that is $g_2 = [0,0,1]$ this is a second vector field. We have two vector fields g_1 and g_2.

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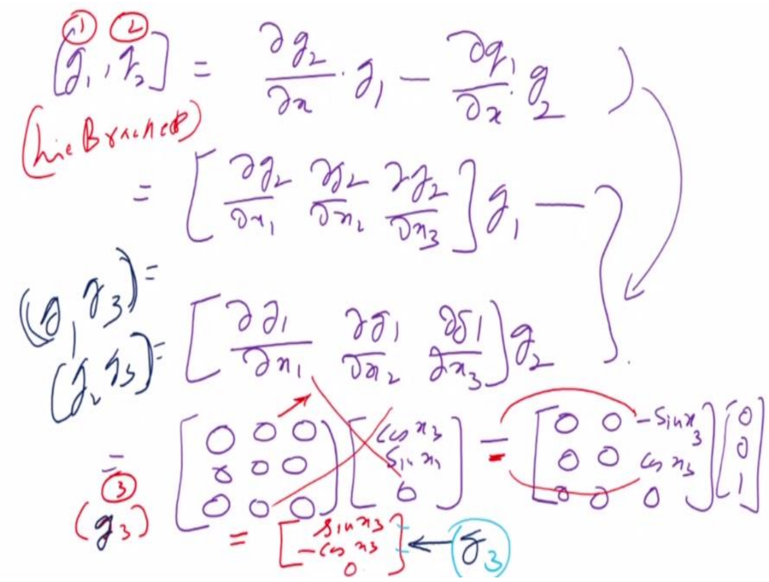

So, if I take the Lie bracket of these two vector fields, what we get is a third vector field, $g_3 = \begin{bmatrix} \sin x_3 \\ -\cos x_3 \\ 0 \end{bmatrix}$. And if you look at it this sine x_3 –cos x_3, in the top view when you are looking at this, this is my x_1, x_2, x_3. So –cos x_3 would mean that it is in the y direction, right. So if this is if you can imagine this to be x, that to be y or this is x_1, that is x_2 when it is in this direction that means it is in the negative direction of, so it is being able to come this side now.

So, this basically tells us that wherever the robot was or what this wheel was, it can go in the negative the other side of it or it can go and park sideways, so it can go and park here now like this. So this is mathematically telling us that taking the Lie bracket of g_1 and g_2 you are generating a third vector field which is g_3 and this is basically telling us that it can have a velocity in that direction and that is what allows the wheel to actually go and park sideways. So, this is also allowing the car to go and park sideways.

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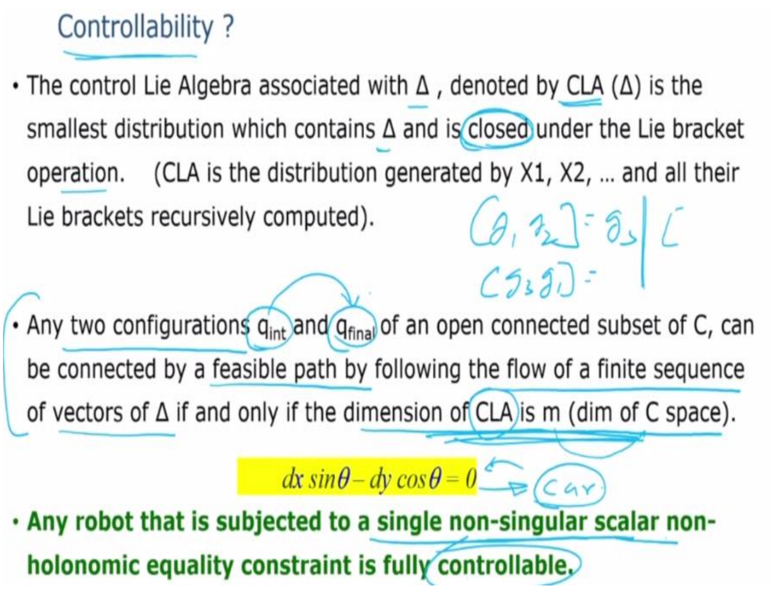

Now, from the point of controllability, we say that the control Lie algebra associated with delta denoted by CLA is the smallest distribution which contains delta and is closed under the Lie bracket operation. What is the meaning of closed under Lie bracket operation? That means you can keep doing g_1, g_2, you get g_3. And you can keep doing g_3 and g_1 you will get something else. Now, is this closed or is it open?

Will you generate each time a new direction or you will end up getting directions which are not independent that is the meaning of the word closed here, as we go along we will see this as we see our next example. Now what is the point of this controllability business? The point of this controllability business is any two configurations $q_{initial}$ to q_{final} have an open connected subset of C can be connected by a feasible path by following the flow of a finite sequence of vectors of delta.

If and only if the dimension of CLA is m and m is also the dimension of the C-space. So, if I want to figure out if a system is fully controllable, I need to check this condition that the dimension of the CLA is equal to the dimension of the C-space. And if that is the case, then we can say that the system is fully controllable. And that also means that from an initial point you can always find the part to the final point that is the meaning of controllable, right.

Now, as a corollary of this, we have that any robot that is subjected to a single non-singular scalar nonholonomic quality constraint is fully controllable. Now, if you remember this is the car, the car is subjected to this constraint and this is a single non-singular scalar nonholonomic equality constraint and hence a car is fully controllable.

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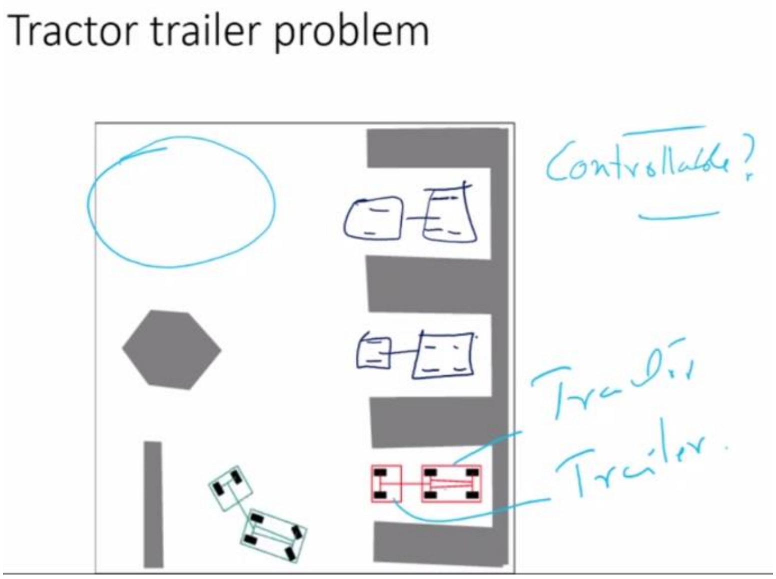

Now, let us proceed further a little bit more tricky, the tractor trailer problem. In the case of a car, we understood that it is completely controllable and you can go and park it wherever you like. Now, what about the case of a tractor and the trailer problem? Now suppose I want to figure out if this fellow from here can go and park there, then the first question I need to ask is, is this fully controllable? Is the system fully controllable?

If it is fully controllable, then only you can go and park like that, otherwise you cannot. So how do we figure that out? We go back to our definition and see that if the dimension of the CLA is equal to the dimension of the C-space, in that case the system is fully controllable, not otherwise. So let us look at this issue.

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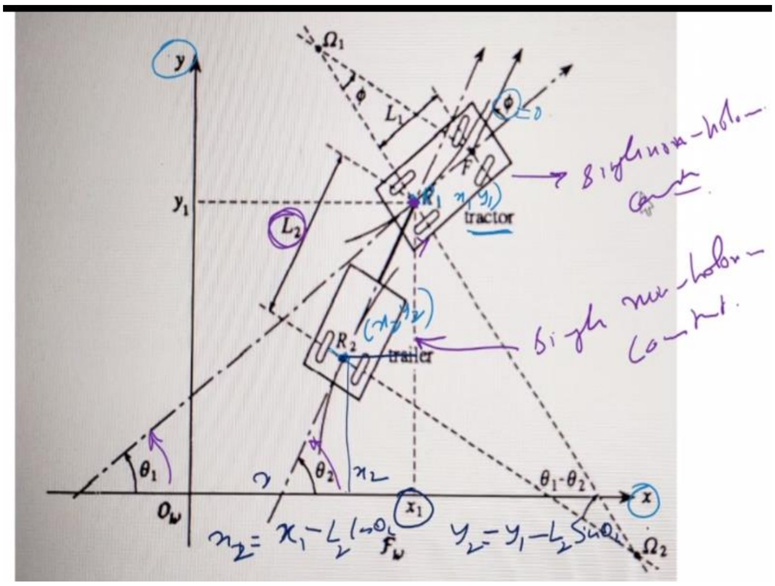

Look at this problem. Now this is basically showing us the tractor and the trailer. So please note that this is my tractor and this one is my trailer that you must have seen this tractor trailers in the roads carrying hay or carrying all kinds of goods. Now, this is the setup of the problem. So first of all is that this is my x axis that is my y axis. This is my tractor, it is shown here. The tractor has four wheels. The centre point between the two wheels is R_1 and it has coordinates (x_1, y_1).

So, R_1 is the centre point between the two wheels R_1 and (x_1, y_1) are the coordinates. Now, the front wheels have a steering which means that there is a phi there. So, depending on the turn that you want to take or the turn that the tractor wants to take you can change that Φ angle and the tractor can take a turn. So, when you are going straight then the phi can be equal to 0, in that case is going straight.

Now, how is the trailer attached? So, the trailer is attached by a revolute joint here. So there is a revolute joint here so it can rotate like this at that point R_1 and it is fixed to the point R_2. R_2 has coordinates (x_2, y_2). Where is R_2? R_2 is the midpoint between the two wheels of the trailer. So this is the setup of the tractor and the trailer. Now a couple of things to note here. The first one is that the tractor is making an angle θ_1 with the x axis that means I am going to assign a local frame there.

So, this is my local frame which is x that is y that is a local frame. Now that the tractor is making an angle θ_1 with the x axis, now the trailer is making an angle θ_2. Now because this is a revolute joint there it can rotate like this or it can rotate like this, but there has to be some relation between θ_1 and θ_2, is not it? Now the first thing to note here is that when this is taking a curve, say for example is taking a curve like this, the tractor is taking a curve like this.

So the velocity at that point is going to be tangential to the curve. So, let me draw it again. This is my curve that the tractor is taking. So, the velocity at that point has to be tangential to the velocity in this direction. What about the trailer? The trailer is taking care of like this and because the velocity will be tangential to the curve at the point. So, both of velocities this one and this one is tangential at R_1 and at R_2. Please note the directions in which the tractor is going and the direction is the trailer is.

And the other thing to note is that L $_2$ is the distance between the point R $_2$ and the point R $_1$, this is the R $_2$. So now let us proceed and see now the tractor is subjected to a single non-holonomic constraint. Similarly, the trailer is also subjected to a single non-holonomic constraint that means both of them cannot go sideways, this cannot go sideways, that also cannot go sideways. However, the trailer is related to the tractor because it is connected here with a revolute joint. Now let us proceed and see how do, we proceed to solve this problem.

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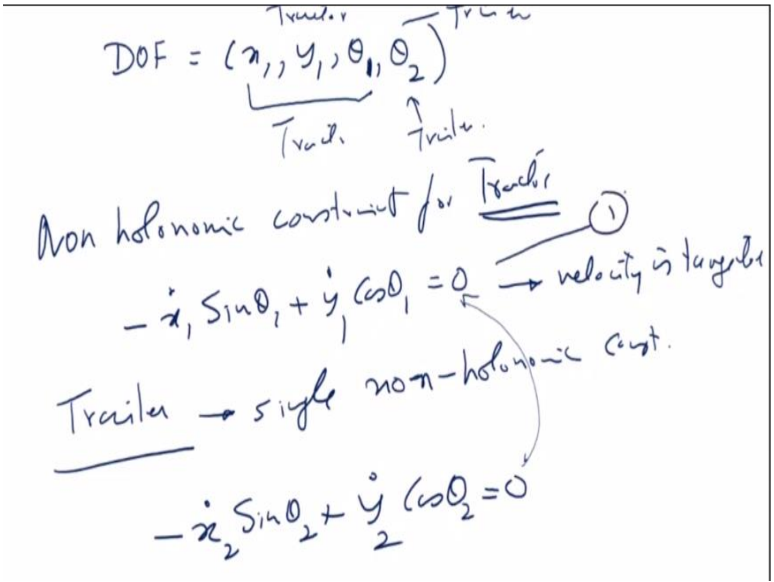

The first thing that you see is that how many degrees of freedom was there in the system. So, degrees of freedom are x_1, y $_1$, θ_1 this is for the tractor, this is the same as that of a car. Now there is a θ_2 there, this is θ_1 here and a θ_2 two there this is for the trailer. So this one is for the tractor that is for the trailer. Now, the non-holonomic constraint for tractor, $-\dot{x}_1 \sin\theta_1 + \dot{y}_1 \cos\theta_1 = 0$.

This is the same non-holonomic constraint that a car has because the velocity is tangential, this we had derived in the last class. So, the tractor is behaving like a car. So, this is for the tractor. What about the trailer? The trailer is also subject to a single non-holonomic constraint. Now what is the non-holonomic constraint is $-\dot{x}_2 \sin\theta_2 + \dot{y}_2 \cos\theta_2 = 0$. You see that this is almost similar to this.

It is also velocity constraint and it is also saying that it cannot go sideways. So, both these constraints are saying the tractor cannot go sideways and the trailer also cannot go sideways. Now is there some other relation between x_2 and x_1? Let us go back and look at the figure again. Now, this is my x $_1$ projection of and this is my x $_2$. So, this is x $_2$, y $_2$, so projection is there.

So, which means that $x_2 = x_1 – L_2 \cos \theta_2$, this is my $\cos \theta_2$. So, x_2 which is this point where is x $_2$, x $_2$ is this point $x_2 = x_1 –L_2 \cos \theta_2$ there is one geometric constraint we have and $y_2 = y_1 – L_2 \sin\theta_2$. So, we have these two conditions also, these two conditions are from the geometry.

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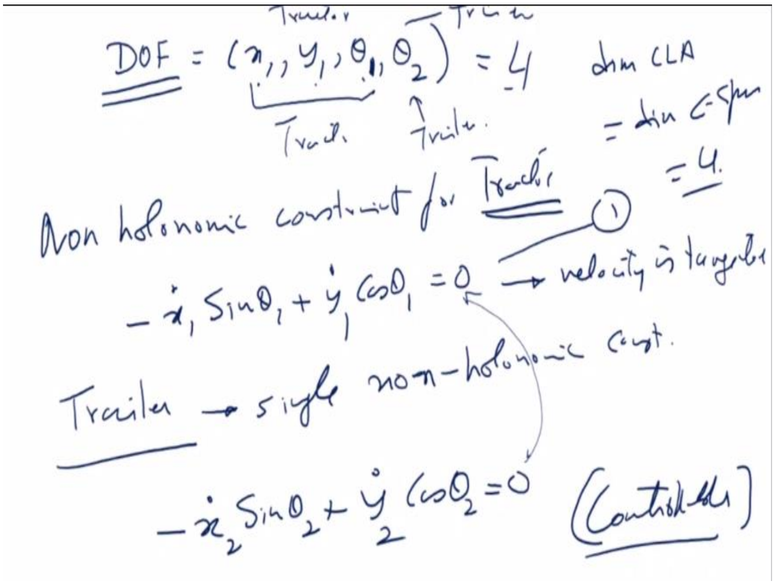

We have $x_2 = x_1 – L_2 \cos \theta_2$ and $y_2 = y_1 –L_2 \sin\theta_2$ from geometry. Now, if I differentiate an add to the first equation that is equation 1, so this is my equation 1. Then what we do get combining these equations we get $-\dot{x}_1 \sin\theta_2 + \dot{y}_1 \cos\theta_2 - L_2 \cos\dot{\theta}_2 = 0$. Combining these equations this is what we get. Now what we see is that if I want to prove that this system is fully controllable, I have to find four g $_1$, g $_2$, g $_3$, g $_4$ four motions.

And show that the determinant of these four vector fields is not equal to 0 which basically means that all of them are independent vector fields. So, what we are doing here is like in the previous case what we did in the previous case, if you remember in the case of the unicycle, I wrote two vector fields, one was going straight which was cos and sine and the other was 0 0 1. So, here also I am writing a vector field for going straight which is x $_1$.

This is for going straight with unit velocity this is equal to this is one vector field $\vec{X}_1 = \left[L_2 \cos\theta_1 \quad L_2 \sin\theta_1 \quad 0 \quad -\sin(\theta_1 - \theta_2) \right]$. So, this is one vector field for going straight. Now another vector field for turning can be written as 0 0 1 0, this vector field is only for turning because this is my angle θ, sorry this is my angle θ_1 for the tractor.

Now, if I use Lie brackets, then what we get is $\vec{X}_3 = \left[\vec{X}_1 \vec{X}_2\right]$ using Lie brackets and this generates another vector field which is $\left[-L_2 \sin\theta_2 \quad L_2 \cos\theta_1 \quad 0 \quad -\cos\left(\theta_1 - \theta_2\right)\right]$. So it generates another vector field like this. Now, thus I have x_3 by taking the Lie brackets of $\vec{X}_3 = \left[\vec{X}_1 \vec{X}_2\right]$. I can get another one x_4 if I take the Lie brackets $\vec{X}_4 = \left[\vec{X}_1 \vec{X}_3\right]$ because if I take the Lie brackets x_1 and x_3 now what we get here is $\left[0 \quad 0 \quad 0 \quad -1\right]$.

Now, if I find the determinant of all of these four, $\det\left[X_1 \quad X_2 \quad X_3 \quad X_4\right] = L_2^2 \neq 0$, which means that these are independent vector fields and because there are four independent vector fields the degree of freedom of CLA the dimension of CLA is equal to 4 and we have seen that if the dimension of CLA is equal to 4 and here also the degrees of freedom is four, this x_1, y_1, θ_1, θ_2 these four.

So, the dimension of CLA is equal to the dimension of C-space, which is equal to 4 which means the system is fully controllable. So, this tells us that this tractor trailer problem this is very interesting because just by looking at it, it is very difficult to say, but once we do the analysis, we find that this system is fully controllable which means that you can actually take the trailer the tractor and park it anywhere you want and that is what is shown in this simulation also.

Even sidewalks parking is possible provided you can perform a number of manoeuvres. Now, this is very interesting problem that you can have a tractor and a trailer and this is a fully controllable system it can go and park wherever you want. So, you can go and park here for example, to be able to go and park here. So, that is the interesting part of this problem that we can prove the controllability of this system simply by using Lie algebra.

And proving that the dimension of the C-space is equal to the dimension of the control Lie algebra. So, we do not go further here and we will stop our discussion of controllability here. And we move on to the next topic which is multifinger robot hands.

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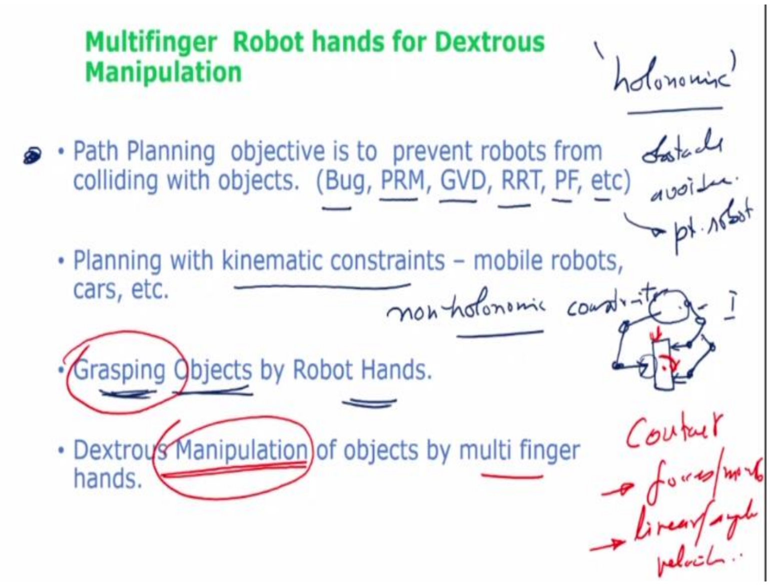

Now, multifinger robot hands are normally used for dextrous manipulation. Now in whatever we have studied in path planning so far, path planning objective is to prevent robots from colliding with objects this is something we have seen obstacle avoidance and whatever we have done like Bug algorithm, probabilistic roadmap method, generalized Voronoi diagram, RRT, potential fields or potential function, etc. the whole objective was obstacle avoidance.

Then here we made the assumption that it was a point robot and can move in a direction. Then we talked about planning with kinematic constraints in the case of mobile robot cars where there were non-holonomic constraints. Here the constraint is holonomic. This also has constraints, but they are holonomic no constraints, here it is non-holonomic constraints. Now let us look at a case where we are not saying we are going to avoid obstacles.

But we are saying that the object has to be grasped by robot hands that is I have an object like this and it has to be grasped by two fingers in two directions, two or three fingers let us say. It has to be grasped by a robot that is basically called grasping objects by robot hands. Now, this robot hands can be looked upon as manipulators. So, this is something like a manipulator and that is my robot hand.

So, it is like this a serial manipulator and this is making contact at this point. Now in this particular application grasping you are seeing that the robot is actually making contact with the object and it is constraining the object such that it does not move that is the meaning of grasping an object that is one. But after grasping an object, you may also want to move the object just the way I showed in the beginning of the class where I held the mouse like grasp the mouse and then a manipulated the mouse and moved the mouse.

It means I want to impart some motions to this object. So, I also want to manipulate the object by multifinger hands. So, I need to perform some motions this side or maybe the side, whatever. So, we need to also look at grasping and we need to look at manipulation. Now grasping and manipulation are a little different, they are also path planning because this object must move in a particular way and hence there has to be some path planning.

Now, it is a little different, in the sense of obstacle avoidance we do not make contact, here we are making contact. So, we also have to worry about the velocities and forces that are being applied because there is contact here. So, the moment there is contact it basically means that you can apply forces, moments and they can be linear and angular velocities.

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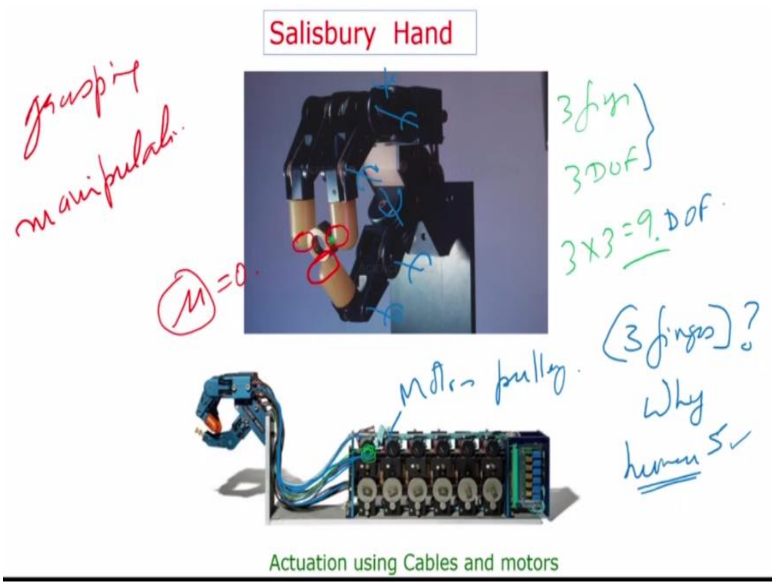

So, this is what we will be looking at in multifinger manipulation, grasping is one and number two is manipulation. Now, this is a very famous hand called the Salisbury hand. It has 9 degrees of freedom, 3 degrees of freedom per finger and you can see that it has three degrees of freedom per finger there is one here that is one degree of freedom, this is second degree of freedom and there is one degree of freedom there.

Similarly, here there is one, there is two there is three. So, 3 degrees of freedom per finger and there are 3 fingers, so it is 3 into 3 that is 9 degrees of freedom this hand has. Now this hand can grasp, as you can see it is grasping the nut there but it can also manipulate. That means it should be able to rotate the nut or maybe move the nut in some way. How is it actuated? Basically, each joint is actuated by cables.

So, these cables run down under the fingers and into the drives which are located a little bit further away. These are all cables which are moving to be, so the motors are here, motors and pulleys and when the motor rotates, the tension in the string changes and that is how the fingers move.

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This is showing the internal construction of the finger motion. So, there are cables. These cables are going through the pulleys and then are connected to particular joints, like this is connected here. This one is connected through the cable is connected here. So, when this cable tension changes, what will happen? This joint will move that is how the fingers of a multifinger hand moves.

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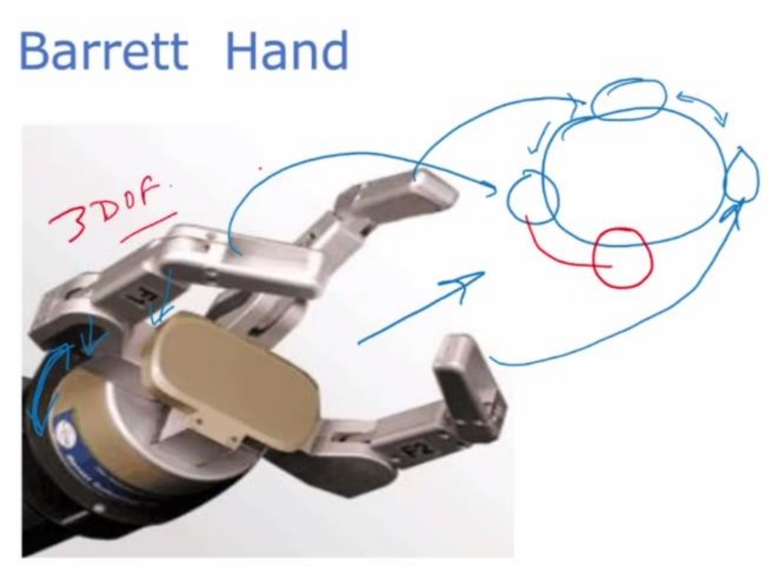

Another famous hand is the Barrett hand. So, the Barrett hand, there are 3 fingers. Something I forgot to mention here is that there are three fingers. So, this question may come to your mind, why 3 fingers? Why do humans have 5 fingers? We have 5, robots only have 3, 2 or 3, but humans have 5. So how many fingers do you need actually that is an interesting question. So, the Barrett hand also has 3 fingers.

It is a little different in construction in the sense that it has a degree of freedom here, it has a degree of freedom here and this full finger can rotate. So, if I draw the side view of this there is one finger here, one finger here and one finger here as shown here. So this is here, this fellow is here, this one is there as shown in this figure but this can rotate this way. So, there is a degree of freedom there and it can change its position.

So, the fingers can rotate and this finger can come here and come here. So, depending on what kind of grasp you want, the finger base can rotate on the wrist and you have one degree of freedom there. So, each of the fingers also have 3 degrees of freedom.

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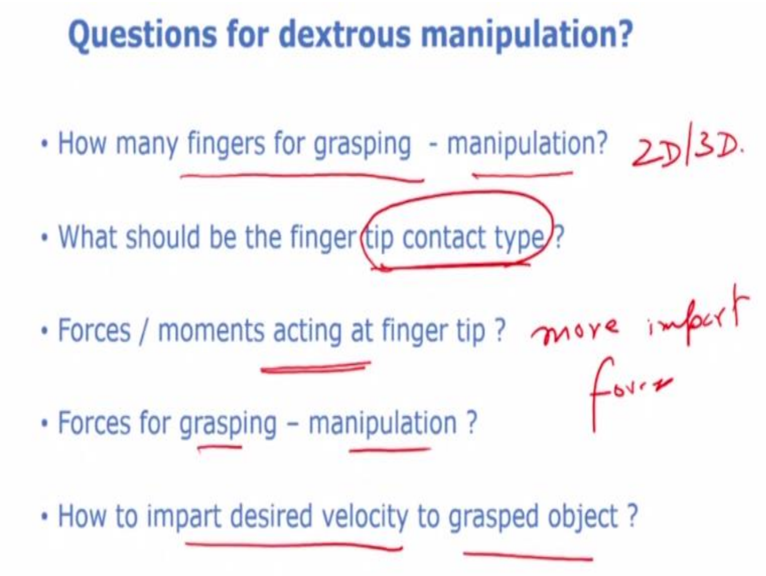

Now, we are going to grasp an object and we are going to manipulate the object means we are going to have some kind of motion plan for the object now. I want to move the object in the particular way. Now, some of the questions that we need to answer the first question is how many fingers for grasping and how many fingers for manipulation? We just want to grasp an object, but then again the question whether you are talking about 2D or you are talking about 3D, is it planar or is in 3D.

So, how many fingers are required for grasping and how many fingers are required for manipulation? What should be the fingertip contact type? We saw that the fingertip contact has a particular shape. So this has a particular shape, the previous one had a different shape. This one fingertip has a spherical shape. The human fingers have a different shape again. So, what should be the shape of the fingertip and what kind of contact because that is the one that will determine the contact type.

Forces, moments acting on the fingertip, now if you want to move an object, you have to impart forces to move an object. So what are the forces, moments acting at the fingertip? What are the forces required for grasping? How, what are the forces required for manipulation? And lastly, how to impart desired velocity to the grasped object? So I have grasped an object and I want to move it along a path, then how do I impart a desired velocity to the grasped object?

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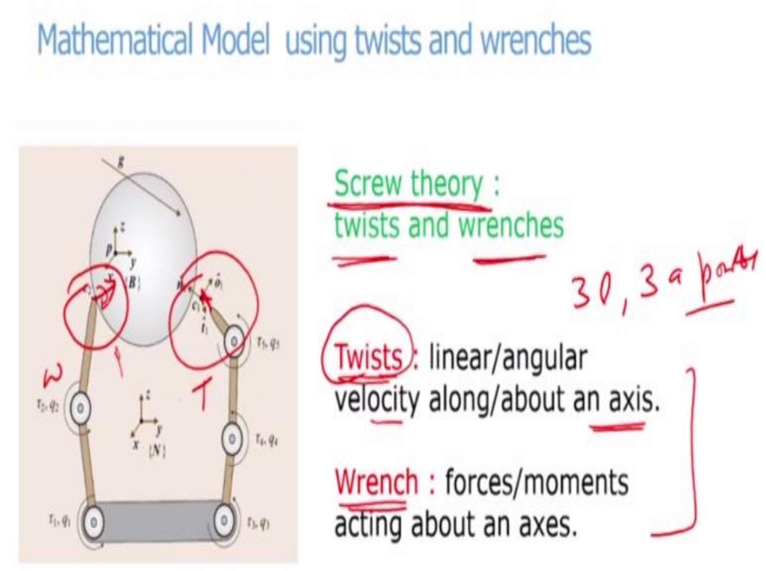

Now here again we need to look at the mathematics behind here, but unlike in the previous case where we talked about motion planning where the robot is not making contact with the object, here the robot is making contact with the object. So, in terms of screw theory, we are going to analyse this and here we deal with twists and wrenches. Not twists are basically the linear and angular velocity along an axis.

Say for example in this case this finger is not making contact, so it can move, it can have linear velocities or it can have angular velocities about an axis simply because there is no contact with the object. So it is not constrained, so these are called twists. So, there are three linear velocities possible and three angular velocities possible. So, three linear and three angular velocities possible when there is no contact, so these are called the twists.

Now, wrenches are the forces, moments which are possible to be applied to an object about an axes. In this particular case so here we talked about twists, here I am talking about wrenches. So, in case of wrenches, this finger can apply some force, it can place a moment. So, those are the forces, moments acting about an axis which the finger can apply to the object. So, the motion of the object is going to determine on what.

It will depend on the forces and motions being applied and what linear and angular velocities may be important to the object. And hence we use twists and wrenches for looking at the types of contact and then we try and decide how to impart a particular motion to the object.

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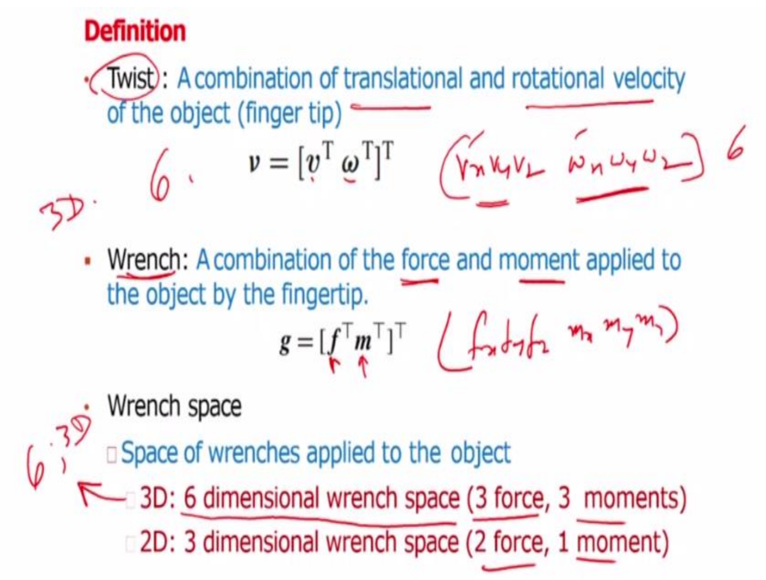

Now, let us come to some definitions before we go into the mathematical analysis. So, the definition of twist is a combination of translational and rotational velocity of an object. It can be defined this v^T and ω^T. So, how many v_x, v_y, v_z and ω_x, ω_y ω_z. So, these are angular velocities about an axis and this is linear velocity along an axis, so there are 6 here. So, these are called twists.

Now of course twists are possible if there is no contact. The moment the object makes contact, the moment the fingertip makes contact with the object, it will get constrained, it may get constrained. Now, what is the wrench? The combination of the force and moment applied to the object by the fingertip is called a wrench. So, a combination of force and moment applied to the object by the finger tip that is called a moment.

Now you may be able to apply some forces in some directions, you may not be able to apply some force in some directions it depends on the type of contact. So, there are forces and there are moments. So, there are three forces f_x, f_y, f_z and m_x, m_y and m_z. Now what is the wrench space? The space of wrenches applied to the object. In 3D there are 6 dimensional wrench space, 3 forces, 3 moments.

In 2D there are 2 forces and 1 moment. So, we are seeing that in case of twists there are 6 **six** in 3D. In case of wrenches also there are 6 in 3D. So, one force and moment along every axis. Similarly, here one linear velocity and one angular velocity about every axis. So, the wrench space and the twist space are 6 and 6 in 3D and it is 3 and 3 in 2D. Now, how do we grasp an object that is the first question. This Salisbury hand is grasping the nut.

So, there is some contact being made between the fingertip and the nut, one here, one here, one here. Now depending on the type of contact that is being made, there will be forces and moments that can be applied. What do I mean? For example, depending on the kind of contacts, if no contact is there then you cannot apply force, you can only have twists, right. So, again the kind of forces that can be applied will depend on for example μ the coefficient of friction.

If μ = 0 then you cannot apply forces which are orthogonal to the normal force there. So, that contact conditions determines what are the twists and the wrenches acting at a contact point in a finger.

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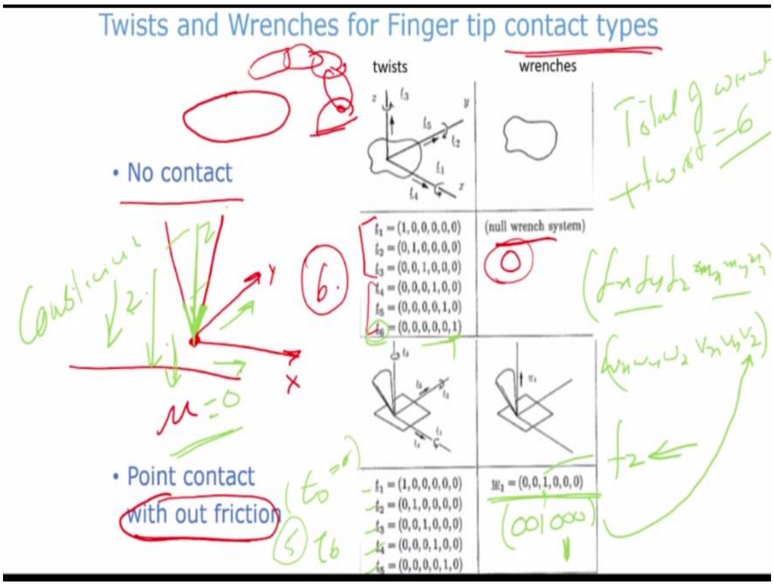

So, twists and wrenches for fingertip contact types. So, if there is no contact, so we have an object here and the finger is here, this is my finger, here also finger and there is no contact. So, in that case how many twists and how many wrenches are possible. Wrenches, if there is no contact you cannot apply force, so there is no wrench possible so it is 0, so null wrench means 0 wrench, you cannot apply, any force any moment because there is no contract between the fingers and the object.

Now, what about twist at the fingertip? You can apply 3 linear velocities and 3 angular velocities, so that is 6 here now. So, when there is no contract, you can apply 6 twists but 0 wrenches, now so no contact when the object and the finger are not being contacted. The moment you make point contact with friction. Now, let us look at this example here. This is point contact with friction. So, this is an object and let us say this is point contact here with friction, so, μ is there.

Let us write this one axis x axis, y axis and this is my z axis, x y z. So, now the moment there is; this is without friction right, so there is no friction. Now, if there is no friction means you can apply a normal force along the z direction. So, you can apply a force in this direction, but you cannot apply a force in that direction and that direction why because $\mu = 0$, there is no friction.

So, this is my normal force in the z direction, please notice it is in the z direction, but there is no force in the other directions. What about moment? Now, you cannot apply moment also because the point contact and there is no friction, so $\mu =0$, so it would not support a moment. So, in that case what are the twists possible and wrenches possible? Let us look at the wrenches first. So, in that case, we can apply a wrench, this is my f_z.

So, this is my f_x, f_y, f_z and m_x, m_y, m_z because these are three, these are three. So, in this particular case only f_z is possible, the others are all 0 that is the meaning of writing it like this 0 0 1 0 0 0. What twists are possible? Now, the twists that are possible are shown here. The first is that one twist will not be possible, which twist will not be possible? Because the fingertip has made contact here like this, contact has been made it cannot go in this direction it is constrained now.

So, it is constrained to move in this direction in the z direction. That means you see here that t_6 which was there here, the last one that is not there that is a linear velocity. So, it is ω_x, ω_y, ω_z, then in terms of the linear velocities it is v_x, v_y, v_z. So, in this particular case that v_z fellow is the one that you cannot have, you cannot have a velocity in the z direction because it is constrained, the object is there, the finger cannot go through the object.

So there are only t_1, t_2, t_3, t_4, t_5 and $t_6 = 0$ we do not write it. Now there is a relation between the t_6 and this ω_1. You can see the velocity which is not possible is in the z direction and the force that you are applying is also in the z direction, please note that. So this is my z direction, so I am applying a force in the z direction and the velocity is getting constrained in this direction only.

That means if the system is constrained there, if the finger is constrained to the object there only then you can apply force, not otherwise, and there is correlation that the total of the wrench and the twist must be equal to 6. So total of wrench plus twist must be equal to 6. So, if you have 1 wrench here, there will be 5 twists that means one twist is gone. So, this is point contact without friction.

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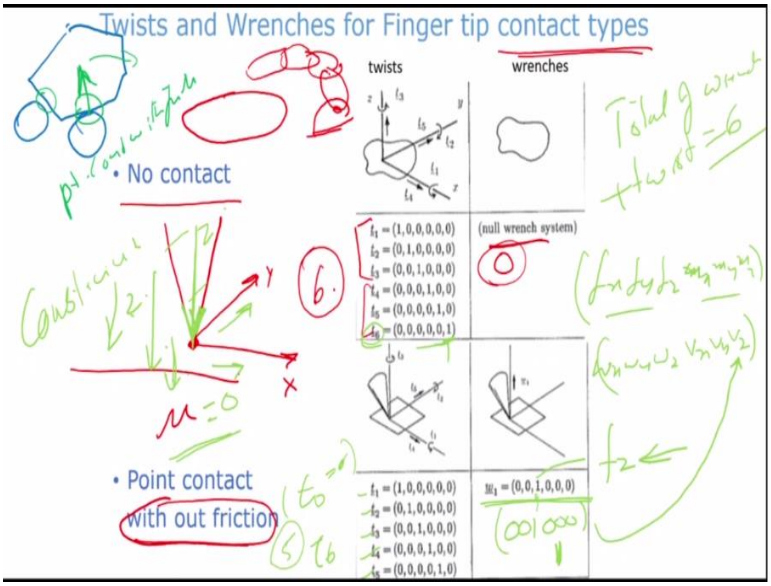

The next is point contact with friction. Now, as you can see that is my x, y and z. Now, there is friction, so μ is there, let us say that is a constant value. Now because μ is there, we can apply three forces, one forces normal force and because of μ there is going to be a $\mu \times N$ there, this is in this direction and this direction. So $\frac{F}{N} = \mu$. So, there is going to be a force in this direction and this direction which is proportional to the normal force into the friction coefficient.

So, I have three forces which are possible. So, you can see that w_1, w_2, w_3 this is f_x, f_y, and f_z are it possible. What about twist? In case of twist because you can apply forces you cannot have a linear velocity, it only allows you to move in that direction. But the angular velocities are there. So ω_x, ω_y, ω_z are there. The v_x, v_y, v_z all become equal to 0. So we have 3 here and 3 here, totally 6 again.

Now, please note that this is point contact with friction and most robot fingertips are this type, point contact with friction. Now, in order to have a little bit better grasp, sometimes the robot fingertips are coated with a rubber pad or they are slightly soft. And also the human fingertips you see we have the bone inside the fingertip. So, we have the bones inside the fingers and there is tissue on top because this is my nail and there is tissue on top.

So, this is soft, tissues are soft and they also have friction. So, they are soft plus μ. So, this is called soft finger contact and in soft finger contact, please have a look at this figure here and try to understand this very clearly. So, this is my x axis, y axis and z axis. Now, in the soft finger contact let us look at the forces because there is μ, μ is there so three forces we can apply like in the previous case. Here also we are applying three forces.

So, w_1, w_2, w_3 is there because we can apply forces there and there is μ and we have one additional moment which we can apply that is w_4 which is in the z direction w_z. So, I have three forces f_x, f_y, f_z and a moment which is my well let me call it m, I have been using m, so m_z. So, I can apply three forces and one moment which moment? Moment about this y. If I draw the top view of this again, what you can imagine is if I draw the cross section of the finger, finger is like this.

So, it can resist a moment about that, there is a surface contact now. So, the finger is making contact with the surface, this is my surface, so the finger has deformed, it is my deformation. So, the finger can resist a moment in that direction because there is friction and there is surface contact now, so it can resist a moment. It can resist moment, but it cannot apply moment, so it is a passive contact there.

Now, because of this there are four here now, if there are four here, then there are only two left here, so which two are left? There are; only two of this left. So, if there are two left, which two are left? So ω_x and ω_y are left, so moment about this and moment about that there is only one left and motion is constrained, linear velocities are constrained in the x direction, y direction, z direction. They are constrained, it cannot move.

And there is one moment in this direction, so it will not allow you to rotate about z. So, it will only allow you to rotate about x and about y and that is this one, so w_x and w_y. So, it is allowing rotation about x and y axis only, but not about z axis. And again this is 2, so it is 4+2 = 6. So, before we are analysing any multifinger manipulation, now let me make a point here. If you are wondering what is the relation with motion planning or path planning.

In this particular case of application, I want to impart a particular motion to this nut, for example I want to tighten this nut. So, I have to rotate it about an axis. To rotate this nut about an axis I have to impart forces and moments and rotation means I have to impart some desired velocity. So, that is what this analysis is going to tell us that what is the relationship between the fingertip forces moments and the resultant velocity of object that is one.

Think of it this way. I have two manipulators, they are holding an object. So, this is one manipulator it is holding an object here and there is another manipulator here. This manipulator is fixed, something like the hand. Now, I want to impart some motion to this object that means I want to give it some velocity and I want to move it in some direction. In that case, I have to impart forces here, right.

So, I need to figure out what are the forces to be applied in the fingertip such that the object has a resultant motion, right. Look at it, so this is for the case of a serial arm. Look at the case of mobile robots. We have multiple mobile robots which are catching, this is what is called swarm. So, we have multiple mobile robots which are catching this object, let me have this object. These are small robots which are catching this object and pushing it and taking it somewhere.

So, these are small robots which are, this an object, it is capturing this object and pushing it somewhere. This is we can call swarm robotics, we can call multiagent system, all of them use the same logic. So, this is in a way a multiagent system also. So, depending on how the contact condition is there between the robot and the object, you can apply forces that means wrenches or twists, right. So, whenever two surfaces are making contact, in our case the robot is making contact with the object.

For example, I have this object and there is a circular robot which is trying to push it and take it somewhere. So, what is the contact condition would be depending on any of this. It could be point contact, it could be point of contact with friction. And if it is point contact with friction, then we can apply three forces there. And if it is point contact without friction you can apply one force only that is the normal force.

So, this is what determines how many forces and moments can be applied and that depends on the contact condition. So, the final resultant motion of the object would depend on the forces, moments being applied by the multiple robots which are pushing it or capturing or doing whatever. So, it can be robotic fingers, it can be serial arms, it can be mobile robots, multiple swarm that can be UAVs catching an object and pushing it somewhere. In all cases, it is the contact conditions that determines.

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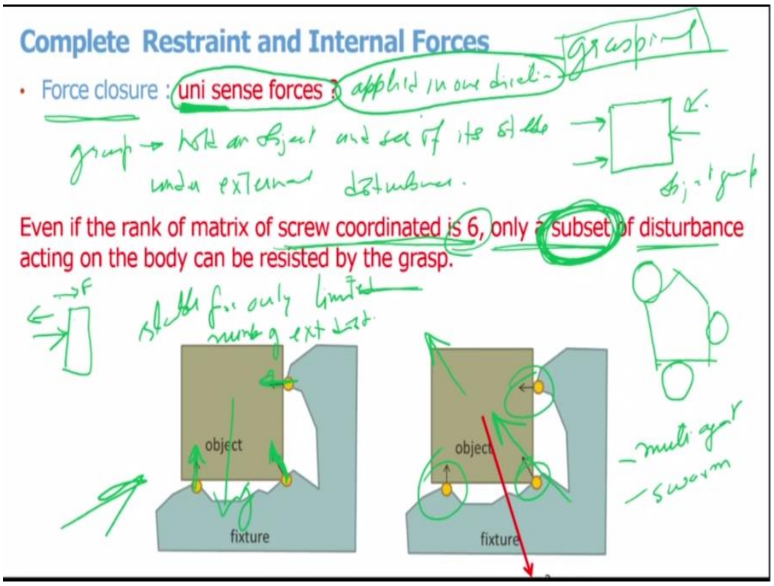

Now first let us look at the grasping. Now this is an object which is being grasped for an object here which is shown in the figure, this is an object and it has 3 contacts which are there; one contact here, one contact here and one contact here. Now we can assume this to be something like a box which is caught with two fingers, three fingers like this, the same thing or let us look at an object with multiple mobile robots.

One robot is here, one robot is there, one robot I there. So, this is a multiagent swarm, they all mean the same thing. Then this is just simple object grasping. The theory and the math is the same. Now in this particular case of grasping, grasping basically means you want to hold an object and ensure that object is stable under external disturbances. So what is grasp is to hold an object and see if it is stable under external disturbances.

Now here we use two terms, one is called force closure, another is called form closure. Now when you are touching an object by using multiple hands or multiple robots or multiple swarms, we need to analyse and understand something that the forces are unit sense forces, forces can be applied only in one direction. This is something please keep in mind. So, when you are looking at this contact condition here, the force can be applied only in this direction.

There cannot be a force in this direction, it is not possible. It is like pulling the object, right? I hope you understand that. So, if you have an object here and you are pushing it from here, you cannot pull it like that why because the contact will break immediately. So, these forces are in one direction only and this is what we call uni-sense directional forces, they are not bidirectional. So, these are unidirectional forces.

Now, in force closure like here, even if the rank of the matrix of screw coordinated is 6, only a subset of disturbances acting on the body can be resisted by the grasp. That means if you get an object like this and you are under forced closure that means I can apply forces like this and the rank of the matrix of screw coordinated is 6, why 6 because at most there can be 6, degrees of freedom are only 6, 6 forces, 6 moments.

So, if the rank of the crew coordinate matrix is 6, even then only a subset of disturbance can be acting on the body can be resisted by the grasp. This means that in this left side figure here, this grasp in a way is not stable, it is stable only with a limited number of forces or disturbances, stable for only a limited number of external disturbance forces. Now, what this means is it has caught, suppose you can argue that this is gravity.

So, this is having a force here, this is having a force in this direction, this is having a force in this direction and is stable. But the moment I use a force in this direction, if I push it in this direction, it will immediately come off. Why? Because it cannot resist forces in this direction. There is no contact which can resist forces in that direction. So, this contact cannot resist force in this direction, this fellow cannot and this fellow cannot because they are uni sense direction of forces.

So, this is something to note that when you are catching an object like here, in this particular case I was showing are you under force closure or are you under form closure. In force closure you can apply, you are subjecting the system only to unidirectional forces and that cannot resist all forces that can come on the object. It can resist only a subset of forces. It can resist only a subset of disturbances that can come on the body, so this is the catchword subset of disturbance forces. In such cases, it is under force closure.

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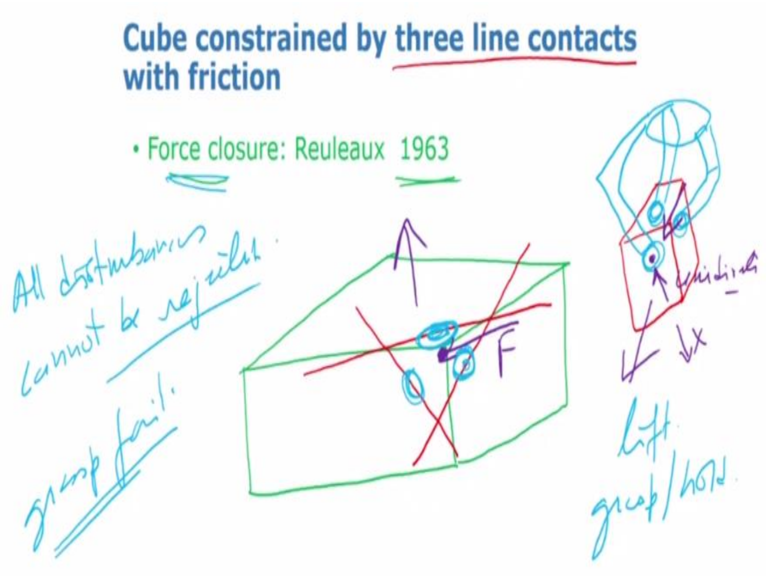

And for example, this is the example he also gave. So, this was invented first way Reuleaux in 1963 very long time ago. If you take a cube, this is a cube and we are holding it by three line contacts. So, a cube constrained by three line contacts with friction. So one line is like this, one line is like this and one line is like this. I can also give the idea that if this is a cube, let us say it is a cube, I am sure you must have felt this or seen this or experienced this somewhere.

If you place one finger here, one finger here and one finger here this is equivalent to saying you are holding it here, here and here. You can still hold the cube, right. If this is a finger like this, another finger here, another finger here, these are three fingers and you are holding it like that. You can still lift the cube, right? You can lift that means you can grasp the cube, grasp hold the cube just by holding it like that using your three fingers you can do that.

But you are under foreclosure, why? Because all disturbances cannot be rejected which means that there will be disturbances which is going to destabilize the system, destabilize here means basically the grasp will fail. So, what I mean basically here is grasp fails that is what I mean by stability here. So, in this particular case this is under force closure. So, I will be able to hold it and probably lift it by holding on these three sides either by line contact or by soft finger contact.

But the moment there is a force in this direction, there is a force in this direction now pushing it in that direction. So, the moment there is a force which is pushing it as you can imagine from this point it is pushing. So, let me draw it like this, this is a force, it will immediately go that side. Similarly, here if I now apply a force from here like this, what will happen? The cube will come off this side because these fellows are unidirectional.

They can only resist forces in this direction, but they cannot resist forces in that direction because of the unidirectional forces because of which this term was called force closure, which means that although you are grasping an object, but it is not a stable grasp, it can get destabilize very easily. If the force comes in another direction it will destabilize.

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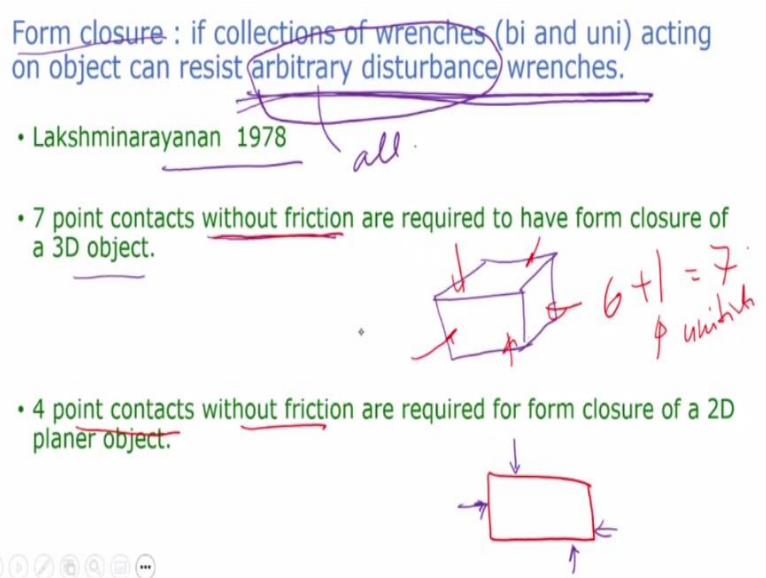

Now, so that is one force closure, the other is what we call form closure. Now, if collection of winches bidirectional, unidirectional acting on object can resist arbitrary disturbances. Now, if you can resist arbitrary disturbances that mean you can resist arbitrary means all disturbances, then we say the system is under a form closure. And for form closure this was invented by Lakshminarayanan in India itself 1978.

And he proved that 7 point contacts without friction are required to form closure of 3D object. So, if I have a 3D object like this, this is a cube, so I will be requiring 7 point contracts one here, one here, one here, one here, one that side. So, there are 6 phases, right? So there are 7 plus you will need one, this one because some of the forces are all unidirectional, so it is 6 + 1 = 7.

And he is the first one who proved that 7 point contacts without friction are required to have form closure of 3D object. And 4 point contacts without friction are required for form closure of a 2D planar object. So, if I have a 2D object like this, I need four to have form closure.

That means I can put two here and I can put two here. Now this is completely constrained, it will not get destabilized anyway due to any disturbance also. So, it can reject any arbitrary disturbance and this is what we call form closure.

So today we looked at multifinger manipulation. And in the next class, we will be moving on to looking at how do, we apply the forces in order to move an object in a particular direction. Then we will look at multiagent system where there are multiple robots which are trying to push an object in some direction and imparting some motion. Now this is applicable also to multiagent systems to unmanned aerial vehicles which are trying to push an object somewhere. So, we will stop here today. Thank you.