Robot Motion Planning
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Lecture - 03
Work Volume and Rotation Transformation

Hello and welcome to this lecture number 3 of the course robot motion planning. In the last lecture number 2, we looked at degrees of freedom and the various joints that are there. So, today we will very briefly revise what we are doing in the last class. And then, we will move on to the next topic. That is after you assemble the various joints and make a robot then what is the work volume or the area inside which the robot can move its gripper.

And then, we will move on to transformations; that is a robot essentially moves in space. So, we need to know the relation between one joint and the other joint with respect to the base. So, we will first start off with the revision of the last class. Then we will talk about work volume, the shape of the work volume when you make robots using different kind of joints. And then, we will talk about the basic transformations; that is the rotation transformation.

And then, we will move on to homogeneous transformations. So, we are talking about work volume and then transformations.

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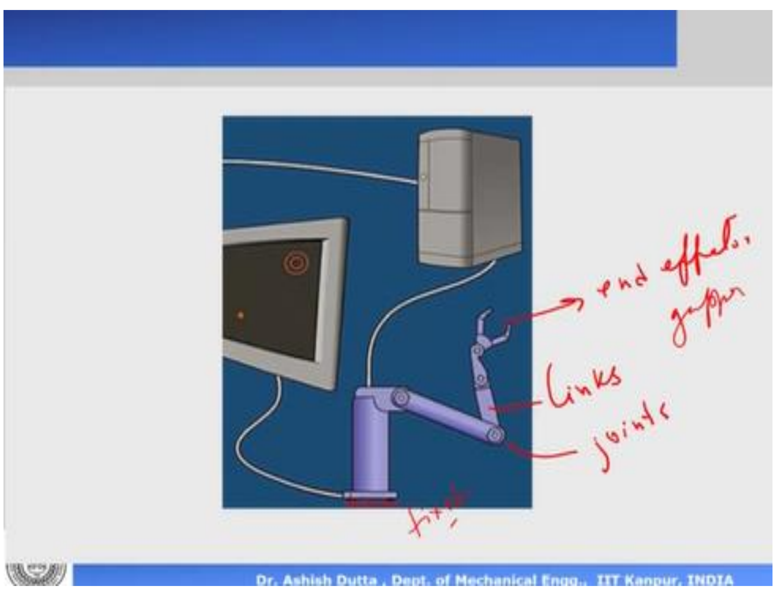

So, very quickly revising about what we did in the last class. We said that a robot is made up of links. So, these are the links. And the links are connected by joints. And at the end, we have the end effector or gripper. And the base of the robot is fixed in this case we say that the base is fixed it does not move. So, this is a fixed base robot. So, these are the different parts of the robot and we looked at the various joints, of the various types of joints which are possible.

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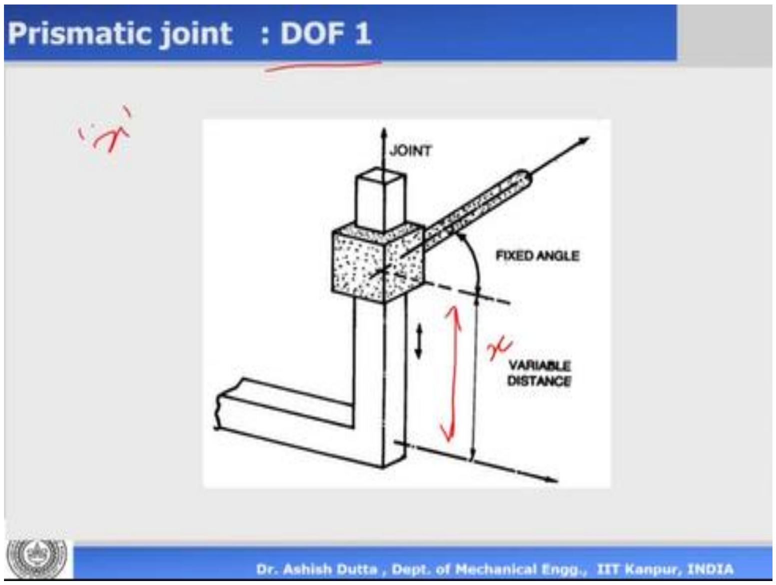

We saw that we have a prismatic joint in which case we have the variable which is the variable distance which is x and it has 1 degree of freedom and the variable is the distance x.

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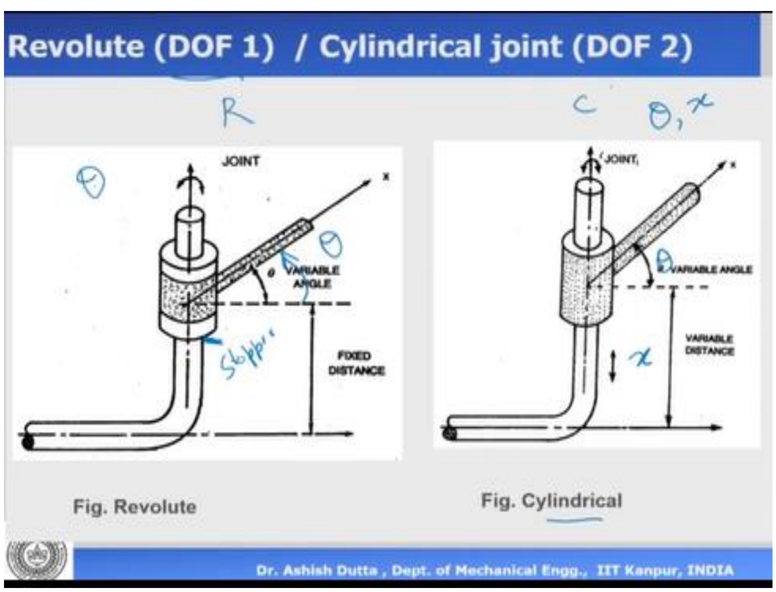

Now, this is a revolute joint, the one on the left side. And in the revolute joint, we saw that this has a rotating degree of freedom which is θ and there is a stopper there. So, it cannot move up and down, so, there is a stopper there. And it also has 1 degree of freedom and the variable is θ. It also has 1 degree of freedom. The other kind of joint is a cylindrical joint and the cylindrical joint has 2 degrees of freedom. In this case, there is no stopper.

So, there is no stopper here so, it can move up and down. And this is up and down x and there is a variable θ. So, there is this has 2 degrees of freedom and the variable is θ and is the translation distance x. So, this is what we can call a prismatic joint. It is also c-alled the translational joint that the variable distance is a translation. And it is represented as a P it is also called a P joint this is called an R joint this called a C joint, cylindrical joint.

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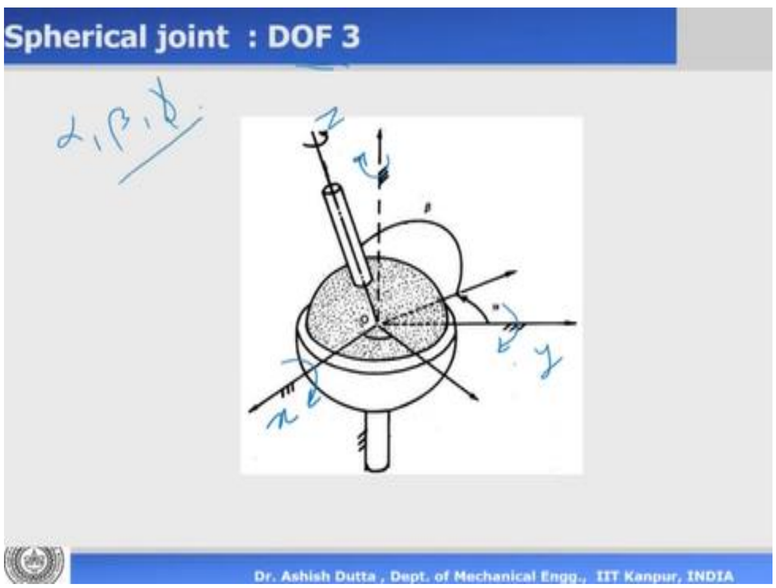

And then, we also have a spherical joint which has 3 degrees of freedom which means that it can move about the 3 axes. If this is my x y and z axis then it can rotate about the 3 axes and, we have 3 degrees of freedom so that we can call it α, β, and γ. Now, we also saw that most robots are made up of the revolute joints and 99% robots are made of revolute joint but when you want to do assembly application where you are assembling 2 parts.

And you need a straight-line motion then, we normally use a prismatic joint. So, prismatic joint is basically used for straight line motion.

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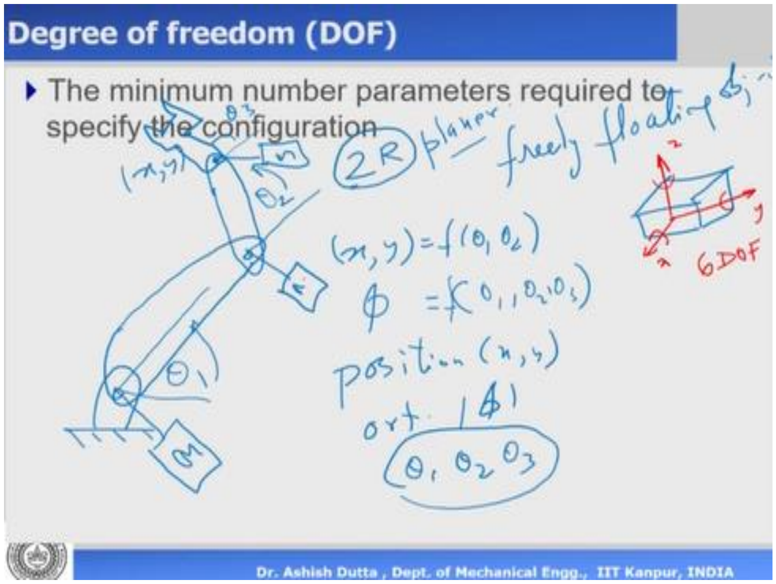

The minimum parameters required to specify the configuration of a system is called the degrees of freedom. And we said that for a freely floating object in space so, freely floating object. We have this freely floating object if we assign an access there let us say I am going to assign an axis some other colour. This is my x axis that is my y axis, and that is my z axis, x y and z. Then, it has 3 translations in the x direction, y direction, z direction.

And it has 3 rotations about the x y and the z. So, this one has 6 degrees of freedom with 3 translations and 3 rotations. Now, when we talk about a robot a robot is fixed at the base like the one, we just saw. So, this is fixed at the base it does not move. Then, we have the first degree of freedom of the first joint. This is the second joint and that is my third joint this is my second joint. So, this is my fixed joint and the variable here is θ_1, the variable here is θ_2.

Let me draw it here. So, the variable here is this variable here is θ_2 and this variable is θ_1. So, to define the position of any point x and y there we need 2 independent parameters which is $\theta_1$and θ_2. So, x and y are functions of $\theta_1$and θ_2. Now, suppose which is 2 degrees of freedom. So, in this case, we can also say that if you want to specify x and y this is a planar 2R manipulator which is planar, that means it can move only on this plane.

So, to specify any point x and y, you need 2 variables $\theta_1$and θ_2. So, this is my θ_2. Now, suppose I add a gripper in here. See, I am putting one gripper here like this so, this is a gripper now and this gripper angle now is my θ_3 here. Then, we can say that the final angle that the gripper is making, $\phi = f(\theta_1, \theta_2, \theta_3)$.

So, in this case, we are defining the position which is given by x and y and I am defining the orientation which is given by Φ. So, in this case, how many degrees of freedom are there? 3. So, θ_1, θ_2 and the third one there is θ_3 also there. So, what we see is that in this particular case, we are having 3 degrees of freedom. And, we can also see that there are 3 joints. So, there is one motor here, one motor here and one motor here.

So, in robotics, we can say the degrees of freedom is equal to the number of motors or the number of actuators provided each joint is actuated by a motor and, they are not coupled in any way.

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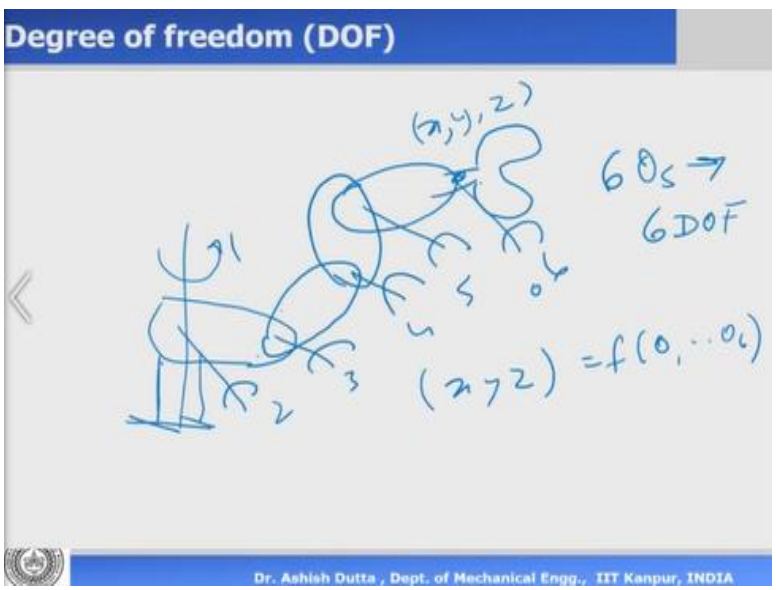

In the case of a mobile robot, can we have more degrees of freedom? Yes, you can have a robot a serial link. So, this is not a serial link manipulator. So, this is a robot which is like this it has multiple degrees of freedom. So, this can rotate about this axis. Then, there is one more axis there, 2, 3, 4, 5 let us say there is an axis here which is so, this is my 1, 2, 3, 4, 5, 6. Now, this point x and y is still can be x y and z and x y z is a function of 6 θ's now.

So, x y z is a function of 6 θ’s, θ_1 to θ_6. Now, how many degrees of freedom are there now? There can also be an orientation. The last orientation of the end effector is also there; $(xyz) = f(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6)$. So, here, we are seeing that there are 6 θ’s and that means there is a 6 degree of freedom system. This is a 6 DOF system again. Again, each joint is actuated by 1 motor and they are independent.

And there is no coupling between the joints. So, I hope you understand what is the meaning of degrees of freedom of a freely floating object in space which has 6 degrees of freedom 3 translations, 3 rotations? In the case of a robot, we talk about the minimum number of parameters required to specify the configuration. And we also see the number of joints that are there. So, in this particular case it is a 6 degree of freedom system.

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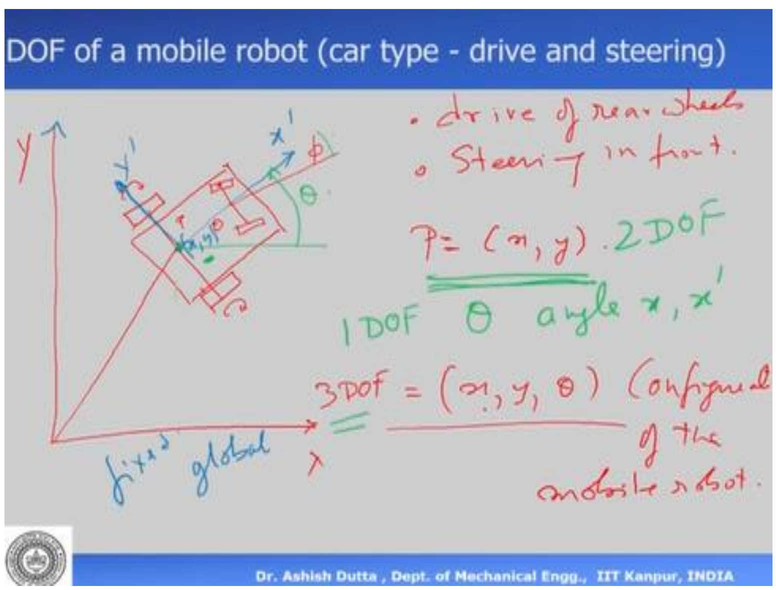

In the case of a mobile robot what we do is in this case we take a point P. So, this is my global frame x y is my global frame of fixed frame which is not moving. So, global or fixed frame and what we say is that I take this point P in between the wheels. And then I say this is my coordinate x y and I assign an axis here. Suppose, this is my x axis and that is my y axis, this is a local axis so, x', y' then the angle between the global and the local axis.

So, the angle here, this angle is my θ whereas Φ here is my turning angle of the wheel. So, in this particular case, we see that because there is an x y I need to define the position of this point P that is by means of 2 points x and y. So, this has 2 DOF plus, I have to define the θ. So, there is a θ which is the angle between x and x'. So, I have 2 DOF and 1 DOF here which is defining the angle of the body.

So, I have a total of 3 degrees of freedom this is x y and θ. This is also called the configuration of the mobile robot, position and configuration.

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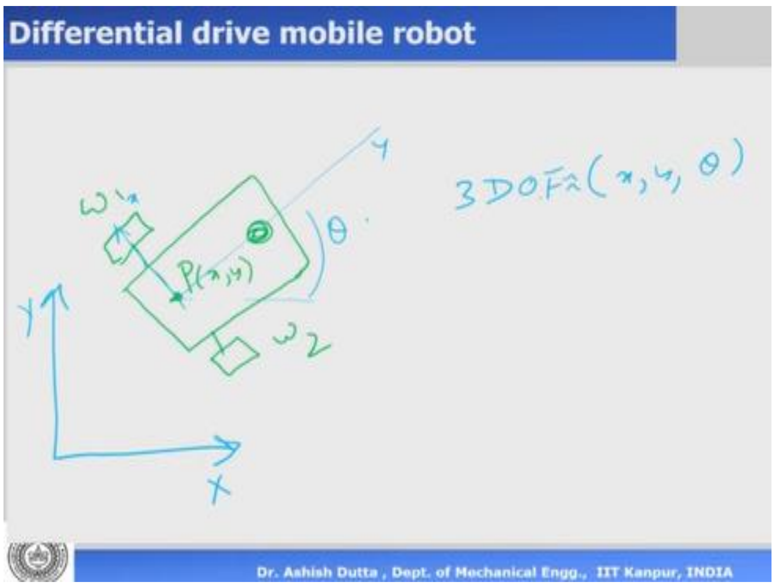

We have seen different kinds of robots and as we go along, we will study different kinds of robots. For example, we have a robot here and we have 2 wheels which are independently controlled and there is a caster wheel there. So, this is ω_1 and ω_2. So, again, I define my point P having coordinates x y. And we can take a local axis I can put a local axis here like this. And this local axis is making an angle with the global axis. So, this is my X Y.

This is my x, y. So, we can say that this angle is my angle θ. And again, this is a 3 degree of freedom system which is x y and θ. So, now let us move on to making different robots with using different kind of joints.

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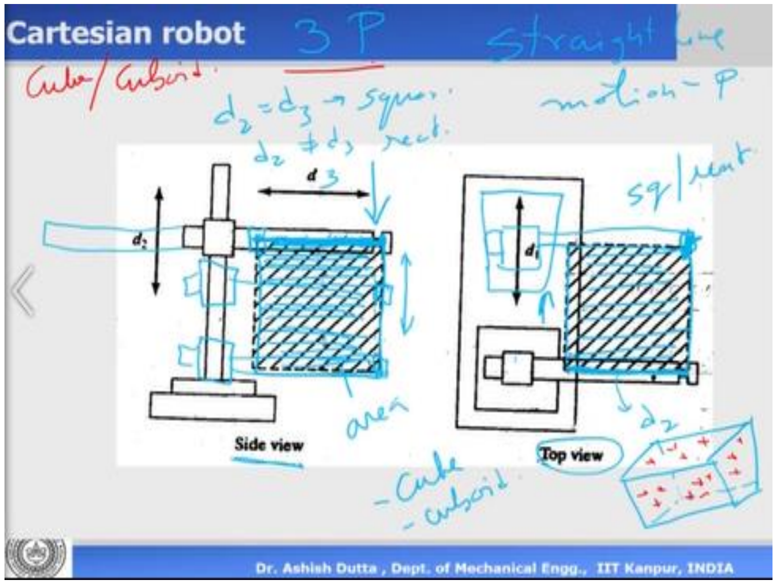

So, we have seen that you can have a prismatic joint; you can have a revolute joint. You can have a cylindrical joint and you can have a spherical joint. So, when we are making robots, we can make robots by using different kinds of joints and obviously, the structure would be different. For example, the structure of a revolute joint and that of a prismatic joint are different. So, the first kind of robot that we will see is a Cartesian robot.

Now, this is making up of 3 prismatic joints. You know a prismatic joint move in a linear direction. So, it has now, this is the side view of the robot. So, this is joint number 1 joint number 1 and 2. And if I see the top view this is the top view. So, the robot can also move like this. The full body is moving the base. So, we have 3 degrees of freedom. And they are represented by this d. this is a translational degree of freedom. That means it moves in a straight line.

So, it is straight line motion at the joints because they are P joints. So, what we can see here is that there are 3 degrees of freedom and 3 P joints. So, this is the first one. Let us call this this is my d_1 which is the first degree of freedom d_2 is this one and d_3 is this one. So, these are 3 degrees of freedom, 3 translations which it can do. Let us start off by looking at the side view. Now, let us assume that this link is fully extended as shown here.

So, this is the maximum condition in which it is extended. So, it is till here it can come till here. Now, I can move it backwards the link when it goes back like this. It will come to here this point.

So, when I am moving this joint number 3, I am retracting the joint in this direction. What is happening this? End effector tip is going like this and it is having a straight line. So, which means that; any point in the straight line it can go to. Now, I will repeat again how did we start?

We said that the joint number 3 was fully extended. That means this is maximum it can go. So, it is reached this point then, I move it backwards. So, when I move it backwards it is going to draw a straight line and, this link is moving backwards. You can imagine that it just come till here that is maximum it can go again. So, it has come till here. What was this geometry of this shape? It is a straight line now. I can look at joint number 2 that is d_2.

Now d_2 also, we can say is maximum external condition is here and minimum it can come down like this. So, this joint is moving down if you can imagine. So, it has come down here like this. Let us come down more and finally, it has come down right till the bottom. So, what we can see is that this straight line is moving down like this. That means joint number 1 can move in a straight line.

It can go to any point on the straight line and joint number 2 is moving downward. So, the straight lines are moving down like this and what you get is this shape. What is the shape? It is a square or a rectangle. So, if $d_1 = d_2$ it is a square. If $d_2 = d_3$ then the shape is a square and if $d_2 \neq d_3$ it is a rectangle. So, please draw this by yourself and understand how this is a rectangle in the side view.

Now, let us look at joint number 1 that is shown on the right hand side and this is the top view. So, this says that this full base can move from this position which is the extreme position to the next position here. So, this my full base has moved and this is the other position here. So, this full motion is d_1 which can move fully like that. So, this point if you look at the left side view, the side view from the top, what would you see? You will see a straight line.

This area would appear as a straight line when you are looking from the top and that is this straight line. So, when I sweep the straight line from this end to that end it is a straight line which is moving from one end to another end. What shape do we get? Again, we get a square or a rectangle depending on the dimensions depending on the motion of this straight line, depending on d and this length which is d_2 which is going to be deciding whether it is a square or it is a rectangle.

So, if I combine both of the side view and the top view what will be the shape of the volume? So, the shape of the volume is going to be a cube or a rectangle, something like this. So, this is my cube or rectangle or cuboid. So, we can have a cube or a cuboid depending on whether all the sides are equal or they are not equal. So, please understand how did we get this cube or cuboid shape?

So, we got it essentially because there are 3 degrees of freedom and there are 3 prismatic joints. Now, if there are three prismatic joints then we get a cube or a cuboid. And this essentially means that the end effector can be placed anywhere inside this cube or cuboid. That means every location inside the cube or cuboid is accessible. So, this robot can position its end effector anywhere inside the cube or cuboid it cannot come outside.

So, obviously, it cannot come outside the square sorry outside this cube or cuboid. It can be anywhere inside there. So, what is the shape of the work volume of a Cartesian robot? It is a cube or a cuboid, partition robot, 3 prismatic joints. So, please draw it by yourself and understand how did, we get this cube or cuboid.

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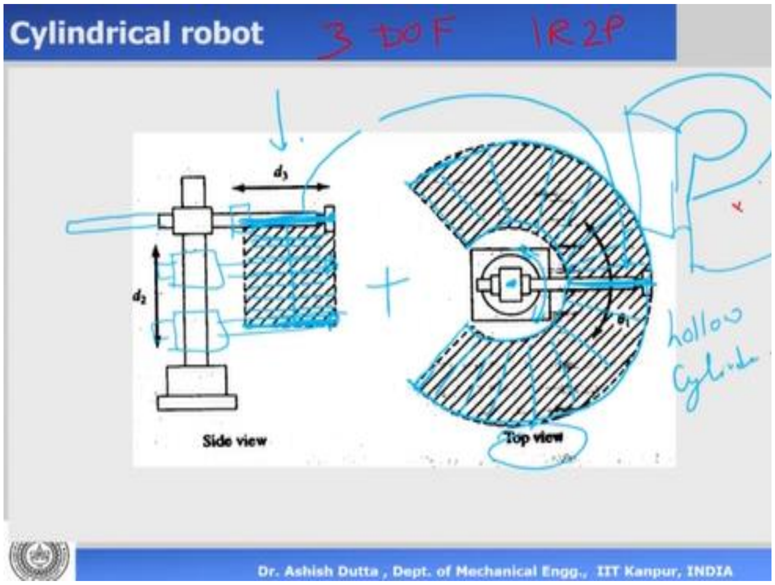

Now, let us have another robot which is made up of a cylindrical joint and a revolute joint. So, this is called a cylindrical robot. This also has 3 degrees of freedom. It has 2 revolute joints and 1 prismatic joint. Now, you remember what are cylindrical joint is from the last class. So, this is a cylindrical joint. So, it has 1 θ which can rotate and it has 1 prismatic distance which is d. So, it can rotate and also translate at the same time.

So, this is my variable R this is my variable P plus there is 1 prismatic joint. So, this has 2 prismatic joints and one revolute joint. Now, let us look at the side view again. Now, the side view has two prismatic joints and it is exactly same as the previous one. So, from the extreme condition here, it can move back this much. So, it comes here so, this is a straight line. So, we get a straight line here.

Now, the joint number 2 can move from here. It can move down like this, and then, it can move down like this. So, this is a d 2 so, it is moving down in a straight line. So, basically, this line can move down till here. And once it can move down till here this is basically again a square or a rectangle depending on the dimensions of d 2 and d 3. Now, when I look at this from the top view, on the right side is my top view.

Now, on the right side this is fully stretched and this is my straight line. So, this straight; line on the top view. Now, this joint is a revolute joint. So, it can rotate like this is the revolute joint of the prismatic axis, sorry, is the revolute joint of the cylindrical joint is the revolute, sorry, revolute axis of the cylindrical joint. So, it can rotate. So, when it is rotating it can rotate like this right from here till here, till here. So, this straight line can move like this because this is rotating.

So, when it is rotating you can imagine this is a straight line which is moving and it is being rotated. So, it is going from one end to the other end like this and what do you get is the semi-circular area with a small circle cut inside. So, if I combine the side view and the top view what would I get? I will get a cylinder. I will get a semi-solid cylinder like this. So, this is my cylinder. So, you can imagine that this is the structure that you are going to get.

So, what you will get is a hollow cylinder. Now, in this hollow cylinder which is the shape of the work volume of a cylindrical robot you can position your end effector anywhere inside. But you cannot go anywhere outside. So, you can position your end effector point here is this one anywhere inside this work volume. But you cannot go anywhere outside the work volume. And it is interesting to note that the shape of the work volume of a Cartesian robot is a cube or a cuboid.

And the shape of a cylindrical robot is a hollow cylinder. So, I have been using this word work volume very often. Now, what is the work volume? So, the work volume is the area inside which or the volume inside which the robot can position its end effector. So, anywhere inside this volume the robot can go.

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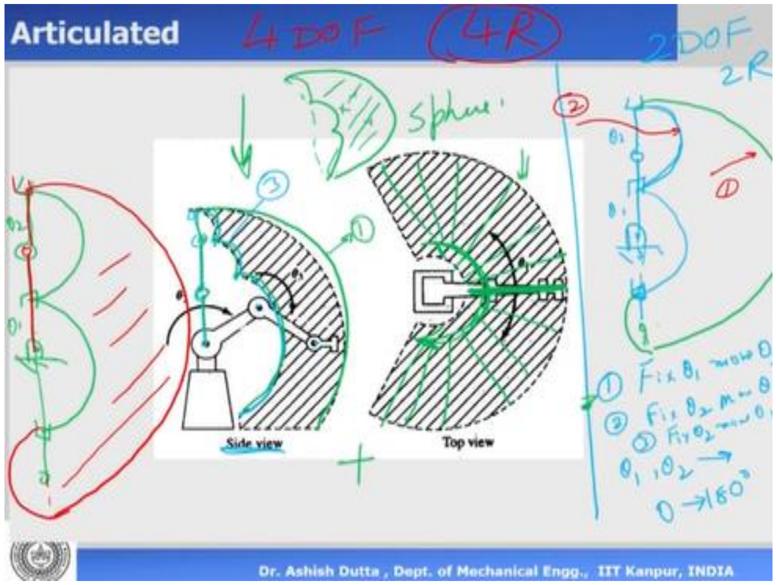

Let us look at an articulated robot. An articulated robot has 3 revolute axis or 4 revolute axes. Let us call it 4 DOF, 4 R axis. Now, let us see how we are going to draw this? Now, for simplicity, let me assume that we are drawing the case for a 2 degree of freedom system just for simplicity before we go so 2 DOF, 2 R manipulator. So, this is 2 R planar manipulator this is planar. So, this is one link, this is another link. This is my joint θ_1 this is my joint θ_2.

Now, let us say that θ_1 and θ_2 can move only 0 to 180^0. Just for assumption say it can move only 180^0. So, how do we start drawing the work volume? Please note this first is fix θ_1 move θ_2 by 180. So, I move this θ_2 by 180 it comes here. What did I get? I got 1 circle. Then point 2 what do I write fix θ_2, move θ_1 by 180. So, this one moves now so, it is going to go like this. So, what has happened?

This fellow has come on the back side like this now. So, this is the back side of this. So, it is sitting like this now. I hope you can imagine what we have done. So, we have two circles now. Now, on the outer circle we have fix θ_2 and move θ_1. So, what do we get? From the fully stretched condition I am going to fix θ_1. So, it is fully stretched like this. Now, I am going to move θ_1.

So, I fix this one what I get is this outer circle. So, this is the shape of the work volume. So, what you can see is that I have 2 circles inside and 1 circle outside because that is the shape of the work volume sorry, this will come on this side. That is the shape of the work volume. Now, some things we have seen. The first thing we have seen is that there are 2 circles inside and there is one circle outside for a 2 degree of freedom system.

Let me maybe, explain it one more time. So, we understand that this is a planar manipulator. So, this is planar 2 degrees of freedom. One link here, one link here. Now, this is my θ_1 that is my θ_2. So, first what do we do? We fix θ_1 and we move θ_2 by 180^0. So, it comes like this so, I got 1 circle. Next, I fix θ_2 and I move θ_1. So, what will happen is? This one will move down like this. So, it has become like this now. So, this is stage 2.

Third is what I do is I take position number 1 again θ_1 and θ_2 are here. So, third let me draw it with some other colour I start off by saying that this is here and this is there. Then, I move like this and this has come backward now. Now, if I open it what will happen? This will come here. So, this is the shape of the work volume. So, what you see is that there are 2 circles inside and there is 1 circle outside.

So, if I follow the same procedure for this 4 degree of freedom system let us start off with the side view. So, let us look at the side view. In the side view, we have 3 degrees of freedom. There is 1 degree of freedom here, here and here the planar. So, I start with the fully stretched condition like this, like this and like this fully stretched. Then, I move the first one, then I move the second one, then I move the third one it comes down like this. So, how many curves did I get?

I got 3 curves and then I come back to this position again. So, stretch like this, stretch like this, stretch like this and then I move all of it like this. So, I move only θ_1 keeping θ_2 and θ_3 fixed. In this particular case, it is θ_2, θ_3 and θ_4 I keep fixed and it moves like this. So, how many did I get circles there? 1 circle here and 3 circles inside. Now, if I look at this from the top view you can see this funny kind of shape there are 3 curves inside.

And there is one outer circle outside. So, when I see from this on the top what I see is that it will be a straight line. We are going to look at this profile this is a planar area. So, when we see from the top it will appear as a straight line. So, this is my straight line. Now, the degree of freedom 1, the first one is a revolute axis. So, it can rotate the full axis from this side to that side. So, the straight line can move like this.

Something similar to the previous one in the case of the cylindrical robot. So, now, if I combine the side view and the top view then what will be the shape of the work volume? So, the shape of the work volume will be something like it will be a spherical part with shapes which are cut inside. It will be a funny spherical kind of shape. It will be a sphere with sections which are cut inside because of this multiple so like this, like this and like that.

So, that will be my sphere lower volume. So, everywhere inside the sphere it can go, anywhere outside it cannot go. So, the human work volume is something similar to this because we have 7 degrees of freedom. In our shoulder we have 3, in the elbow we have 2 and wrist we have 2. So, our work volume is also spherical. So, if you rotate your hands from one end to the other end and from the top to the bottom what is the shape of the volume that you are going to carve out in space?

So, it will be spherical so, it is very similar to this. So, I would suggest that please do it yourself. Draw this by yourself draw the 2 degree of freedom first. Then, draw this 3 or 4 degree of freedom. And, it will become clear how did the shape come. So, we have seen that, depending on the kind of joints that we are using, the shape of the work volume is going to be different. So, this is a Cartesian robot. This is a cylindrical robot. This is an actuated robot. We have other kinds of robots.

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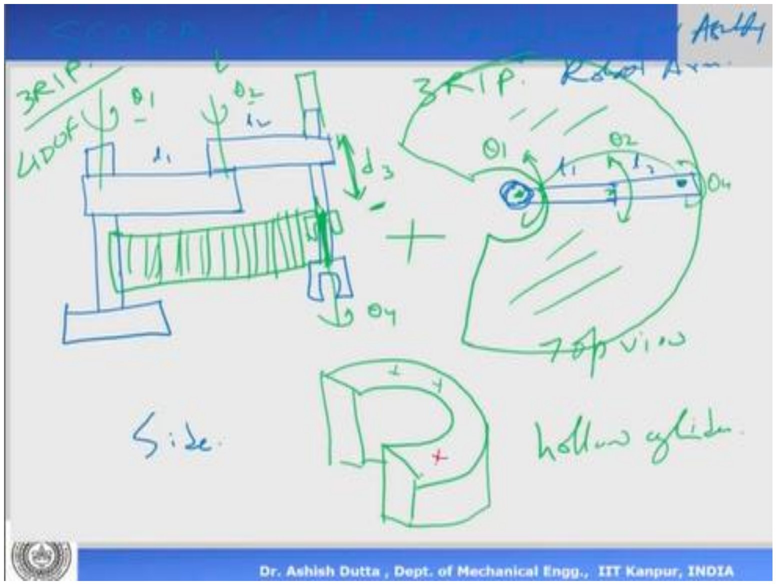

For example, we can talk about SCARA robot. So, I will very quickly draw a SCARA robot. And then, we will discuss so, this is another kind of robot which is very commonly used, it is called SCARA. So, it stands for selective compliance for assembly robot arm. Now, this SCARA robot basically has degrees of freedom like this. So, this is one if you can imagine this is one link it is fixed to the ground here. There is another link sitting on top like this.

And there is a prismatic link here. So, this is prismatic. Now, this is the side view. Now, in the top view we have one here, one here, another one sitting on top which is here. This is the top view. So, this gripper will be there. So, this is my l_1, this is my l_2. Now, what are the degrees of freedom? So, SCARA has 4 degrees of freedom and it has 3 revolute, 1 prismatic. So, the revolute joints are here, here. And there is 1 revolute joint here for rotation of the axis.

And there is a prismatic joint here which is moving which is my d_3. So, this θ_1, θ_2 and θ_4 , d_3 and θ_4. Now when, I look at it from the top view from here, what you get is the top view. And the top view is like this. So, this is my θ_1, this is my θ_2. And d $_3$ is perpendicular to the plane. So, you cannot see it and similarly, θ_4 is down there. So, this θ_4 at the gripper. So, if you look at this top view it appears like a 2 link manipulator only.

So, what you will be getting is, the work volume will be, one outer volume like this. So, one outer shape like this because it is a 2 link manipulator and this will bend like this till here maybe.

And there will be another cut out section like this. Is it okay? So, this is looking like this now from the top view and this full area, there is the work area. Now, when I look from the side view what we can see is this can move up and down.

That means this end effector can go up and down. So, this can come till the top here. So, it can come to the top this much it can go on top this can come on top this is a straight line. Now, the straight line can be swept when θ_1 and θ_2 moves. What will happen the θ_1? This will come like this. So, what you get? An area like this. It is a rectangle because the straight line when you move θ_1 and θ_2 if you can imagine this straight line can go to different positions like this.

Now if I combine this plus that what will we get? We will again get a cylindrical kind of shape. A hollow cylinder. Again, we get this kind of a shape. So, we get a whole cylinder. So, what we do get is? This is called the selective compliance assembly robot arm. And this is very, very commonly used in electronic assembly. This is used because there is a prismatic axis there and this prismatic axis is the one that moves in a straight line. So, it helps in assembly.

All electronic assembly other kinds of assembly is done using this kind of robot. And this has 3 R and 1 P joint which has 4 degrees of freedom. Please draw it by yourself and try to understand where are the degrees of freedom and how did we get this shape of work volume? And again, any point inside this is okay. The end effector can go there any point outside there the end effector cannot go and it is out of the work volume.

So, this is very, very important to understand this concept of work volume because if your task is outside the value. For example, you are trying to do assembly here which is outside or you are trying to pick up this object and put it there. Now, this is outside the work volume. Obviously, the robot cannot catch it. This is outside the work volume. So, the task always has to be inside the work volume. Whatever I am doing I want to pick a place from here to there.

This has to be inside the work volume it cannot be outside. That is why the shape of the work volume is very, very important. Now, please try to think out of this which one would you prefer. Which shape would you prefer and which type of joint would you use? For example, this one is a

Cartesian robot. This one is a cylindrical robot. This is an articulated robot and the other one was a SCARA robot and they all have different volumes.

So, which one would you prefer? Because if you are buying a robot or you are designing a robot you have to decide which volume, do you need. It is an interesting question? Please think about it.

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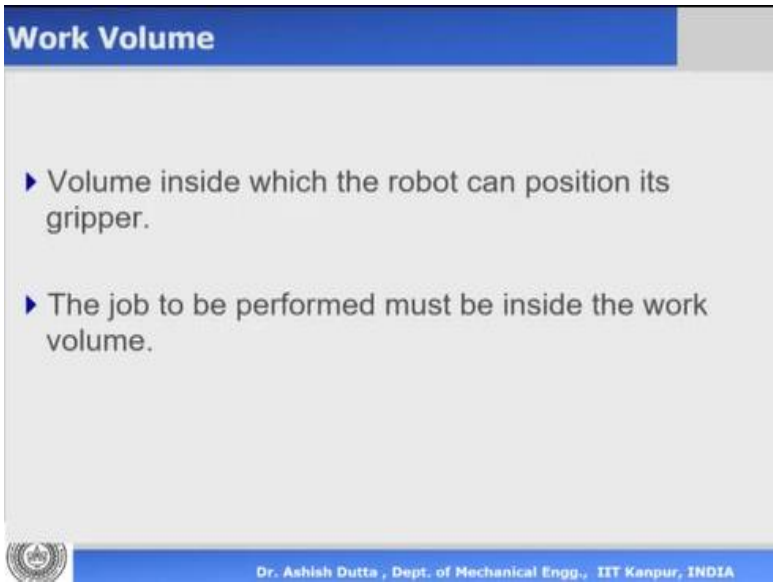

Let us move on further. So, we have said that the work volume is the volume inside which the robot can position its end effector with a gripper or end effector. Now, the job to be performed must be inside the work volume. Of course, you are trying to do a task outside the work volume you cannot do because the base of the robot is fixed. So, it cannot move.

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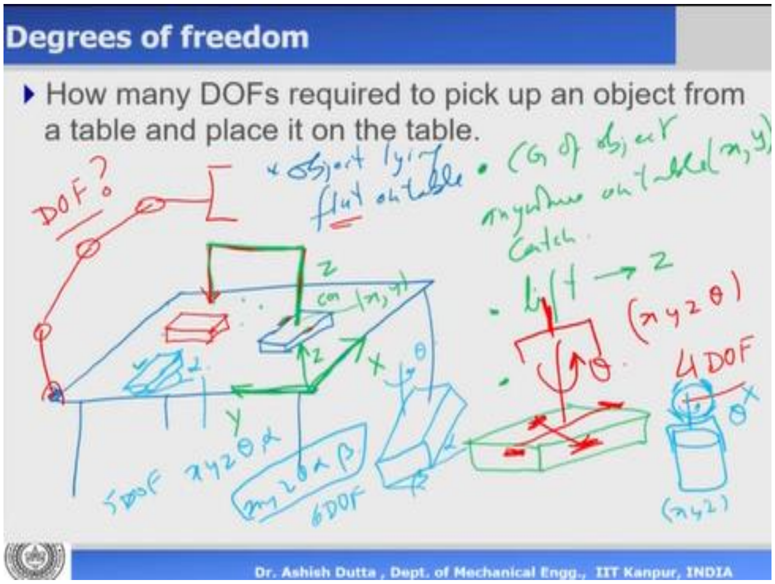

Now, let us move on further and try to answer this question about degrees of freedom required. Now, suppose you want to make a robot or you want to buy a robot, serial arm. We will talk about mobile robots later. Now, you want to pick up an object from a table and put it somewhere else. So, this is a table and a robot has been placed there. The robot has to pick up this object. So, there is an object which is lying there on the table object lying flat on table.

So, I want to pick up this object I want to pick up this object and I want to place it somewhere else. So, this one object is the table I want to pick it up and place it somewhere else. Pick it and place it like this. I placed it like this now on the table flat on the table. Please note that it is flat on the table. So, I want to design a robot. Let us say there is a robot here. It has some degrees of freedom. It has to go and catch this and pick it up.

The question is how many degrees of freedom are required? I hope the question is clear. I have placed an object on the table, flat on the table. You can imagine this is your mobile phone maybe. It is a rectangular object, cuboidal object. I want to pick up the mobile phone by catching it here. So, I catch it here and I catch it here, pick it up and maybe, I will move like this, like this, like this, like this, like this, like this. That is my motion profile. Catch it, go up, go sideways and put it down.

How many degrees of freedom are required? How do you answer this question? Now, how do we answer the question is that the first thing we need to do is we need to analyse the task and before doing that we put our axis. Let us say I have my x axis here I have my y axis there and I have my z axis here. So, x y z are the three axes. Now, the object CG, let us say this is the CG. It could be anywhere on the x y table. So, CG of object anywhere on table.

What is the meaning of this English language? So, anywhere on table means the object CG has a coordinate x y. let us say it is on the ground. Let us ignore the z now let us say it has coordinate x and y which is lying on the table. So, that means this fellow has two parameters x and y because the object could be anywhere on the table. This x y could be anywhere. So, it is all different x y. so, x and y are both variables. So, I need to catch the object somewhere near the CG.

And I need to lift the object so this is the first thing catch there. Number 2 is lift the object. So, if I lift the object, what it means? That I go upwards. So, this is my z now. So, lift the object means z. So, now I have x y and z required. Now, I am moving this side. This is again x y. Then, I am moving down this is my z and I am keeping the object. So, how many degrees of freedom are required? You might think it is 3 but it is not 3 it is 4.

Why is it four? Because at the; end effector you are catching the object here and here. So, I have an object. So, this object has been caught by the gripper. So, the gripper is catching either here or is catching that side so, it has to rotate. So, it needs this degree of freedom θ at the end effector. So, this is my gripper. So, I need to rotate the gripper about that θ axis in order to be able to catch it along different diagonals. This is one that is the other one.

So, I need x y z and I also need the θ. This is 4 degree of freedom. So, this indicates that I need 4 degrees of freedom to perform this task that is minimum. So, I need 4 degrees of freedom to specifically perform this task of lifting an object placed on the table and then, placing it somewhere else. This gives you an idea of degrees of freedom that the minimum is 4. Now, suppose I want to place an object at an angle to the table. My object was kept like this.

Let me change the colour and put some other object. So, my object had an angle here. So, this is my object. You can imagine the same object. But the object has an angle there with the table. I hope you can imagine that. Let us see it I have already used. Let us call it α. Now, in this case, if you want to catch it like this and like this you will need x y z θ and α for placing it at an angle.

Now, suppose I have 2 angles then it will become α + β, x y z θ α β. Suppose the object had another angle. So, it was like this, let me say the object is like this. So, there is an angle β here and there is an angle α there now. Now, it has α, it has β also plus it has a rotation angle θ. So, in this particular case, it is 6 degrees of freedom. So, I hope you understand the difference between 4 degrees of freedom catching and placing object which has 4 degrees of freedom 5 degrees of freedom and 6 degrees of freedom.

This is 5 degree. Now, you should be able to correlate what we said in the beginning that an object has 6 degrees of freedom in free space. It has 3 translation axis which is x y z and it has 3 angles which α, β, γ. These are also can be called as direction cosines. So, this basically shows us that if you want to catch an object and place at any angle on the table then you need 6 degrees of freedom.

And that is why in general we say a general robot requires 6 degrees of freedom. So, please again try to understand this carefully by placing different objects. We look at another interesting concept suppose the object is a cylinder, now the cylinder, the top view is a circle. So, in a circle you can catch at any angle in any rotation angle. So, in this particular case, you do not need that θ. You can catch using the gripper at any angle. So, this θ can be anything.

So, in that case, x y z is enough. So, when you are catching a symmetric object like a sphere or you are catching a cylinder then you do not need to have 4 degrees of freedom. You can do with 3 also because the symmetric object. There is no need for the θ rotation at the top.

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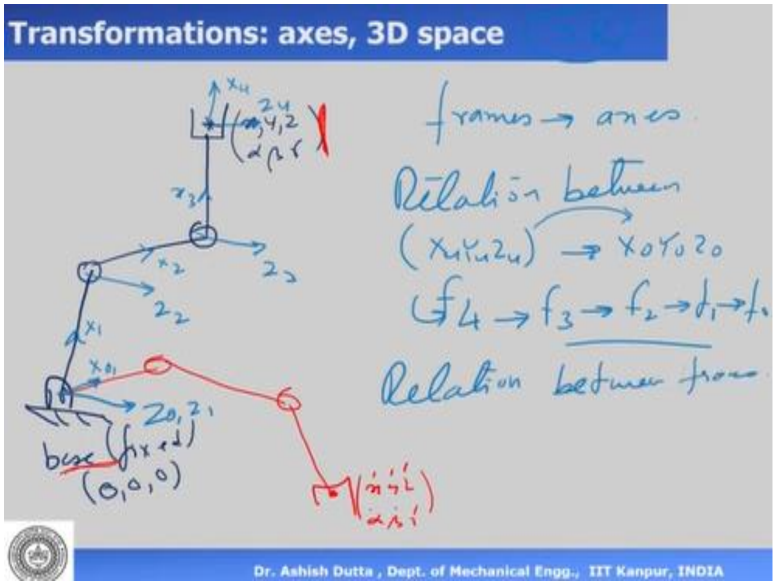

So, let us move on transformation axes and 3D space. So, we have seen that when we are looking at the study of robotics. A robot essentially has a base which is fixed. So, we have the base of the robot which is fixed. So, I have the base which is fixed and then, I have linkages like this, like this, like that. So, this has 3 joints θ_1, θ_2, θ_3. Now, this one is base. It is fixed, does not move. So, I can always assign 0, 0, 0 there because it is not moving.

In the case of mobile robot, I need to position a global 0 0 0. In the case of this kind of robot, I can write 0 0 0 and that is my base, fixed base of the robot which is not moving. We have the end effector and suppose, this is x y z and it has also other 3 parameters α, β, γ that means the position of this point is x y z and the orientation of the end factor is α, β and γ. Now, I want to find the position of x y z because it is moving in space.

So, when it is moving, I need to find its relation with respect to a fixed frame. Suppose the joint is moving and the joint has moved and it has come there just come here. In that case, what we can say is I need to find the relation of x y z which has come here x y z as α', β', γ' in the second position. Now, as it is moving in space, I need to find the relation of a frame or this point with respect to the base frame always. Why? Because the base is not moving.

Now, in order to find the relation of this point with the base I cannot go from there to there in the base in one shot. So, what we do is we assign frames. We assign frames or what we call axis. So, I assign an axis here and call it Z_0 and this is my Z_0, Z_1, this is Z_2, Z_3. This example is 3 3 3 R joints, 3 revolute joints. Then, I can assign my x axis. This my x-axis x, x_0, x_1 is here, x_2 is there, x_3 is there. Now, I can assign1 more axis here. Let us call this Z_4 and this is my x_4.

Now, what we do is we assign the relationship between frames. So, if I want to find the relation between frames X_4, Y_4, Z_4 and X_0, Y_0, Z_0. Then, I cannot come from here to here in one shot. So, what I do? I go from frame 4 to frame. So, I go from frame 4 to frame 3 then from frame 3 to frame 2 then from frame 3 to frame 1 and then to frame 0. So, this basically means that I need to find the relation between frames. So, I am going to find the relation between frames.

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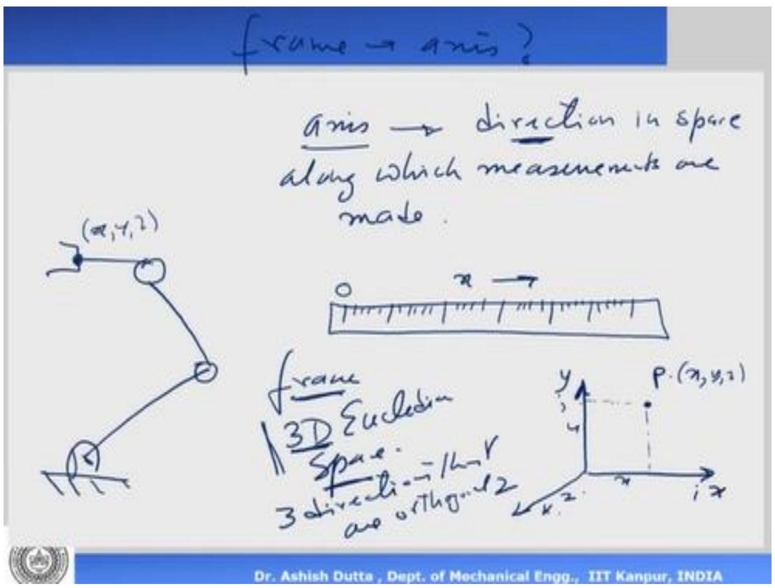

Now, let us again go to the next page when we are looking at that robot I just drew. This and let us say this one. Now, I need to find the corner of that point x y, is it okay? So, the first thing that we need to do is assign frames and then, I can find the relation between the frames. So, if I want to assign a frame over an axis the first question that comes up is what is an axis then? In the previous example, I simply wrote x y z I did not write y but x.

You can write y that provide this side I wrote x y z and assigned frames. We always do x y z but what is this x y z and what is the frame or what is an axis? So, an axis is a direction in space. So, it is a direction in space along which measurements are made. So, axis is a direction in space along which measurements are made. So, if you say x axis then the measurements are made along an x axis.

For example, your scale when we say this is your scale if you are used to the scale. This is your scale by which you measure distances. So, if I say this is my x axis then I am measuring along the scale and whatever is the distance if this is my 0 then that is my axis. This is my x axis. Now, if I add one more then I can get y axis and if I add one more I can get z axis. So, axis is a direction in space along which measurements are made. What is the direction?

Why is x pointing this side? Can it be pointing that side? Why is x in this direction or in this direction? It can point any direction, actually, it does not matter because it is only a direction in space. Now, when we say a frame then we basically mean we are talking about 3D Euclidean space and in 3D Euclidean space it is 3D. It is saying it is 3 dimensions. So, there are three directions possible. So, what we do in 3 D Euclidean space?

We define 3 directions x y and z and a few properties of this directions are that they are orthogonal. So, frame in Euclidean space, three directions that are orthogonal. So, we can also call them i, j and k. So, when we are saying the coordinate of a point what we basically mean that this is a point P. I am drawing a perpendicular of this point on the various axis. You know what is an axis? It is a direction in space.

How many axes are here? 3, why 3? Because it is 3D equally in space and this the axis are orthogonal. So, I drop perpendiculars onto the 3 axes. And I measure the distance I write x y and z this is x, that is y and on z direction will be z. So, basically, we see what is the meaning of orthogonal axis? And we also understand why there are 3 axes. What is this x y z?

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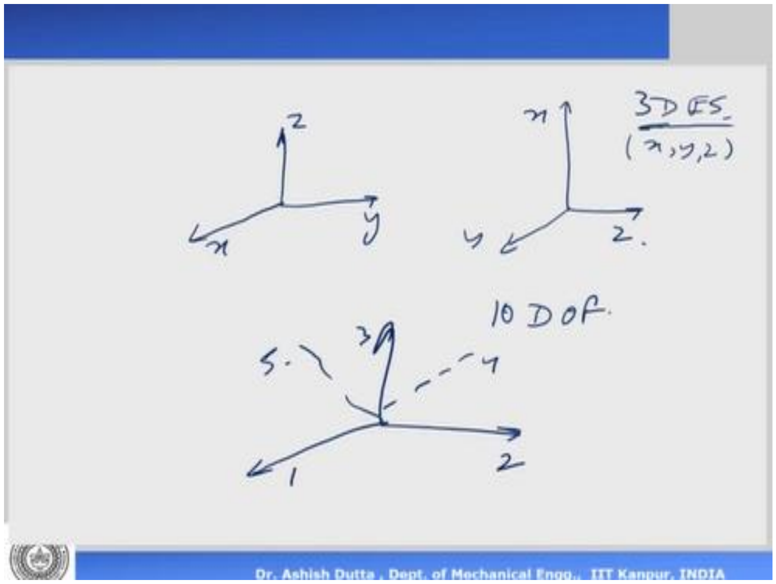

Now obviously, I can take my x y z in any direction. So, my x could be this way, y could be that way and z could be that way it is still my x could be this side. My y is this side, x y, and z is that side, this is also. That is for right hand rule. So, just for convenience, we normally put the x y z as per our convenience. Something to remember is that this is 3D equivalent space and because it is 3 D there are 3 axis x y and z.

Can you have 4 axis or 5 axes? Can you have a high dimension space? Yes, when we move to homogeneous space you will see that we will talk about 4 dimensional space. And can you have even higher than that? Yes, you can. But can you draw it? Well, there is a problem, why? Because we; are living in a 3Dimensional world. So, if you want to draw higher number of axes you will not be able to. Now suppose, I want to draw I have 10 DOF space.

So, I can make some assumption like this X_1 first dimension, second dimension, and third dimension and the other dimensions are coming in between. So, this is my fourth dimension. This is my fifth dimension. So, I can do that now. Let us go back to what we were talking. So, what we have seen is that we are assigning, frames that means fixing different axis at the joints and finding the relation between them.

Now, what is the relation between the 2 frames here? Let us say frame 3 and frame 4 and frame 3 and frame 3 and frame 2, frame 2 and frame 1. What is the relation between them? Now, if I think about that I try to imagine how did the last frame go there? Suppose, I have a frame which is here I can say that my Z_0 was here and Z_1 is here X_0 is here and X_1 is here, that is Y_0. So, these 2 have the origin to be the same point. Now, I have another frame there which is Z_2.

This is my X_2 and I have my x y, y is there. We do not draw the y. So, X_2, Y_2 and Z_2. Now, what is the relation between the frame 2 and frame 1? How did frame 2 go there? We can assume that the frame 2 and frame 1 were initially coincident at the origin. This is my 0 0 0 then frame 2 has translated by this distance there. And, it has also rotated you can see that X_1 and X_2 there is an angle there. So, there is a translation between the origins.

And there is a rotation between the x-axis. So, there are two transformations which are involved here. One is a translation the other is a rotation. Let me explain this in a bit more detail.

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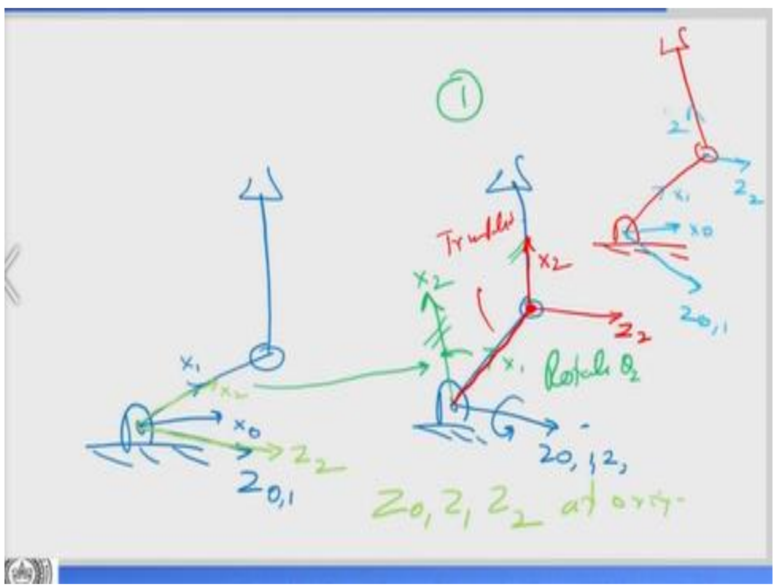

Let me draw in terms of robot manipulator. So, initially we say that we have Z_0, z-axis. Let us call it Z_0 and Z_1 here. And, there is another axis. This is X_0; this is X_1. Now, let me say initially that there is another axis y. now, the y axis sorry there is axis 2 so Z_2 is also here. And X_2 is that side, the green colour. So, we have Z_0, Z_1, Z_2 all the same point at origin. Next, what we say is that I rotate. Let me draw the robot again. So, this is what I was drawing.

So, what I do? My Z_0 and Z_1 remain but to my X_2 what I do is I rotate it about this axis so that Z_2 is also here. I rotate it and my X_2 comes this side. So, I am rotating it by an angle θ and it is now aligned. It is perpendicular to this plane; it is perpendicular to that axis. So, that is my X_2. So, X_2 becomes get rotated like this and comes here. So, it has rotated by that much. So, my X_2 axis is this side now. And X_1 axis is here.

So, what did I do? I rotated so; I rotated this angle. I rotate it by what angle by θ_2 by angle θ, let us say. And then what I do? So, this is my first transformation rotation. Then, what I do is I translate the origin of 2 here, so Z_2 has gone there, Z_2 has gone there and my X_2 is this side, now. Now, what did I do here? I translated. Now, if I draw this robot again what will it look like? It will look like this. So, I have X_0, Z_1 here, Z_2 has gone there.

This is my X_0; this is my X_1 and this is my X_2. I hope you understood how X_2 got there? So, initially they were coincident the origins were same of 0, 1 and 2. Then, I rotated the second frame wide angle θ and I translated it by distance. Let us say l 1 along that x along the link. So, the origin has gone from here to there. And that is doing that I am going to go from link to link or frame to frame so, in the previous case, where I drew all the frames.

So, from this frame I went there, there and then there. So, I can find the relation between this frame to this frame then this frame to this frame this frame to this frame. I can find that in that order so the end effector frame. I can always find with reference to the base frame by coming from the end effector frame to the previous frame then to the previous frame into the previous frame.

Because at most, there is a rotation involved between frames and number 2 there is a translation involved between the frames. So, if this rotation and translations were not involved, sorry, so, we have only 2 transformations involved rotations and translations. And this can give you the relation between any frames to the base frame in 3D space. So, basically, what we do? We find the rotation transformation. And then, we find the translation transformation.

And then, we go from one frame to another frame. And by doing this, we can find the relation between the end effector frame. The position and orientation of the end effector with reference to the base frame. So, this is basically to explain about axis and to explain about how do we go from one frame to another frame. And what are the various transformations involved? So, we looked at the rotation transformation and the translation transformation.

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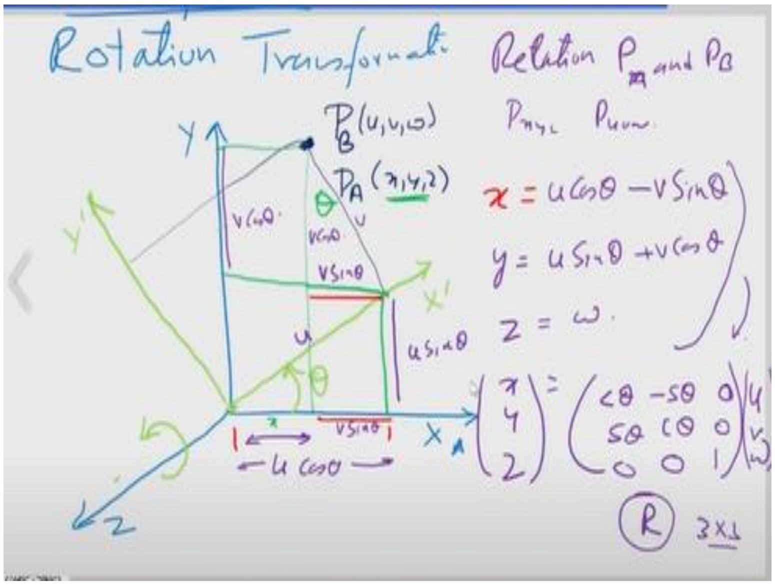

So, the first one that we will see the first transformation that we will see is the rotation transformation. So, listen very carefully to this. It is very, very easy. Once you understand it you have to do it by yourself and you do not have to remember anything. It is so simple once you understand how it is done. So, initially, I have 2 frames. This is, let us say, my frame which is fixed. I call it the A frame and there is another frame which I call the B frame.

So, this is my X, Y, Z sorry this is my X, Y and Z frame. Then, I have another frame which is called the B frame. So, initially, B frame and A frame are the same frame. Same frame means the origins are same. Then, what I do? I rotate the B frame by an angle θ. I rotate the B frame like this by an angle θ about the z axis. So, this is my θ. So, my X has come there, X' to Y' has come there, Z remains same because it is not moving.

Now, I am saying that if I am given the co-ordinate of a point here P, u, v, w P_B in frame B and P_A the point. So, this is the point P in frame A as x y z. What is the relation between u, v, w and x, y, z? What is the meaning of u, v, w? u, v, w is the projection of this point on frame, this frame here and here. What is x, y, z? x, y, z is the projection of the point. So, is this projection on the x-axis and the y axis? So, this is my x, that is the meaning of x, y, z.

So, this is the point P is the same point, please note. But because it is expressed in 2 different frames the coordinates will be different. Now, I want to find what is the relation between these 2. So, I make this very simple construction. I make this construction and this is my angle θ. So, what you can see is that this distance let me draw it in red colour. This distance is equal to this distance. So, this full amount is equal to what? So, x is this distance. How much is x?

So, this is my x. So, x is equal to and how much is this? This is u. So, $x = u\cos\theta - v\sin\theta$. So, what is u cos θ? u cos θ is this distance. This full distance is u cos θ. And, what is v sine θ? What is v? This is my v because this is θ. So, this is v sin θ. So, you can see the total distance $x = u\cos\theta - v\sin\theta$ that is the red one. What is y? $y = u\sin\theta + v\cos\theta$.

What is z? z = w. So, what is the relation between P(x y z) P_A and P_B. That is what we are trying to find or P (x, y, z) and P(u, v, w). So, you can see geometrically we have found the relation between x, y and z in terms of u, v, w. So, this is very, very simple. Now, if I write it in matrix form, I can write this as, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix}$

And this side is u, v, w. So, from this set of equations, I am getting this. This is called my rotation matrix. It is a 3 by 3 matrix. So, I would suggest that you write this down. You derive this matrix geometrically the way I have done it here. And once you do it you never forget for the rest of your life. So, it is extremely important because this is the very basis of robotics now. So, we need to know what is the rotation matrix.

What is the translational homogeneous transformation matrix? But we really need to know how this is done and how to derive it. Now, I would suggest that please look at, please draw what I have drawn. We will start from here in the next class and we will proceed from here in the next class, look at various properties of the homogeneous of the rotation matrix. But you should be able to draw it by yourself and derive it. There is nothing to by heart here.

Once you derive it you will never forget it for the rest of your life. So, let us stop here today. We will stop and thank you.