Robot Motion Planning

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Lecture – 19
Multifinger Robot Hands

Hello and welcome to lecture number 19 of the course robot motion planning. In the last class we looked at systems which are subjected to kinematic constraints that is like a car which cannot move sideways because it is subjected to the non-holonomic velocity constraint. And then we moved on to multifinger robot hands that is when a robot, multifinger robot hand grasps an object and what are the constraints that are imposed by the fingertip contact on to the object surface, right.

So, we have seen that depending on the kind of the contact condition whether it is with friction, without friction, whether it is soft finger contact that will impose kinematic constraints on the object itself. Today, we will move on from there and see that if you want to manipulate an object by multifinger hand, say for example I have an object like this mobile phone and I catch it with three fingers, so two fingers the side, one finger this side, and I want to do like this.

So, I am going to impart a velocity or trajectory to this object and to get this object to follow this path what is the corresponding forces that I must apply by my fingertips? So that is what we will be looking at in terms of the grasp matrix. So let us start off from there and let us very briefly revise what we have done in the last class regarding fingertip contact conditions and then we will move on to the grasp matrix.

So, we were looking at motion planning with multifinger robot hands and we will very quickly revise and then we will move on to deriving the grasp matrix and also looking at conditions of what are internal forces and what mathematically is the meaning of grasping an object.

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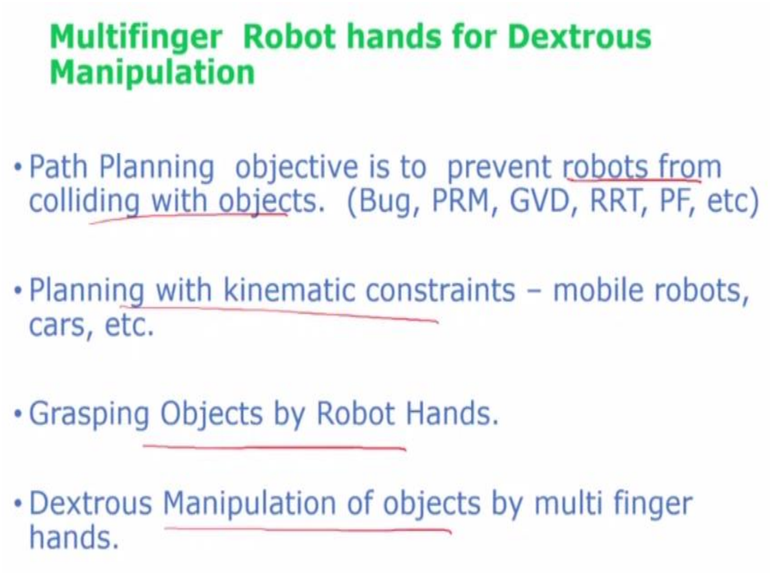

Now, we had seen that multifinger robot hands are basically used for dextrous manipulation that means when you are catching an object and rotating the object or manipulating the object, you are imparting a particular trajectory, you are expecting that the grasped object is going to follow a particular path.

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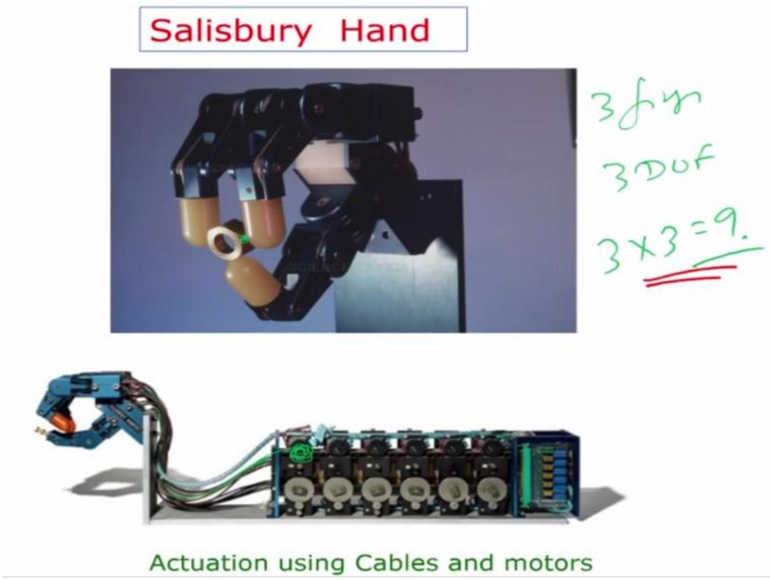

So, we saw that there are different kinds of robot hands. So, they can be three finger robot hand which is called the Salisbury hand. So it has 3 degrees of freedom on each finger and in total it has 9 degrees of freedom and it is actuated by this cable kind of wires.

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Now, as we see further the other kind of hand is the Barrett hand. Again, it has 3 fingers and 9 degrees of freedom.

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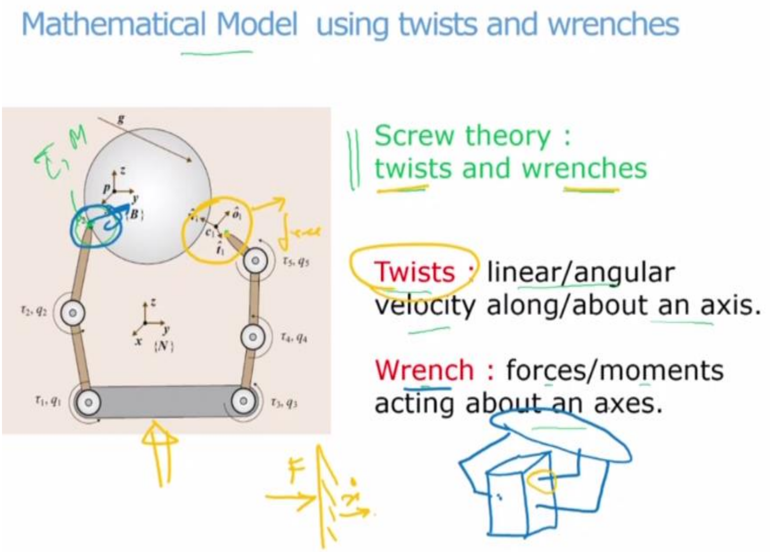

Then we move on and we see that if you want to manipulate an object, for example I have an object like this, I am holding this cube by a three finger hand there are two fingers on this side and there is one finger on the other side and I want to manipulate this object. I want to translate it or to rotate it about an axis. In that case, the first thing that we need to see is what is the contact condition, for example the fingertip is making contact with the object at this point here.

So depending on the contact condition, certain constraints will be imposed on the object now. So to analyse that we basically look at what is called screw theory and we talked about twists and wrenches. So if you look at this two-finger robot hand, on the right hand side this finger is not making contact with the object. Now the moment the finger makes contact with the object, there is a constraint on the motion of the object.

So if the object is not moving, once the contact has been made, for example on the left hand side, you can apply a force, but you will not be able to control the velocity, right? So it is like saying that once you have an object, once you have a wall, you are pressing against the wall, you can control the force that you are applying on the wall, but you cannot control the velocity in that direction anymore, why because you are constrained there now.

Now these constraints would depend on the type of contact that has been made. So what we see here is that when no contact has been made, here the fingertip is free to move like in this case, it is free to move, then we can talk about the linear and the angular velocities about an axis and these are called twists. So this finger which is free it can apply a linear velocity or it can have a moment about a particular axis at the fingertip.

Now when contact has been made like in the case in the left hand side here where contact has been made, in this case we can apply a force and we can apply a moment which are called wrenches. So, wrenches are the forces and moments acting about an axis when contact has been made.

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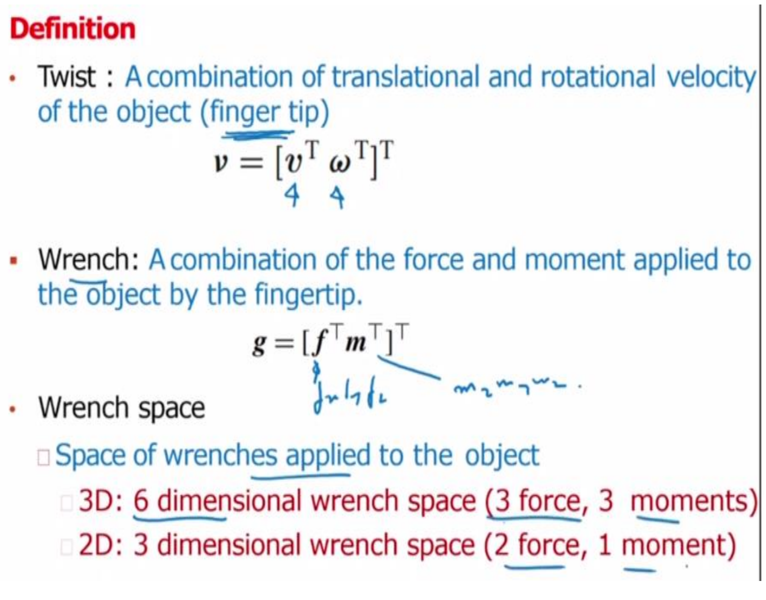

So, by definition, a twist is a combination of translation and rotational velocity of the object at the fingertip, which is given by a linear velocity. There are three components and three angular velocities. Now wrench is a combination of the force and moment applied to the object by the fingertip. Now there are three forces f_x, f_y, f_z and three moments m_x, m_y, and m_z.

Now, in order to get grasp an object, what does this mathematically mean that is something which we will look at today. Now, we have seen that the space of the wrenches applied to the object has 6 dimensions that is in 3D. So, there are 3 forces and 3 moments and in 2D it has 2 forces and 1 moment.

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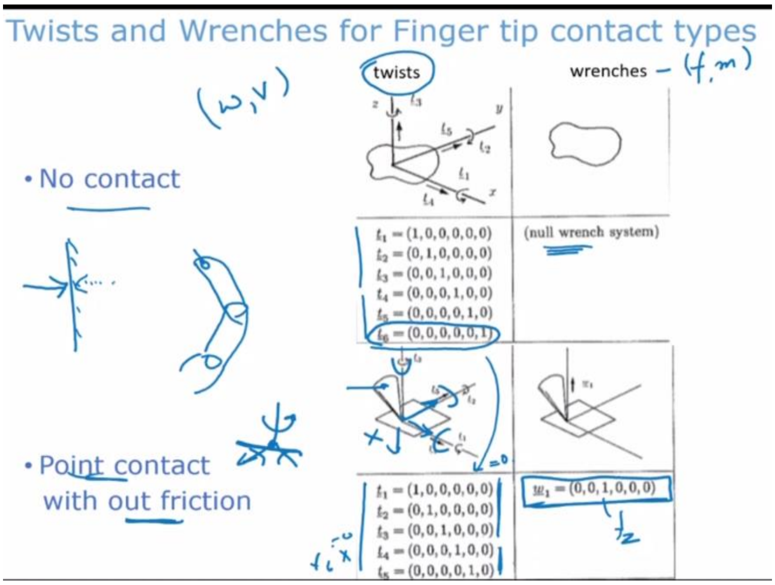

Now, depending on the kind of contact that is made different constraints will be put. Now, twists remember are to do with angular velocities and rotational velocities. So, angular velocities and linear velocities, whereas wrenches have to do with forces and moments. And there are 3 forces 3 moments whereas here there are 3 angular velocities 3 linear velocities. Now, once if there is no contact then we have seen that you can apply it can move in any direction.

For example, I have a finger, fingertip here which is not in contact with any object, in that case this fingertip can move in any direction, So, there can be 3 linear velocities and 3 angular velocities, so there are 6 twists which are possible because there is no contact. Now, because there is no contact you cannot apply a force. So, please note that you can apply only a force if there is an equal and opposite reaction as per Newton's third law which means that you can apply a force only if there is contact.

So, in this case there is no contact and hence no force can be applied and hence the wrench is 0. Now, if we have point contact with friction, for example this is a point contact, you can imagine this to be a robot finger. Now, at the fingertip if there is friction, then what is going to happen is that it will constrain motion in one direction. Now, in this particular case there is point contact without friction.

Now, there is no friction then the fingertip is free to move in this direction, this direction, but it is only constrained to move in that direction that direction you cannot move anymore and it can twist about this axis, that axis and that axis. So, please note it is point contact, it is like you have an object like this and there is a point contact being made this can rotate like this and can rotate like this, it can rotate like that and it can apply force in only one direction.

So, we can have only one wrench, which is about the z axis. So, this is my z axis f_z, you can apply force only in one direction, sorry this is f_z and it can cannot apply forces or moments in any other direction. In terms of twist, it has 5 twists. So, it cannot move in this direction so that t_6 which you have seen that is not there that is equal to 0. The t_6 that was here that one has become equal to 0 in this case and the remaining 5 are there.

So, it has 3 angular velocities and 2 linear velocities. So, this is point contact with friction, now this is a kinematic constraint that means when this fingertip is making contact with the object, it is subjected to that kinematic constraint which has to be satisfied always.

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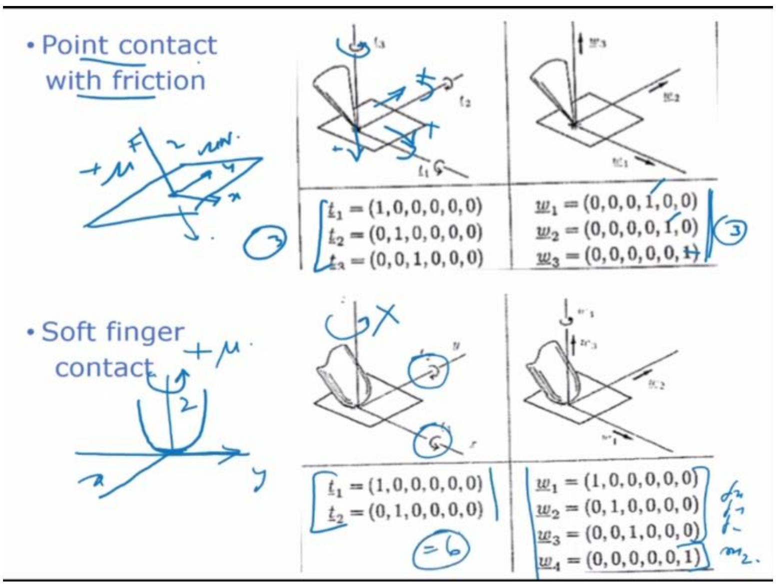

The most commonly used in case of robotic grasping is point contact with friction. So, in this case, there is point contact and the restriction at the fingertip. So, what will happen is it cannot rotate because the restriction, so what would happen is in terms of linear velocities it cannot move in this direction, the restriction so it cannot move in that direction, it cannot also move in that direction. So, you can imagine here if you have a surface like this and I have point contact like this with friction that is μ.

What will happen is it cannot move in this direction it is constrained, it cannot move in this direction or in that direction if this is my x, y and z why because there is a μ F. So, the force of friction, if this is my F, $F = \mu N$, so there is a μ n reaction, μ n reaction on the other side. So, this would basically mean that it is constrained to move because of friction, it cannot move in this direction, this direction or that direction, but it can rotate about this axis, this axis, this x axis.

So, only the rotational degrees of freedom are still left and there are 3 and the forces are all constrained. So, this is my force f_x, f_y and f_z. So, there are 3 twists and 3 moments here. If you have a soft finger contact, which is similar to the human finger context, so if you can imagine that if this is a hard surface and you are pressing with your finger it will deform at the fingertip a little bit. Now, this is my axes x, y and z.

In that case what happens is soft finger contact also has friction. So, what will happen is one more additional degree of freedom will be gone that is you will not be able to rotate it about that axis. In case of point contact, rotation is still possible whereas in the soft finger contact the rotation about the z axis is not possible anymore, but this rotation, this rotation is possible. So you only have 2 twists which are left.

And in terms of forces we have 3 forces and 1 moment. So, this is my moment about the z axis and these are 3 forces so f_x, f_y and f_z and this is my m_z. So, you will always find that the total of this plus this is equal to 6, it just depends on the kinematic constraint that is there.

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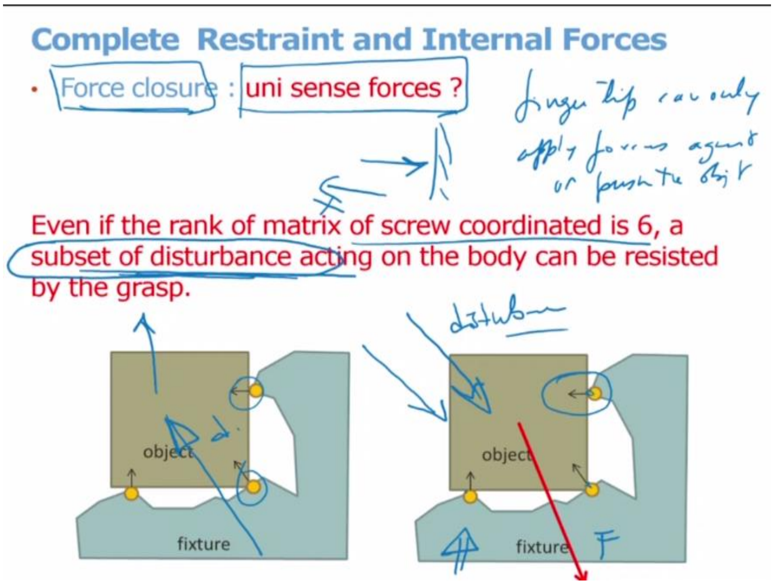

Now, we were talking about complete restraint and internal forces. Now, what is force closure? Force closure is, suppose you are holding an object using a three point contact with friction. And if the rank of the matrix of the screw coordinated is 6, then it will appear that the object can resist all kinds of disturbances that are going to come in the body, but that is not so. So, in this particular case it can only resist a subset of disturbances acting on the body.

What it means is that, you can see this very clearly from this figure on the left and on the right. So, on this figure on the right hand side if I put a force in this direction, let us call this a disturbance force, the object will still be grasped that means look now what we mean by grasp here. So, what I mean here is that there will be no breakage of contact there. The moment the contact is lost, the object is not grasped anymore.

So, in this particular case because the force is pushing in this direction, what will happen is the object will still be grasped, so it is stable in that sense. If I put a force in this direction now, F here this is what my disturbance force d and what will happen? Immediately this will come out, this contact will break. That means this type of grasp with three fingers with point contact can only resist a subset of disturbances acting on the body.

Which kind of disturbance? The kind of disturbance which is tending to force the body into the fingertips, but the one which are going in the other direction will immediately destabilize it. So, this is basically what we call by force closure. So force closure, when we say a grasp is under force closure basically we mean that the object is being held by the fingers and it can resist only a small subset of disturbances acting on the object, not all disturbances.

Why does this happen? Because when we talk about forces, please remember in the previous case when we talked about forces here, we talked about wrenches. The force has direction. So we have force acting in this direction, so fingertip is acting in this direction. So this my fingertip and that is my fingertip. It is applying a force F. It can apply a force in this direction, but it cannot apply a force in that direction. It is like pulling the object, you cannot pull the object, right. So that is what we call the uni sense of direction.

That means when we are saying the forces that a fingertip is applying a force, it is only uni sense force, so fingertip can only apply forces against the object or push the object, so if you have object here can apply in this direction, it cannot apply in this direction. So that is the meaning of uni sense force and because of that we have basically what is called force closure. So, it basically means that because the fingertip cannot pull the object, if there is a force in that direction it will not be able to resist it.

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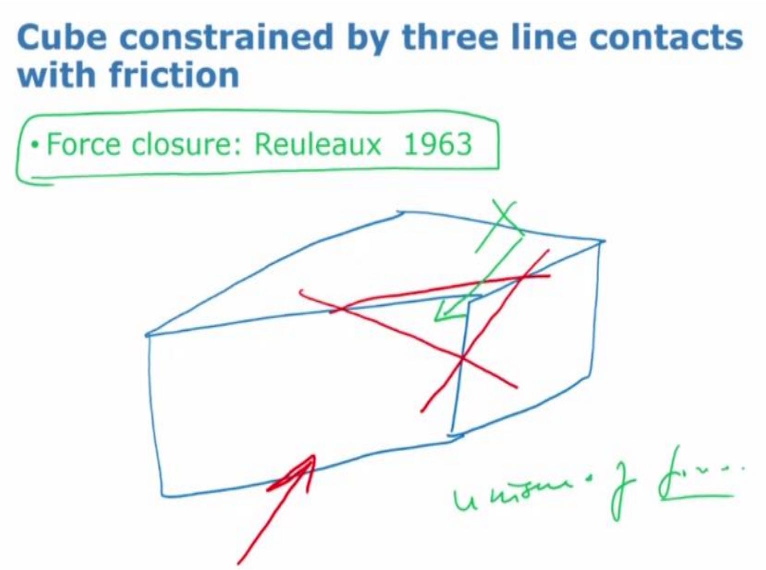

Now, the example is given here by means of holding a cube. So, suppose if I have a cube like this, you might have experienced this yourself, that if there is a cube like this, I have three line contacts when I am holding it like this. So, I am holding this cube this way and if I press it in this direction it is still stable, but the moment I press it from this side or if I give a force on this side, it will immediately displace. This is because of this uni sense of force directions which means that the force can push but it cannot pull. And this term was first used by Reuleaux in 1963 in France when he talked about forced closure of grasps.

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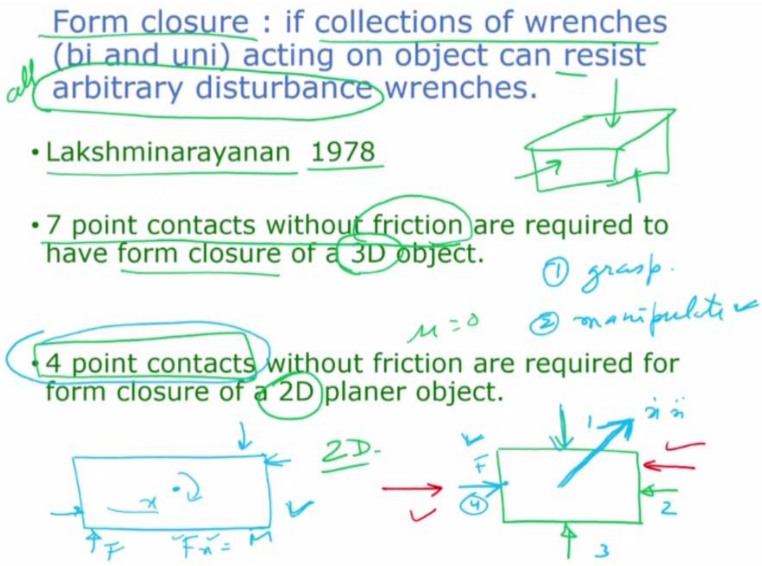

Now, what would be more interesting is that we are interested in holding the object in such a way that it is stable against all disturbances that any disturbance can come from any direction but it will not destabilize and such kind of closures are called form closure. Again, it was basically formed by Lakshminarayanan in 1978. And he basically said that if you have a cube for example, he basically proved that if you have a cube like this you want to hold it in fact any shape, then you need 7 point contacts without friction.

Then if you have contacts like this, then you need 7 point contacts without friction to hold an object in form closure. Now what is the definition of form closure? So, the definition of form closure is the collection of wrenches or forces, moments being applied by the fingertips acting on the object can resist arbitrary disturbances or all disturbances can be rejected in such a case we say it is under form closure.

And he proved that 7 point contact without friction are required to form, form closure of a 3D object and 4 point contacts are required in case of 2D object. So, 7 point contacts for 3D object and 4 point contacts for 2D object. Now, let us look at this example which is interesting and easy to understand. So, we have a cube this is 2D, just to understand, now we need 4 point contacts, right? Now why 4 it is very easy.

So, suppose I have 3 fingers like this, there is no friction, $\mu = 0$. Now, this can be in forced closure, it can resist forces in which direction? It can resist forces in this direction, but the moment I give a force in this direction it will immediately disorganize, it will come off, why?

Because there is no friction force, so it cannot resist that force. So, for doing that I need to add one more finger. So, 3 finger is not possible, I need to add one more finger.

So, this is my 1, 2, 3, 4. Now it is stable, no wrenches, now you can push in any direction you want, the grasp will not come off. This is very easy way of understanding why you need 4 contacts. Now, when we are talking about 4 contacts, the other contacts that comes up is where are you going to place your contacts? So, I just put the fingertips in the centre, suppose I have a cube again like this, what is the best place and the most stable place to have contact or to place the fingers?

There is an interesting question which you will see subsequently. So, I can place it in the centre like this or I can place it in the corners. And if I place it in the corner, what is the advantage? The advantage is that it can resist the highest moment. What is a moment acting on the centre? The moment is this force into this distance x, F x is the moment. So, if my x is large, then you require a very low force in order to catch an object against a moment.

So, this grasp might be better than this grasp. So, we see that the first thing is the number of fingers, the second thing that we have to worry about is where to place the fingers. Now, apart from that we need to see that if I want to move this object now in a particular way. Suppose I want to move the object this side in that way, then the fingertips have to impart velocities to the object.

And basically, it has to grasp the object that means apply force and also move it in a particular direction with some velocity and acceleration. This I can say is the path of the object that I want to take it to. So, one of the objectives of multifinger hand is to grasp that is to hold an object, number two is also to manipulate an object. Now, what is the meaning of manipulating the object?

It basically means moving the object in a desired trajectory or along the desired path. So, I have to figure out what are the fingertip forces that are to be applied if I want to move the object in a particular path and that is something which we will be seeing subsequently.

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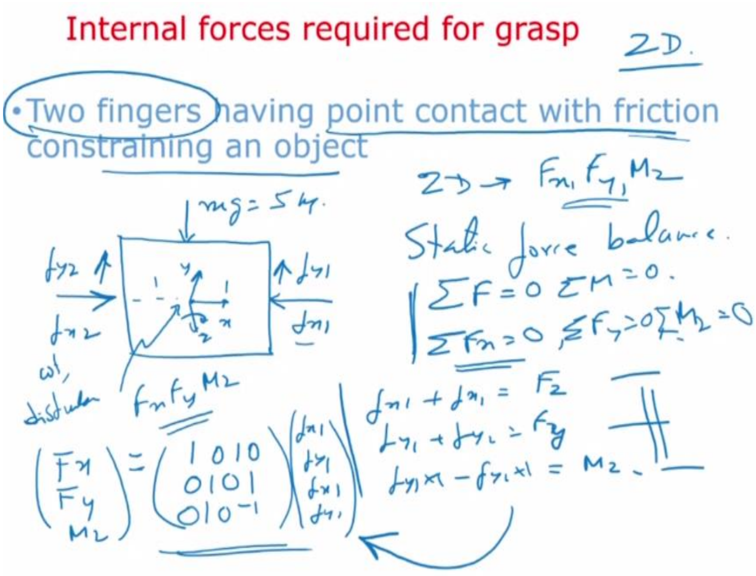

Now, in order to understand the concept of internal forces let us look at a two finger grasp having point contact with friction holding and constraining an object, this is 2D, just to understand the concept of internal force. So, we have an object here this is in 2D. I have one force acting two fingers with friction so there is the force there, force here, force here. So these are two fingers having point contact with friction constraining an object.

Now what are the forces which can act on this object disturbance forces? This is my x axis, y axis, there can be a moment about the z axis. So in 2D, we can have 2 forces. So F_x, F_y and M_z acting. It is in 2D, so there can be forces in the x direction, y direction and there can be a moment about the z axis. This is in 2D. So at most the disturbance forces acting under CG of the object can be F_x, F_y and M_z.

That means to balance this F_x, F_y and M_z I am going to apply fingertip forces which is f_{x_1}, f_{x_2}, f_{y_1}, and f_{y_2}. Now these are forces along the x axis and the y axis. Now from static force balance equations we know that $\sum F = 0$ and $\sum M = 0$ for this to be a static equilibrium. So for this system to be in static equilibrium which is subjected to 2 forces F_x, F_y and M_z in the centre we are applying two fingertips which are applying these forces.

Now I need to find what is F_x and F_y in order to balance a particular F_x, X, Y and Z acting in the centre. So, first thing what I do is I say $\sum F_x = 0$, then I say $\sum F_y = 0$ and then $\sum M_z = 0$. So, this means forces in the x direction are all equal to 0. So, what are the forces in the x direction $f_{x_1} + f_{x_2} = F_Z$ that you can see from here. Similarly, $f_{y_1} + f_{y_2} = F_Y$ and if I say this distance is equal to 1 unit then the $f_{y_1} \times 1 + f_{y_2} \times 1 = M_Z$, the three questions here.

One in the x direction, y direction, z direction. If I write matrix form then I can write $\begin{pmatrix} F_x \\ F_y \\ M_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \end{pmatrix}$. So, this set of equations I have written there. There are three equations. So, on the left hand side I have the forces which are acting disturbance forces, the disturbance force is also considered the weight of the object.

So, in 2D at most you can have F_x, F_y and M_z as disturbance. So there are two forces one moment. Now, this is equal to this matrix into the kinetic forces f_{x_1}, f_{y_1} and M $_z$. So, I hope you understand this equation, right. So, I will go on from here and explain the concept of null space.

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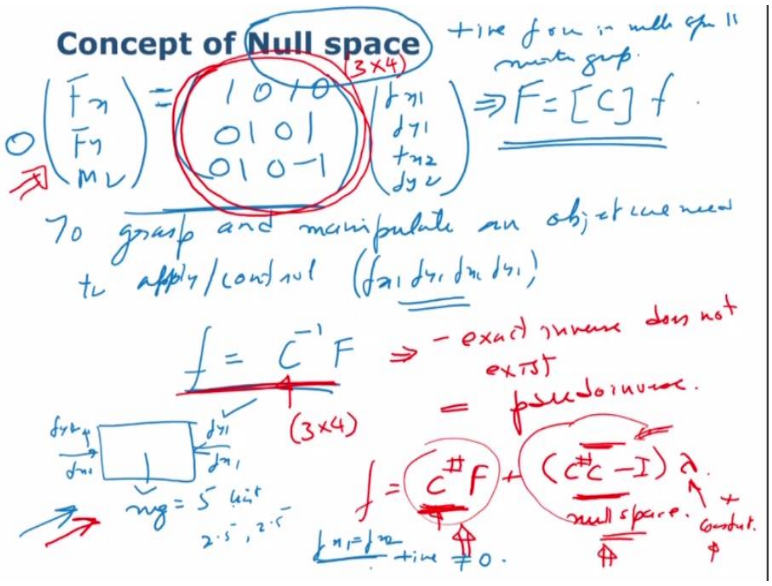

So, $\begin{pmatrix} F_x \\ F_y \\ M_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \end{pmatrix}$. If I write it in short form, then $F = [C] f_x$, let me write it this way. So, what is required? Now to grasp and manipulate an object we need to apply and control the small x. That means I need to control $\left(f_{x_1}, f_{y_1}, f_{x_2}, f_{y_2}\right)$ in order to balance these external forces or to move the object in some direction.

So basically, I need to know what is f for a given F. So that means in this equation, what I need is small $f = C^{-1}F$ that is what I need. Suppose I just ask this question here, we have an object. This object has weight has mg is equal 5 kg, this object with 5 kg, how much kinetic force are you required to grasp this object? How to solve the problem? So essentially, we have to write our equations of statics or static equilibrium, then try and solve what will be $f_{x_1}, f_{y_1}, f_{x_2}, f_{y_2}$ that is how we are going to solve it.

Now, if we just look at this problem, please think about it. Do we have an object m g is equal to 5 units, whatever units. I am asking you what is $f_{x_1}, f_{y_1}, f_{x_2}, f_{y_2}$. If you to just look at this problem you tend to say that it is 2.5 and 2.5. Is that the correct answer, just think about it. Now, to solve it basically I have to write my equation of static equilibrium like this and what you get is these three equations this, this and this and you actually get this set of equations.

In the case when the force is, there is only one mg acting in the y direction, F_x will become equal to 0 and M_z will become 0, but only F_y will be there. So in this particular case, we see that we need to invert this equation. Now what am I driving at? What I am driving at is that this matrix is a non-square matrix. Please note that it is 3 X 4 which means that you cannot invert it directly. If I want to find this one, this is a non-square matrix 3 X 4.

What does this mean mathematically? This basically means that this problem that I just spoke about that you have an object of some weight and I want to grasp it and if I try to solve this equation, solve it by writing equations of static equilibrium what we end up is this equation and if I have to take the inverse and find F then C is not square. So, now what do I do? So, here in this case we cannot find the exact inverse, exact inverse does not exist. So, we find the pseudoinverse.

Now, the solution of this equation is $f = C^{\#}F + \left(C^{\#}C - I\right)\lambda$. So, the solution of this equation basically is; we do not have an exact inverse because this is a non-square matrix, so what we get is a pseudoinverse which is not an exact inverse and so we get one part of solution plus another part of the solution. So, the solution comes in two parts, we call the particular solution and the homogeneous part. Now, why does this happen?

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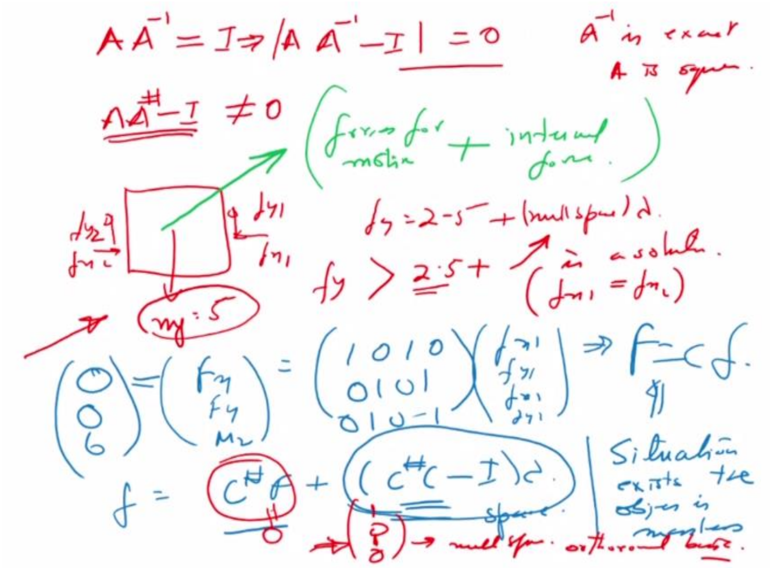

Now, this happens because if you think a little bit if you have matrix $AA^{-1} = I$. That means AA^{-1}– I = 0. If A inverse is an exact inverse. So, if $A^{-1}$is exact that means A is a square matrix. Now, if A is not an exact inverse, then you have $AA^{\#} - I \neq 0$, what does this mean here? Mathematically it basically means that you have four unknowns and you have three equations. So, how many solutions do you have? You can end up with infinite solutions.

So, this actually is one part of the solution and this is the other part which is basically in the null space. This is the concept of null space. So, you have one part of the solution here and the other one there, now this part of the solution is basically in the null space of F and there is a constant here and there is a constant there, the positive constant there. So, which means that your solution now of f consists of two parts; one part is this part, the other part is that one, again there is a constant there.

So, what it basically means is that you have multiple solutions here, how many? Infinite. So, you have one basic solution and then there are multiples of those solutions. Now, please think about it. So, I just talked about catching this object which is having mg = 5 and you said that the force, what forces are required f_{x_1} and f_{x_2} ? Just by thinking you tend to say that F $_y$ = 2.5 that is the basic solution.

And there are multiple solutions, which are multiples of the other fellow which is here that is my multiple solution which is coming from here. So, the other solution is basically coming from the null space which is getting multiplied by λ which is a positive constant. So, you can get multiple or infinite solutions. That means what is the answer to this question that I asked? You have an object of weight 5 kg, you want to lift it, how much force is required?

You tend to say half, half and half of the weight, but that is only one solution. You have infinite solutions, any solution which is 2.5 plus this is a solution. Now, why did this happen? This happened essentially because this matrix here is a non-square matrix, so please think about it. Now, whenever you have number of variables more than a number of equations, you end up with this kind of situation. And if your matrix is not square, then you end up with a pseudo matrix and with a pseudo matrix you end up with a null space.

The solution will consist of two parts or within a solution plus particular integral a particular solution and which basically means that you can have one basic solution and multiples of it. Now, this makes sense in the real world. Why? Because otherwise if you have to lift the object of 5 kg and you have exactly 2.5 which you apply a little bit more it is not right because exact. So, there is nothing like exact in the real world. So, if I apply any force which is more than this, it is fine.

So this is the meaning of the maths part here. But what is more interesting here or more confusing rather at times, if I write this equation again, let me write this equation again, $\begin{pmatrix} F_x \\ F_y \\ M_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \end{pmatrix}$ and in total we said F = C f. Now, suppose we have a situation in which the solution of this, the solution of that $f = C^{\#}F + \left(C^{\#}C - I\right)\lambda$.

Now suppose the situation exists where the object has no mass, so it basically means that this matrix is equal to 0 0 0. So, there is no force when that object is massless. Where? Say for example in outer space there is no gravity, so it is weightless. It is just floating anywhere. So, if you want to catch a floating object and you want to move it somewhere, do you have to apply force, I hope you understand what I am saying here.

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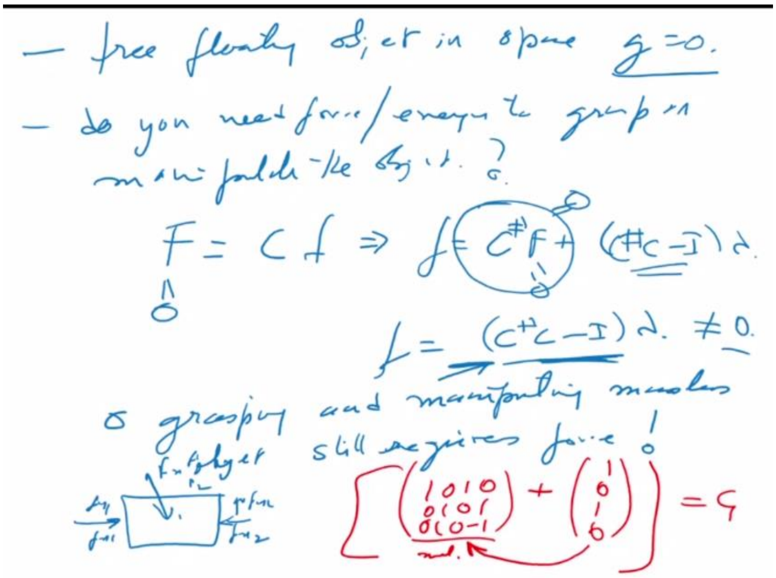

That we have an object which is free floating. Free floating object in space that means g = 0. So, do you need energy? Do you need force? Force means you have to do work, do you have to expend energy to grasp or manipulate the object? This is an interesting question. How do you answer the question? You simply look at the equation F = C f and which says that $f = C^{\#}F + \left(C^{\#}C - I\right)\lambda$.

So, if it is massless means F = 0, that means this fellow becomes 0, but that fellow is still there because $f = \left(C^{\#}C - I\right)\lambda \neq 0$, it cannot become 0 why because these are non-square matrix. That means even if you want to grasp an object which does not weigh anything, you still have to apply force which is against our perception of reality. So, grasping and manipulating massless object still requires force so let us put this exclamation mark there which is very strange, why?

Because this one never becomes equal to 0 and that is in the null space of F. Now, in outer space when objects are free floating even then you have to do it, even then you have to grasp an object you have to apply force, now that force is exactly coming from here. So, please understand this, the concept of null space is very important here that when you are grasping an object and you want to find the forces that are required to grasp the object and to manipulate the object you have to deal with the null space.

Why? Because the matrix involved there is a non-square matrix. So, it basically means that there is a minimum force and any multiple of that is. Now, this will also mean that the solution of this equation here would consist of two parts, this part and this part. Now let us look at what the null space would look like. So, let us look at this example before we move on here, the previous example that we looked at.

So we had $f_{x_1}, f_{x_2}, f_{y_1}, f_{y_2}$ and we had the forces acting there so F_x, F_y and M_z. Now in this particular case, what will be the forces in the null space? Can we guess that? Say for example here suppose this is equal to 0, F_x, F_y, $M_z = 0$, what will be this part? Can you guess? So, this is the definition of grasp. What is the definition of grasp? The definition of grasp means that there must be forces in the null space or there must always be positive forces in null space to maintain grasp.

Let me go back here. So, even if this part were to become equal to 0, then also this part will be there. Now in this particular example, what will be the forces in the null space can you guess? So, simply by looking at this figure here if $F_y = 0$ that is fine you can still hold. That means f_{x_1}, f_{x_2} must always be there and they will be equal and opposite, otherwise the grasps cannot be maintained. So, in order to hold this objects f_{x_1}, f_{x_2} must be positive, it has to be nonzero.

The moment it becomes 0, the grasp will fail. So, what is when we are saying grasping an object that is English language, mathematically what is grasping an object? Grasping an object essentially means that you must have forces in the null space and they must be positive. The moment they become 0, your grasp will fail. So, in this particular case, what will be this part if you can guess?

So, even if this becomes equal to 0, this fellow will be there which $f_{x_1} = f_{x_2}$. So, my vector is going to be 1 0 1 0 that is the force in my null space. So, this is called the orthonormal basis. So, the orthonormal basis is 1 0 1 0 here now, that means even if you do not have any force here, you still require this fellow to hold the grasp. So, if you think a little bit on this, if I want to grasp an object and move it along somewhere, along the direction now, so I have grasped an object, this is grasped and I want to move it like that.

So, to move it you need to apply forces and to grasp it you need to have internal forces. So, the total force will be the forces for causing motion plus internal forces. So, this is basically saying that you need to grasp the object and then move it along in some direction. So, you need components of two forces now. Now, let us just look at an example here.

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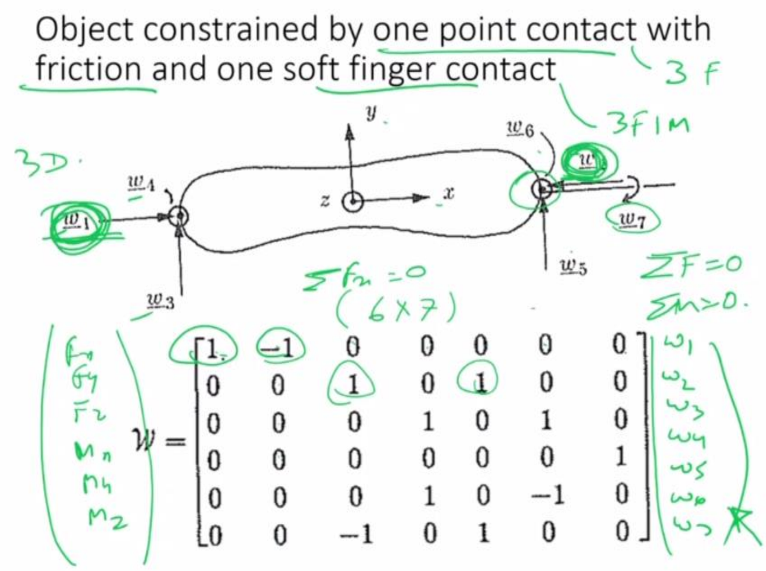

So, this is an object constrained by one point contact with friction and one soft finger contact. How do we solve such problems? So, this is an object which is constrained by one point contact, let us say two fingers in which there is one point contact here. There is one point contact with friction, so there is μ and there is one soft finger contact now. What are the constraints that are there, what are the twists and what are the wrenches that are being applied?

So, point contact with friction you can apply 3 forces, right. So, there is w_1, w_4, and w_3. This is my x axis, y axis. If there is soft finger contact, so here there is 3 wrenches or three forces. Here there is 3 forces plus 1 moment, so this is soft finger contact. So, this is applying 3 forces w_5, w_6, w_2 and there is one moment w_7. Now in this case, this is my x axis, y axis. Now in order to solve this problem, we simply do $\sum F_x = 0$ and $\sum M = 0$ and write our equations.

That means I am going to write forces in the x direction equal to 0. So this is my f_x, we are calling this w here, so w_1, w_2, w_3, w_4, w_5, w_6 and w_7. And on this side we have F_x, F_y, F_z, M_x, M_y, M_z, this is a 3D problem. Now when I read my equation, $\sum F_x = 0$, what do I get forces in the x direction equals 0. What are the forces in the x direction w_1, which is here and w_2. So, $w_1 - w_2 = F_x$.

So I am writing into matrix form. Similarly in the y direction what is there, w_3 is there which I am having here and w_5 is there. So, that is how I get this matrix. Now again note here that this is a non-square matrix. So it is a 6X 7 matrix, this is non-square. So, if I want to find the fingertip forces, again I have to find the pseudoinverse and then find the null space of the matrix.

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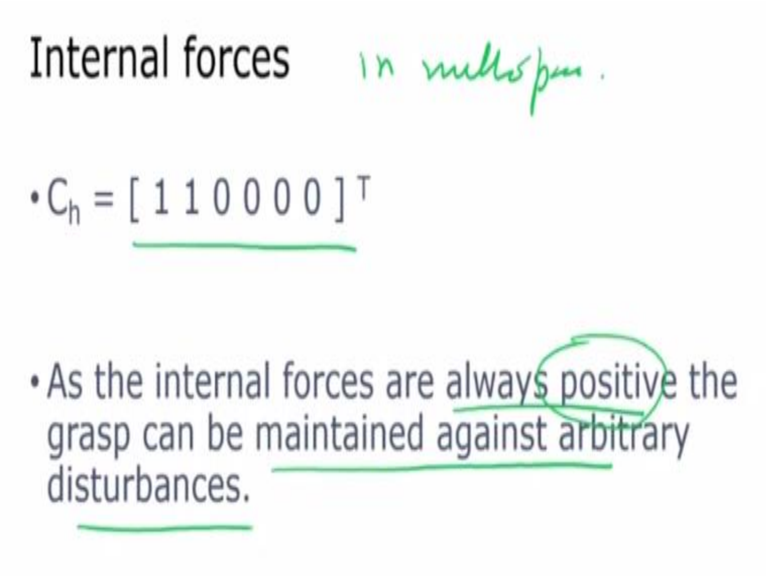

Now in this particular case, what will be the internal forces in the null space? So internal forces in the null space would mean that in order for this grasp not to fail, this force must always be there and this force must always be there, other forces can go. But those two fellows have to be there, w_1 and w_2. So, it basically means that the internal forces in the null space will be 1 1 0 0 0 0.

Now, so this means internal forces are always positive, the grasps can be maintained against any arbitrary disturbance. This means that when we say mathematically that the object has been grasped, we mean that internal forces are there and they are positive that is the meaning of a grasp being there.

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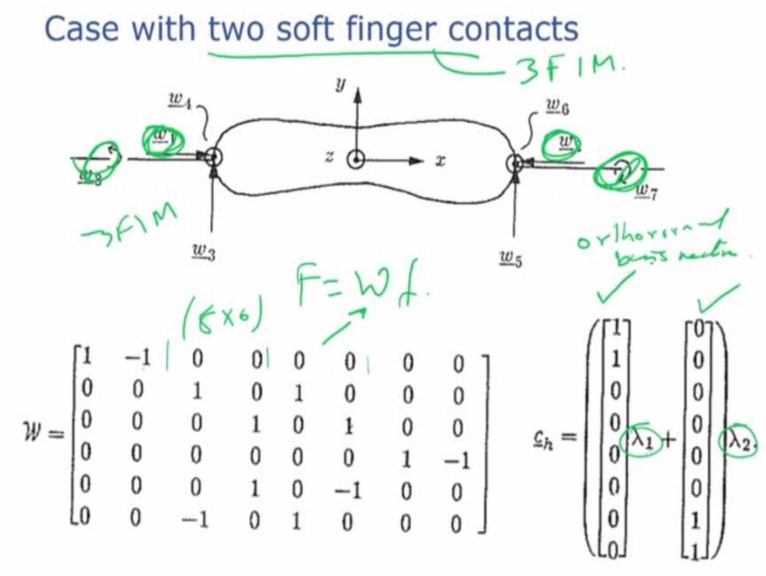

Now, if we think a little bit more there can be different kinds of contacts, say for example here where you have two soft finger contacts, this is just an example, if you think a little bit it will become clear. Now in this, two soft finger contacts, two soft finger means 3 forces plus 1 moment. So in this case, there are 3 forces and 1 moment on each side, so 3 force, 1 moment, exactly the same way. We write our equation F = w into f and we get this matrix in which case this is how, this is 8 by 6, non-square again.

Now, in this particular case what are the forces in the internal spacer or what are the internal forces? In this particular case, we need this fellow, we need this fellow, we need w_m and we need w_8 also. So, this is one orthonormal basis vector, this is another one. So, these are called the orthonormal basis vectors. Now we have λ_1 and λ_2 which are positive constants that means any multiple of this is fine.

So, there is a one basic solution and multiples of that solution are all correct. So, when we talked about multifinger hands, we basically have to worry about this case where we have fingers, we have the fingers which are holding the objects which have positive internal forces and whatever additional forces are required to move the object.

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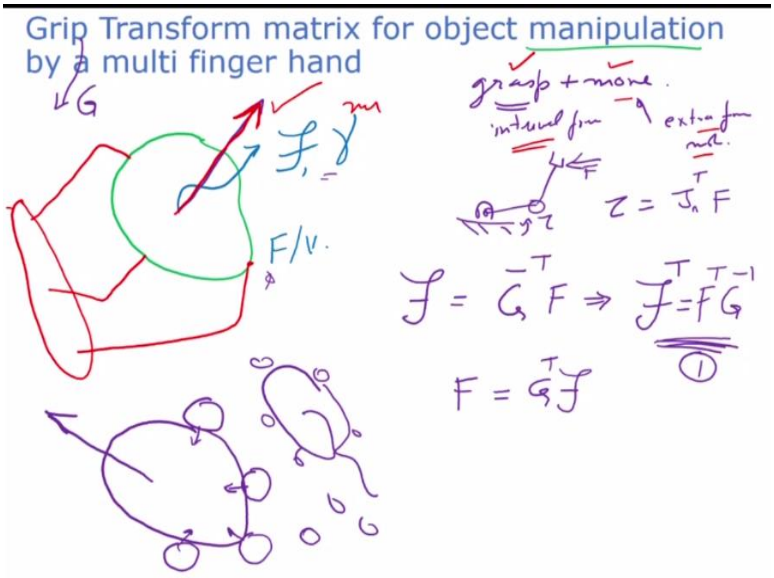

So, next let us look at the grip transform for object manipulation. As I said that, I have let us say this is an object and this object is held by a multifinger hands. We have one here, multiple fingers, let us say there are three fingers. So, this is my robot hand and these are three fingers, this does not look very good, let us say I have one more here, it is better. So, these are fingers and these are contacts. Now we are saying that each of these fingertips can apply a force which is F twist and a wrench which is v.

So, the fingertips can apply a wrench or a twist. Now because of these multiple fingers, each of these fingers can apply a twist or a wrench. What will happen is the centre of the object will have some motion. So let us say the object is moving like that. So, there is a sum of forces which are acting at the CG of the object and it has some twist. So, we have multifinger hand which is holding an object. I want to impart some velocity or move this object along a particular path.

For example, I want to move this object along this path like that I want to move it or let us take it simply I want to move it along a straight line like this. I want to move the object like that. Now kinetics will have to apply some wrench because of which there is going to be a resultant motion of the object and it will go in that direction and it basically means you have to grasp plus move. It is exactly similar to in the case of multiagent systems where we talk about.

We will see this in the next class where we have multiple robots. Multiple mobile robots which are pushing this object in some direction, this is a multiagent system. It is exactly the same problem where these small agents are applying forces and trying to force the object and take it in some direction. We can also talk about capture where we have a moving object which is moving around and I have multiple robots which are going to go and capture the object.

So, this is exactly the same, the multiple robots will apply a force on the object and constrain it and then it can push it in some other direction. So, it basically means you have to grasp the object which means you need internal forces and you need to move so you need to have extra forces for motion. Now how do we solve this problem? If you remember we talked about in the case of a serial manipulator arm if there is a force acting there, then what is the relation between the force and the torques, the joint torques.

So, the joint torques $\tau = J^T F$. What is J? J is the Jacobian in the n^{th} frame. So, similarly this gives us a relation between the joint torques and the force at the end effector. Now, the grip matrix basically gives us a relation between the fingertip forces and the forces of the body centrally subjected to and if I can find that relation then I know exactly how I can move the object in any particular direction. So, we call the grip matrix to be equal to G.

And I write the body centre forces $f = G^T F$ which means that $f^T = F^T G^{-1}$. So, G is the grip transform which relates the fingertip forces to the forces at the body is subjected to at its CG. Now, from here what we see is that so my $F = G^T f$, right. Now, what I want to do is we want to find the relationship in terms of velocities that what should be the fingertip velocity that should be applied in order to have a particular object velocity. Now, how do I do that?

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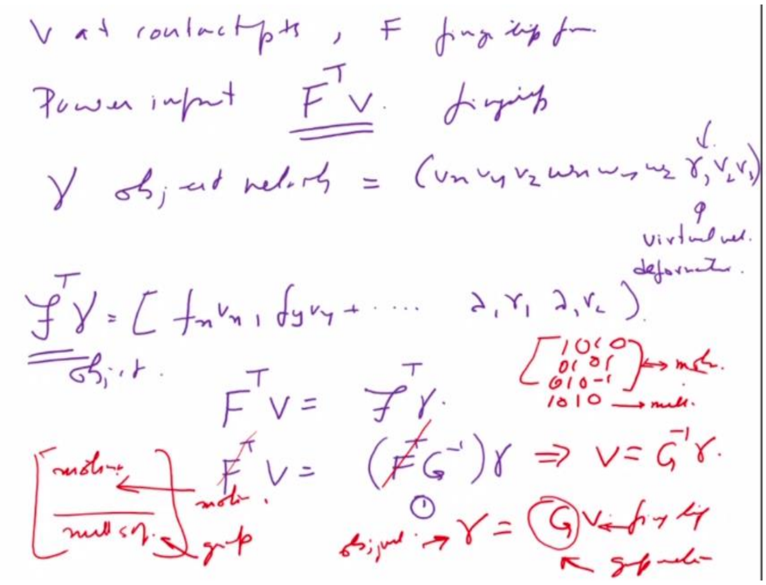

Now, what we say is V is the twist along each of at contact, right, V is at contact points, So, the total power input to the system is equal $F^T V$, F is the fingertip force. So, V is the contact point velocity and F is fingertip force. Now if you say γ is the object velocity then, γ= $(v_x v_y v_z \omega_x \omega_y \omega_z \gamma_1 \gamma_2 \gamma_3 ....\gamma_n)$.

Now, these are what are called virtual velocities or deformations. So, if the object is deforming it is also having a having a velocity of the fingertip that is what is coming from here. Now, in terms of the power at the centre of the body this is, $f^T \gamma = \left[f_x v_x, f_y v_y ....\lambda_1 \gamma_1, \lambda_2 \gamma_2 \right]$. So, power input is $F^T V$ at the fingertip and this is the power at the object.

So, these are the fingertips and these are the object and they have to be equal, right whatever power is given from the fingertip have to be conserved on to the object. So, if I equate these two, then $F^T V = f^T \gamma$ and we have seen that $F^T V = \left(F^T G^{-1}\right)\gamma$. So, from 1 we get this and I can cancel off F^T from both sides, this and this will cancel.

So, what we do get from here is $V = G^{-1}\gamma$ and so I can take $\gamma = GV$. So, this G is what is called the grip transform and it gives the relationship between the object velocity which is here and the fingertip velocity which is there. So, what will this consist of? It will consist of two parts, one is called this will consist of the null solution and this will consist of the homogeneous part.

So, this is the particular solution or this is the actual solution plus the null solution. So, in the previous case, what will we get is the grip transform will consist of; let us take the simple example this one. So, our matrix was $\left[\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}\right] = G$. So in the G matrix this is going to come as this fellow will come to the bottom here that is my null space.

So, this has become a square matrix. Now you can solve it and you can find the relation between what is the object velocity, so here now this is my fingertip velocity and this is my object velocity and this is my grip matrix. So, the grip matrix will ensure that the null space is there, so the grasp is maintained, and this is the one that is going to give you the motion. So, this part of these will cause the motion and the bottom of it will cause the grasp.

So, in the previous case we looked at the solution like this $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$ So this fellow is in my null space and this fellow is the motion fellow. So, whatever direction I want to move the object now, let us look at this example. So, I want to move the object in this direction. So, whatever velocities are required in that direction, whatever forces are required, I can get from the top part of the matrix and to maintain the grasp I am going to get from my null space.

So, I am getting this part plus that part. So, I am getting my internal forces from the null space and I am getting the extra forces for moving the object from here and that is the basic way the grasp matrix works.

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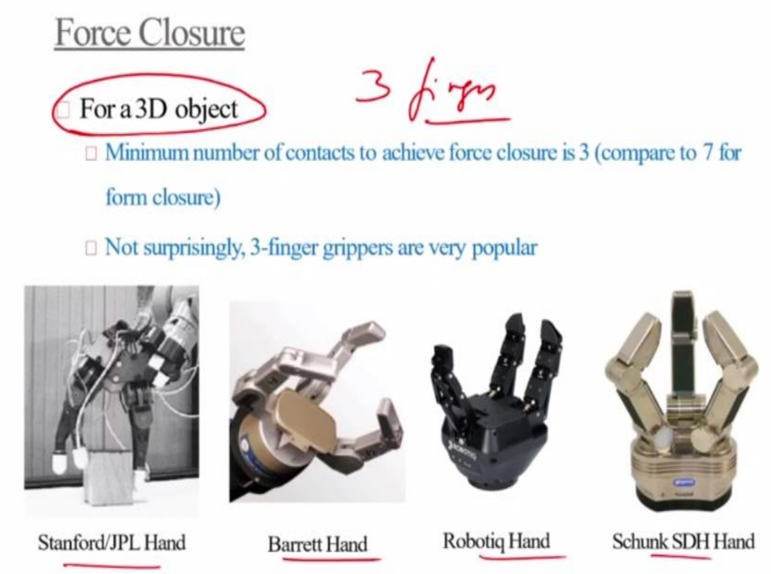

Now, this is also we looked at force closure and we understand that for in 3D you need 3 fingers and that is why most of the robot hands have 3 fingers and they basically work on the concept of forced closure. So this gives us an example that if you look at various robot hands, the Stanford hand, Barrett hand, Robotiq hand, Schunk hand they all have 3 fingers because that ensures forced closure.

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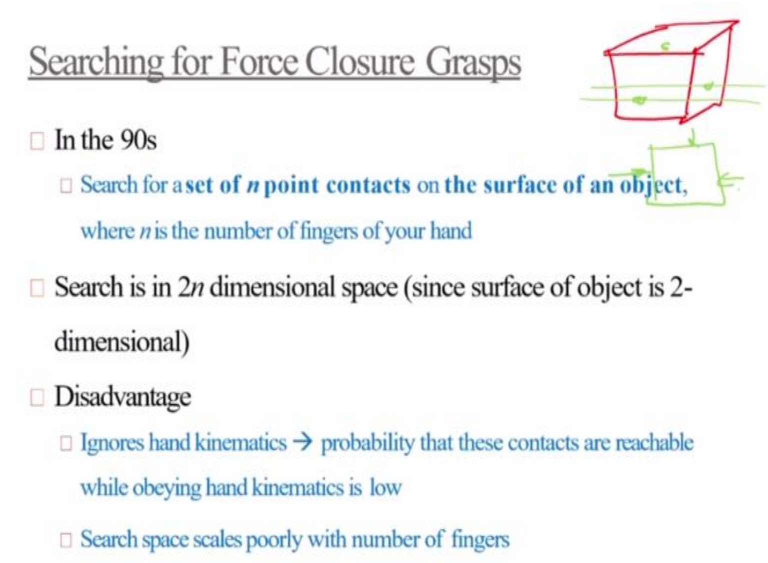

Now it is very interesting to note that there has been a lot of research in terms of how would you grasp an object that is basically called searching for force closure grasps. One way is if you look at a regular object, this is a regular object cube, I can slice this in terms of 2D objects. I can slice this and make it 2D. So if I slice it, it will become like this, then I grasp it by holding it here, here and here. So, I can basically identify these points before I grasp it and then I grasp it. So, there has been a lot of research and interesting things of research is how do you autonomously grasp an object.

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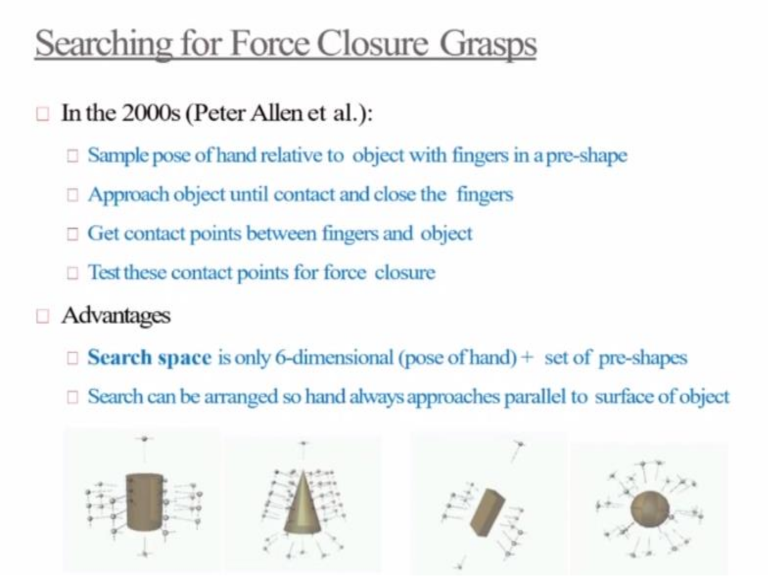

And people have been working for a very long time. This is basically called search for force closure grasps. So, if you have regular objects, it is not much of a problem because you can identify areas where you can grasp and then you can go and grasp it.

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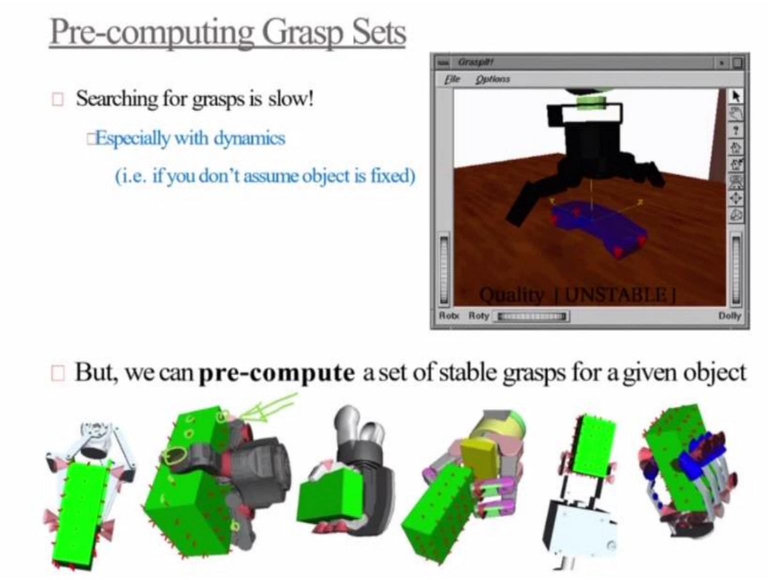

Now, people have looked at pre-computing graph sets. For example, if you are given a cube how to hold it, so this is a cube and you're supposed to hold it, so you can go hold it in that order. Now, there are other solutions also. These are all solutions. We have seen that in the null space you can get multiple solutions. So, these are also solutions. So, the grasps can optimize a function and get into those points.

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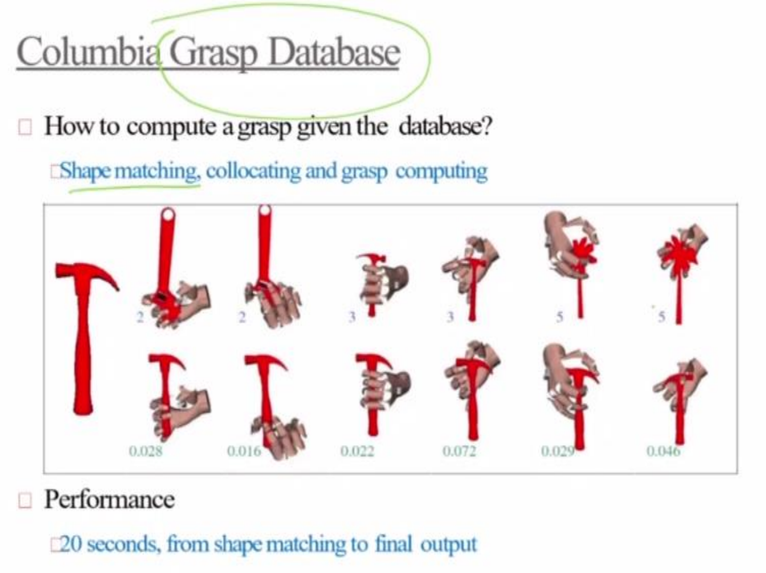

The other from multifinger hand, another way of looking at grasping using multifinger hand is by generating this database of grasps. For example, if you are given an object like a hammer and you want to hold it. Then what they do is what is done is the shape matching between the hand and the object and then an index is optimized, for example how would you hold the object. So, the best way of holding an object to satisfy a particular index, so they basically match the hand pre-shape and the index and they try and proceed.

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Now, recently there has been a lot of interest in grasping and manipulation planning that you need to grasp and then you need to manipulate the object.

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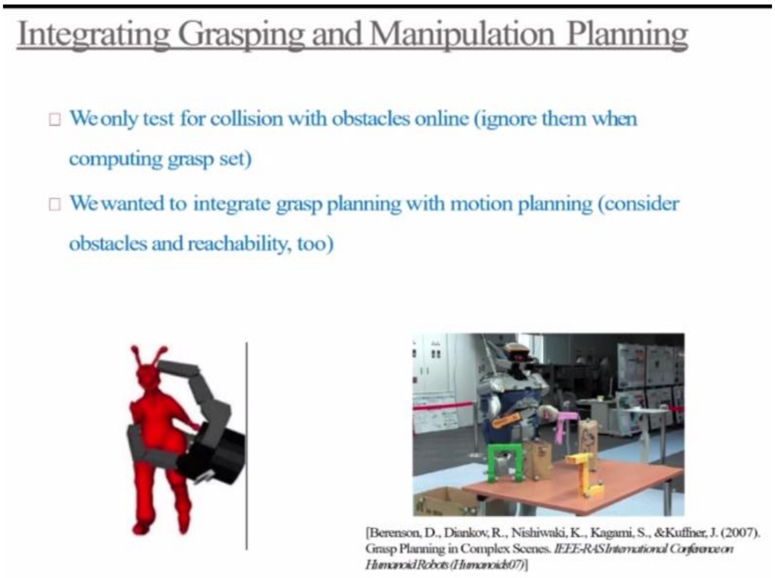

For example, in cluttered environments like this, this humanoid robot has to pick up an object and move it somewhere without hitting something. So that means grasping and manipulation.

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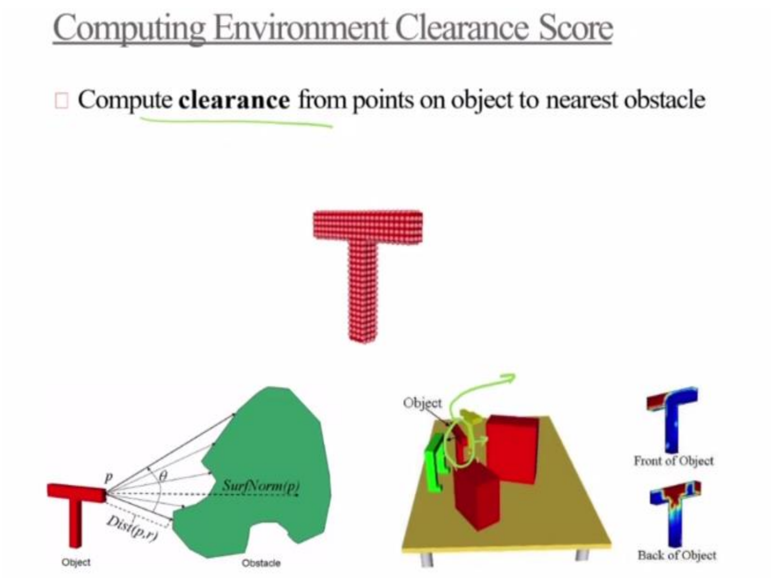

So, what is basically done is we compute matrix. One of the matrices can be computed the clearance from point on object on nearest obstacle. Say for example if we want to catch this object and I want to move it somewhere then I see what is the obstacle distance from the other obstacles and take that as a metric.

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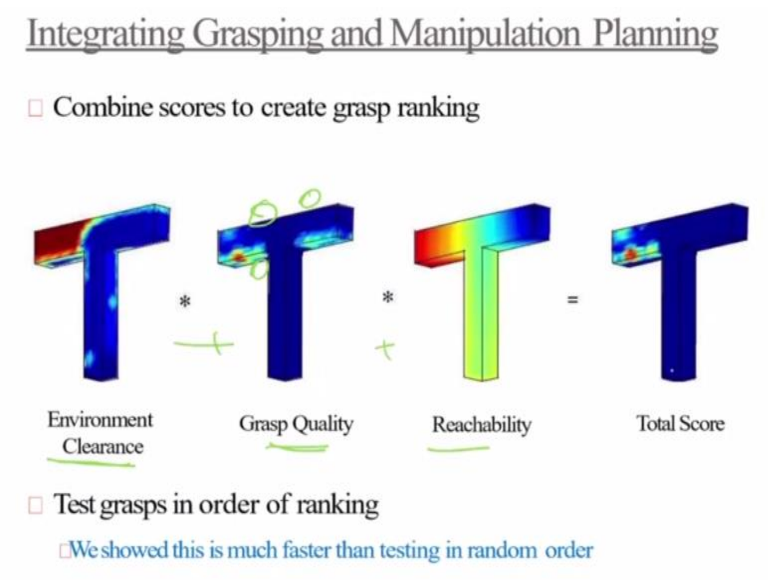

Then, look at the clearance, look at the grasp quality in terms of where can you catch. In terms of quality, the minimum force can be one I can get here, here and here that will give me the minimum force required. Reachability, then add all of these and put some kind of a total score and make that a metric.

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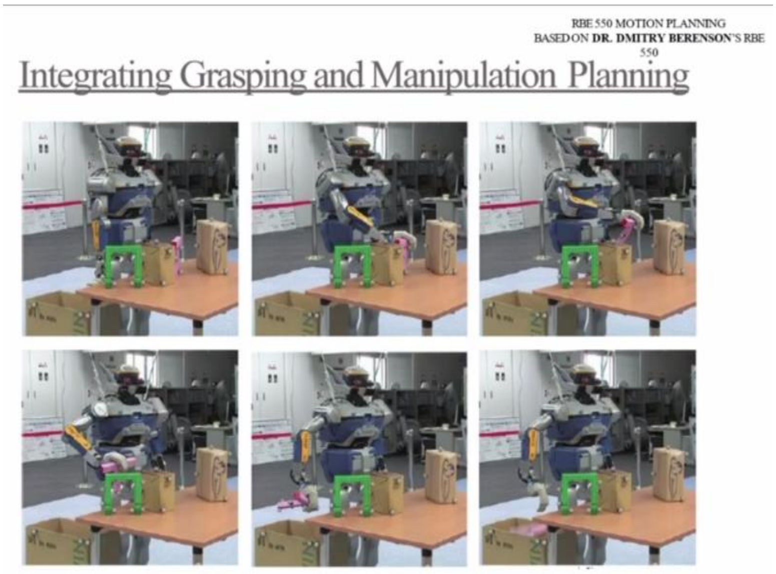

So, these are some of the areas of research which is very active these days. People are also using machine learning that instead of trying to solve this problem in terms of doing the mathematics and the algebra, by using machine learning techniques.

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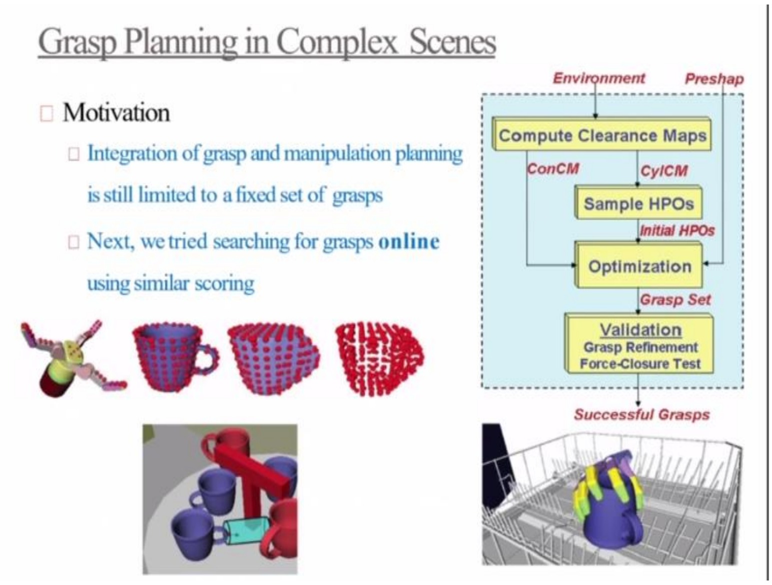

Now in terms of the course, we have studied things like RRT.

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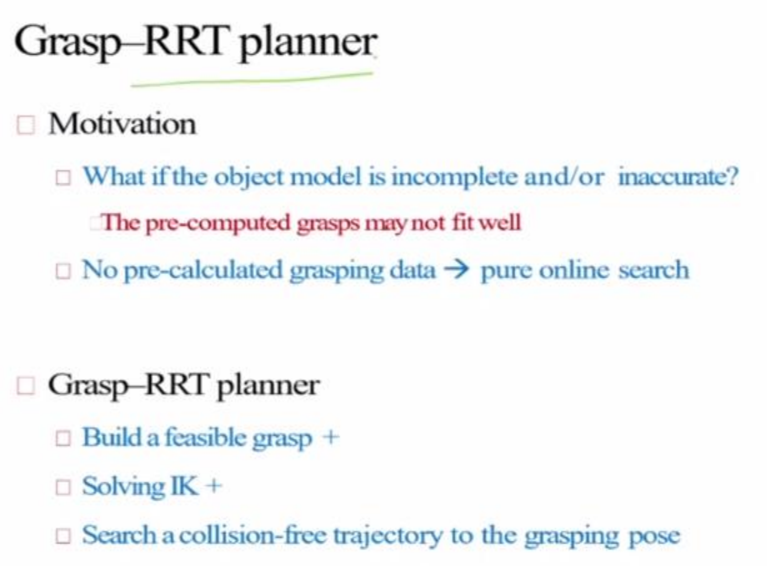

So, people have also started using concept of RRT, the grasp planning using RRT random trees.

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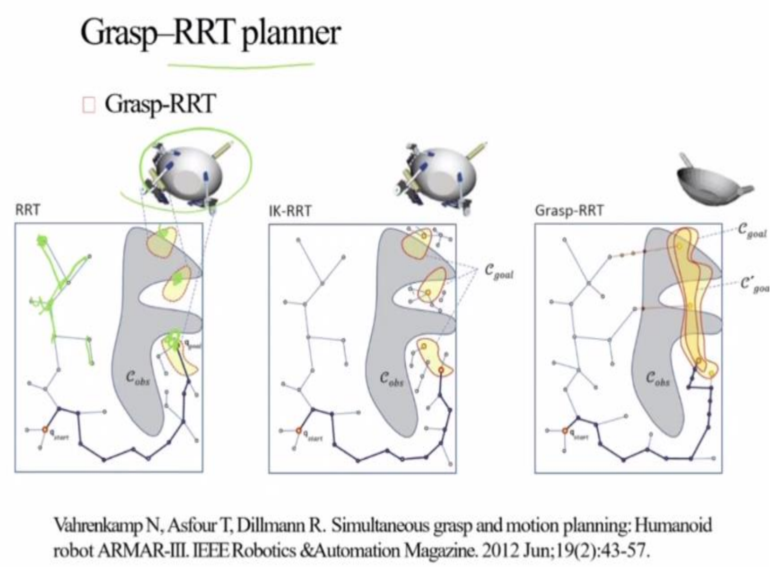

That is suppose you are here and you want to go here and catch an object. Say for example, this is my object and I want to catch this object. So, this my object and I want to catch it using three fingers, so one finger here, one finger here, one finger here. So, what we can do is by using sampling based method I can start from here, start shooting these nodes, make these edges and see by placing different fingers how can I go and catch this object and optimize some metric.

This is by grasping using RRT. But it is extremely difficult because there are 3 fingers, you have to place 3 fingers such that you have to satisfy the grasp and also hold the object. But RRT type planners are also used for these kinds of tasks.

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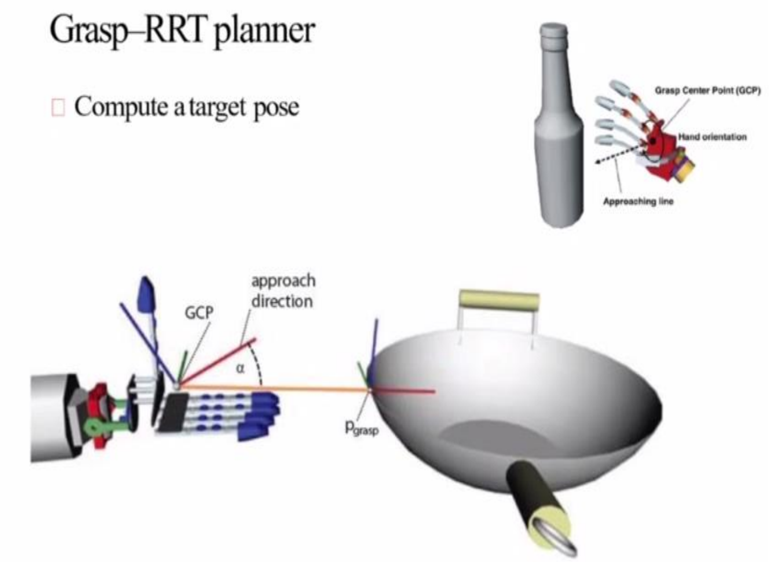

Now, this is the latest area of grasping manipulation where if you are given some kind of object, which is not a regular object like a square or a rectangle, how do you go and catch it by using a multifinger hand. So, people use RRT by manipulating or optimizing some index and then that go and catch this object. So, today we looked at multifinger robot hands and the basic mathematics and the algebra that is involved, the concept of null space.

Because when we are talking about null space, when we are talking about grasping, mathematically it basically means that you must have forces in the null space to be positive, then only you can have a grasp, otherwise the grasp will fail. So, in terms of motion planning, grasping and then manipulating the object is basically meaning that you need to have forces for moving in a particular direction along a particular path, but at the same time you must also maintain internal forces and that is what makes grasping and manipulation very difficult.

Now, as of today there is no robot in the world which can actually take a pen and do very simply like this. It looks very simple, but it is not because of the mathematics involved. So, this is something very interesting to note and please think about it. The human beings can move the fingers and move a pen like this so easily, a robot cannot. Why? Because it cannot solve those kinds of equations in real time and it does not have sensors.

So, there is no way by which it can actually measure the force and apply that force in particular sequence to get this kind of motion. So, we will stop here today and in the next class we will move on to the next topic that is optimization in motion planning that when you have multiple agents or multiple fingers, multiple robots doing a task how are you going to use optimization to get your path results? So, we stop today. Thank you.