Robot Motion Planning
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Lecture – 08
C Obstacle

Hello and welcome to this Lecture number 8 of the course Robot Motion Planning. In the last class, we had looked at configuration space. And today, we will move on from there and look at C-space obstacles. That is when we have obstacles of different shapes then how do we construct the corresponding C space. Then, we will move on to the C space for serial armed robots. Very quickly revising what we had done in the last class.

We were talking about C space. What is C space? And then, we will move on to C-space obstacles.

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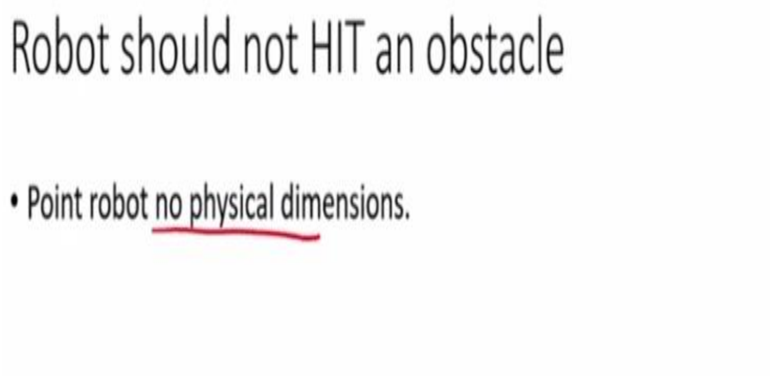

So, in the last class, we saw that when we talk about a mobile robot we essentially meet we mean that a mobile robot should not hit an obstacle when it finds the path. So, and, to do that what we saw the various examples that we looked at. The robot was a point robot. It did not have any physical dimension, for example, in the case of the bug algorithm. Now, real robots have physical dimensions which means that they are not a point.

Now, so, the question is how to specify all the points in a robot. And, that is what we are trying to look at and because to do that we go to the configuration space.

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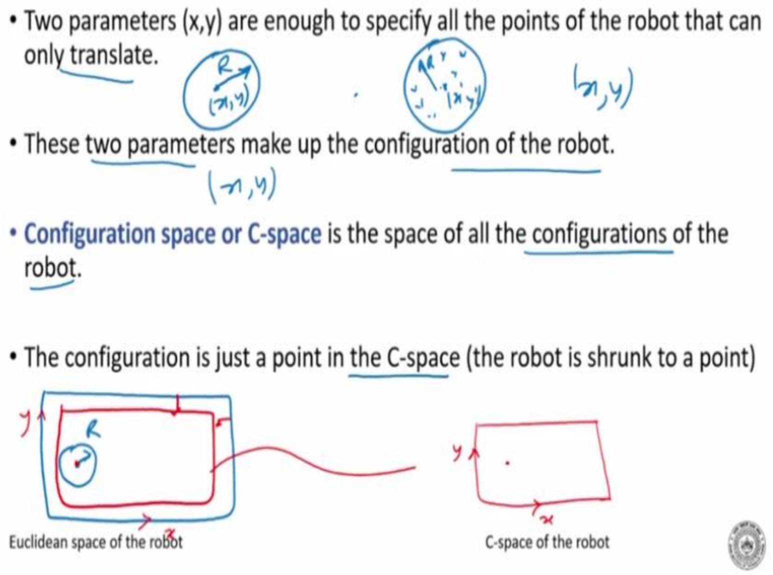

Now, we see that 2 parameters are enough to specify all the points of robot that can only translate. For example, if I have a robot like this and I know its radius I know its center x and y and I do not know the radius R then it automatically means that if I know where is x y and I know what is R. So, I basically know that if the point is here $x' y'$ then I know all the points inside the robot because I know the radius R.

So, indirectly, I know all the points in the robot. So, basically just knowing x and y is enough to find out where all the points of the robot align. And, these 2 parameters x and y in this case, they make up the configuration of the robot. This is for a mobile robot. Please remember, this is a mobile robot that can translate. Now, the configuration space or C space is the space of all the configurations of the robot. What is the configuration of the robot?

The configuration of the robot is x and y because knowing x and y we know where are the location of the other points in the robots. So, the configuration is just a point in the C space. So, for example, if you have a workspace like this and you have a mobile robot which has a radius of R, then, how do we make the C space? We basically shrink the robot to a point. So, the robot becomes a point here. And, this comes inside by R.

So, this becomes smaller by a distance R and that is my C space now. So, this space becomes my C space. And, the robot becomes a point in the C space. Now, this looks like the Cartesian space. For example, this is x and y. Similarly, this is x and y. So, they look similar but they are different spaces. So, the path planning is essentially we are trying to find the path for a point robot now because the robot has shrunk to a point and the workspace has become smaller.

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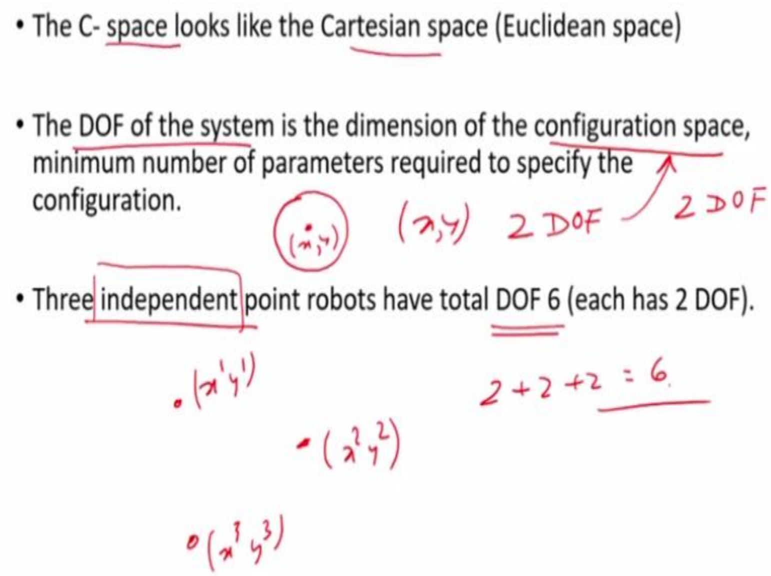

Now, the configuration at the C space looks like Cartesian space but it is not the same. Please remember that. Now, what is the degree of freedom of a system? And, how is it related to the configuration space? So, the degree of freedom of the system is the dimension of the configuration space. In the previous case, we saw in the case of the mobile robot. We had x and y and we said that x and y is enough to specify the configuration of the system.

So, in this particular case, this is 2 degree of freedom for the robot which has 2 degrees of freedom because it can only translate. And, the configuration space also has 2 degrees of freedom. So, the robot and the dimension of the configuration space is 2 which is x and y. So, if you have a point robot its configuration will have a dimension of 2. So, if I have 3 independent point robots, for example, we talk about swarm robots and multi-agent systems.

So, I have 3 robots here now. This is x_1 y_1. This is x_2 y_2. This is x_3 y_3. So, these are the configurations of these 3 robots. Now, they are independent means they are not connected in any way. So, what is the total configuration? What is the total degrees of freedom of the system? Now, it is 2 + 2+ 2 = 6. So, please note that these are 3 points which are independent, each of them having 2 degrees of freedom. So, the total degree of freedom of the system is 6. They are not connected in any way.

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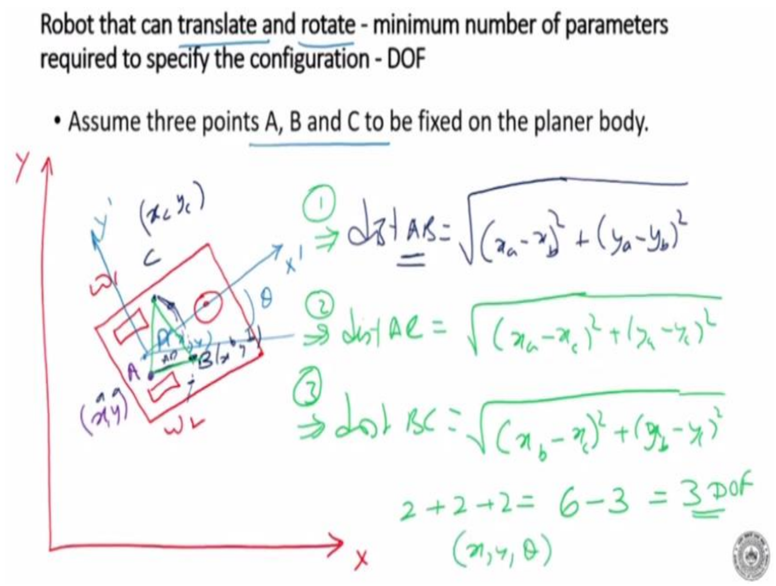

Now, a robot that can translate and rotate minimum number of parameters required to specify the configuration. In the previous case, we said that it can only translate. It cannot rotate. But, if you have a robot which can translate and rotate, then, how many degrees of freedom are required? So, how do we, let me draw such a robot. Now, this is a robot here. And, we have 2 wheels. And, let us say it has a caster wheel. There is a caster wheel there.

So, this is ω_1 ω_2. Now, we need a global frame. This is my x axis. That is my y axis Now, in between the 2 wheels, let us take a point P which has coordinate x y. Now, I put a local frame here which is in this direction x' and that is my y'. And, this is my angle θ. Now, what is the degree of freedom of this system now? Now, this robot can translate and it can also rotate. So, what is the degrees of freedom of this system?

How do you define that? Now, let us say that how many parameters are required to fix the position and orientation of this system of this mobile robot. Now, let us say we have 3 points. Let us assume these 3 points, A, B and C. So, the first point A can be placed anywhere on this robot. So, I can place my point A here for example. This is my point A. How many degrees of freedom it has? It has 2, x and y. So, it could be anywhere on this body of this robot.

Now, next, I want to place the point B. Now, the point B is let me say here. So, this distance is my, this is my point B. So, this is x_a y_a. This is x_b and y_b. Now, the distance between A and B is fixed. So, the distance AB is fixed. And hence, $\text{distAB} = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$. That is the distance between A and B. So, where could this point B lie? It can lie on a radius which is having a distance AB.

So, anywhere on this radius, this point can lie. So, I have fixed my point A which could lie anywhere. Now, the point B must have a just fixed distance from point A which is given by this distance. But, that is not enough to this point B could lie anywhere on the radius here on this circular radius. B could lie here. B could lie here. B could lie here. And, the distance would still be the same.

Now, to fix that that basically means that I need another point. I need the point C. So, I have got my point C. So, this is x_c and y_c. Now, if I say that the distance A to B is this distance, the distance B to C and the distance A to C now gets fixed. The point B, the moment I say that the distance AB is a constant distance and the point B could be anywhere on the circle. Now, the moment I put the third point. That is C.

And, I say the distance between AB, AC gets fixed then the orientation of the body also gets fixed. So, the position has got fixed and the orientation of the body has also got fixed. If I have only 2 points A and B the orientation is not fixed. Why? Because this point B could be anywhere. So, it could be rotating. But, the moment I put my point C, now, the position and orientation of this body gets fixed. Now, what about the distance?

So, the $\text{dist}AC = \sqrt{(x_a - x_c)^2 + (y_a - y_c)^2}$. And, the $\text{dist}\, BC = \sqrt{(x_b - x_c)^2 + (y_b - y_c)^2}$. So, that means you need 3 points to fix the position and orientation of a body. You can get 2 points to fix the position. But, you cannot fix the orientation because it can still rotate and it can still maintain the distance AB. But, the moment I put the point C.

Now, the orientation also gets fixed. So, this means how many degrees of freedom are there? 2 + 2 + 2 = 6. But, now, we have put these constraints. There is one constraint here, second constraint here and the third constraint here. Now, the moment I put this distance constraint between those 3 points, so, it reduces the degrees of freedom of the system. So, it becomes 6 - 3 constraints which is becoming 3 degrees of freedom.

So, this robot which can rotate and translate has 3 degrees of freedom x, y and θ. Now, something to remember here is that this type of constraints are called holonomic constraints.

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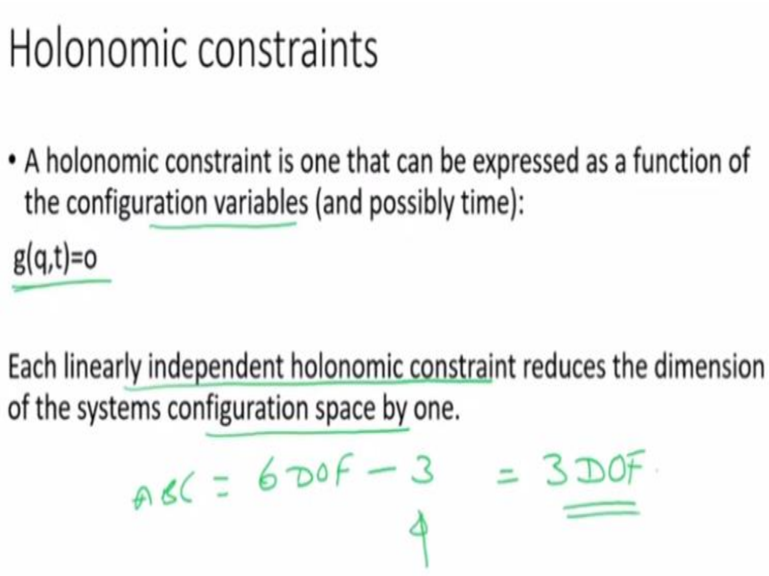

So, a holonomic constraint is one that can be expressed as a function of the configuration variables and possibly time. So, I can write this distance in the form of g(q,t) = 0 or just in terms of g and q which is equal to t. So, this constraints, this distance constraints are called holonomic constraints. And, each linearly independent holonomic constraint reduces the dimension of the system configuration space by 1.

That means in the previous case, we had 3 points A, B, C which had 6 degrees of freedom if they were independent, remember. But, we have 3 constraints now, those 3 distance constraints. So, my degree of freedom of the system has become 3 DOF. So, this type of constraints which are distance constraints are called holonomic constraints. And, they will reduce the dimension of the configuration space by 1.

So, if there are 3 constraints, it will reduce by 3. So, this becomes totally 3 degrees of freedom.

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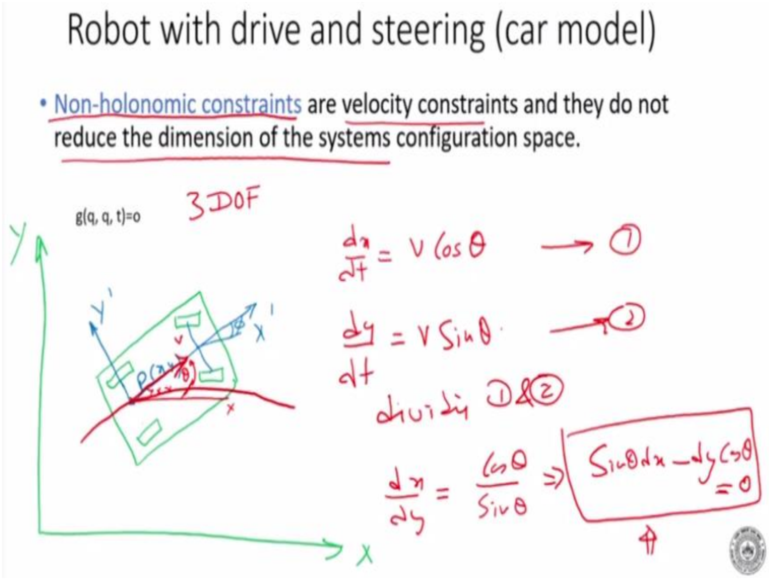

We have another kind of constraint which is called a non-holonomic constraints. Now, suppose, we have a car here, let us imagine this car and we have a wheel and just a turning wheels in front. Now, this is my global frame x and that is my y. This remains same. Now, let me say I fix my axis like this. This is the center point. That is the point P which has x y coordinates. So, this is my x axis. This is my y axis, x', y'.

Now, this is, now, there is an angle there. So, there is my axle and there is the angle. So, that is my Φ which is my turning angle. Now, when the car is turning, please note that when the car is turning, this will is turning now. This point is having a curve. But, at every point on the curve, the velocity vector will be in this direction. That is my v. This is θ. θ is the angle here with the x axis. This is my x.

So, at this instant, when the car is turning although the car is turning at that point every point, the velocity vector is going to be tangential to the curve which means that this velocity vector will have 2 components. So, $\frac{dx}{dt} = v\cos\theta$ and $\frac{dy}{dt} = v\sin\theta$. So, this velocity vector will have 2 components, one along the x direction one along the y direction given like this.

So, when I divide if I call this Equation 1 and Equation 2. So, dividing 1 and 2, what we do get is $\frac{dx}{dy} = \frac{\cos\theta}{\sin\theta}$ which implies that $\sin\theta dx - \cos\theta dy = 0$ which means that at every point on the curve the velocity vector has to be tangential at that point which means that we have put in a constraint like this. Now, this is a velocity constraint.

In the previous case, we saw there were position constraints in terms of distance. Here, this is in terms of the velocity now. And, this type of constraints are called non-holonomic constraints. They are velocity constraints. And, they do not reduce the dimension of the system configuration space. So, in this particular case, although, this robot or this mobile robot has 3 degrees of freedom, we have seen.

But, and, it also has a velocity constraint like this in non-holonomic constraints. In terms of velocity, it does not decrease the dimension of the configuration space below 3. So, there are 2 kinds of constraints. We have seen holonomic and non-holonomic constraints. We will come to this in more detail when we come towards the end when we do motion planning with holonomic and non-holonomic constraints.

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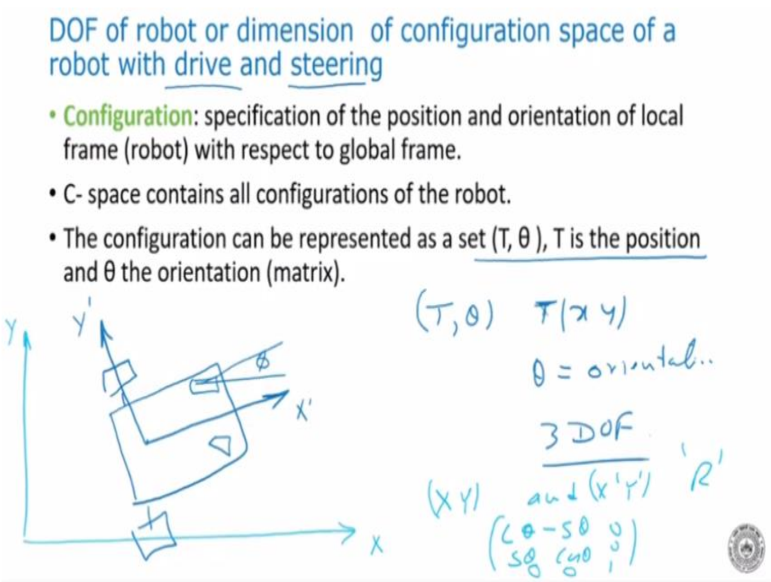

Now, the degree of freedom for robot or dimension of the configuration space of a robot which has drive and steering. Let us have a look at that. So, here, we have seen that the robot which has drive and steering. We just saw that robot. And, it has a steering. So, there is my steering angle here. So, this is my steering angle which is Φ. So, this robot has dimension 3. So, the configuration of this robot is, will be represented by (T, θ) where t is corresponding to x and y and θ is my in this particular case is the orientation.

So, this is my 3 degree of freedom system. Now, although, there is a velocity constraint, it does not decrease the dimension of the configuration space. Now, what is the angle between the global frame and the local frame? That we have seen. We know that there is a rotation matrix there. So, the angle between the global frame, so, if I draw my global frame here, so, my global frame is x and that is my y.

Then, the relation between x y and $x'y'$ is given by the rotation matrix R. You know, what is the rotation matrix? It is the rotation about z. So, it is $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$. So, this is basically to explain the configuration of the mobile robot which has rotation as well as which has got drive and steering.

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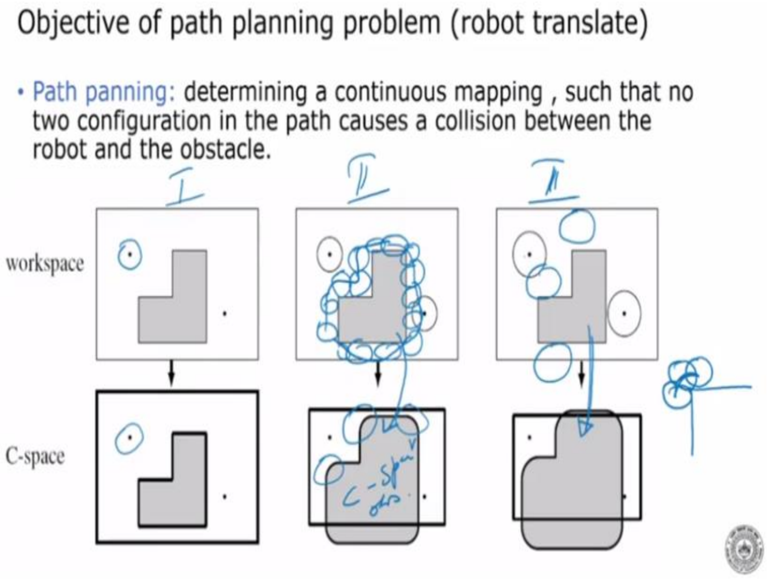

This is something which we did in the last class. How do we draw the configuration space? Today's class is about C-space obstacles. So, we are going to make C-space obstacles for different shapes. In this particular case, we see that because the robot is a point robot the configuration space shape remains the same. In this particular case on the top that is in Case 2, we see that the robot is going to go and sit like this.

And, what happens is the shape of the obstacle becomes enlarged. And, at the corners, I explained the last time that when the robot goes and sits there it is like exactly at that corner point the robot can rotate like that because it is a point contact. So, there is a curve. That is how we get these curves. In the third case, we have seen this robot is slightly larger. So, it is a bigger robot.

So, it goes and sits like this and the shape of the configuration space becomes like this. So, basically, we go and put the robot wherever it can go and sit the maximum it can go. And, we draw the locus of the center of the robot. That is how we get the C space. So, this is my C-space obstacle.

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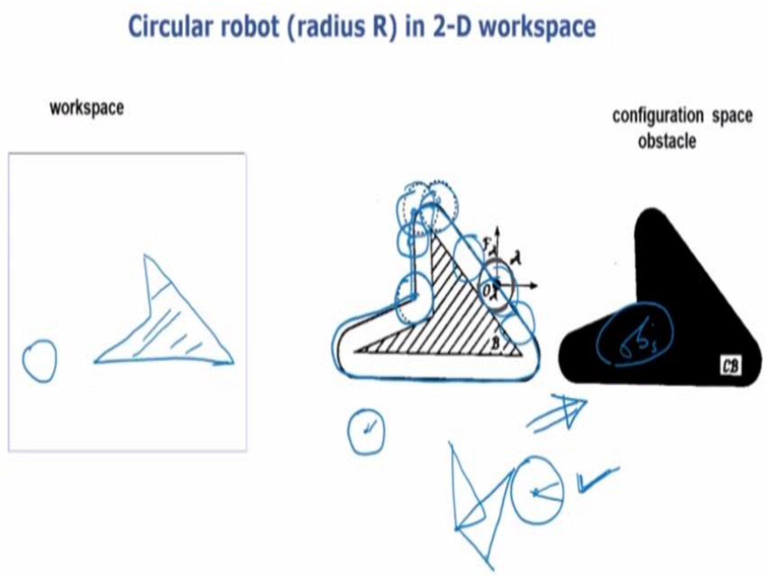

Now, we also looked at the case where we have a mobile robot like this. And, we have an obstacle which is shaped like this. How did we get the shape of the C-space obstacle? You basically imagine the robot to be going and sitting at all these points the maximum position to which it can go and sit. So, it can go here. It can go here. It can go here. And, wherever the center of the robot that is the maximum it can go.

So, we draw the locus of the center points. And, we, our shape of the obstacle becomes like this. It is like the robot is a circular robot. So, it is going and sitting there. And, it can go and hence the obstacle increases to this shape. Now, you can visualize this very clearly. And, this is simplified. Why this simplified? For the very simple reason that it was a circle and the circle is shrinking to a point. So, it is like there is no rotation.

The circle does not have a rotation because even if it does rotate it is still a circle. So, you can see the difference between a triangular robot like this and a circular robot. So, in a triangular robot, if the robot rotates, it will change. The complete shape will change. It will become like this that you can imagine. But, a circular robot, even if it rotates, it still remains a circle. There is no change in the shape.

So, first, we will turn. We will start off by looking at circular robots which is easier to understand. Then, we will go on to triangular and other shapes.

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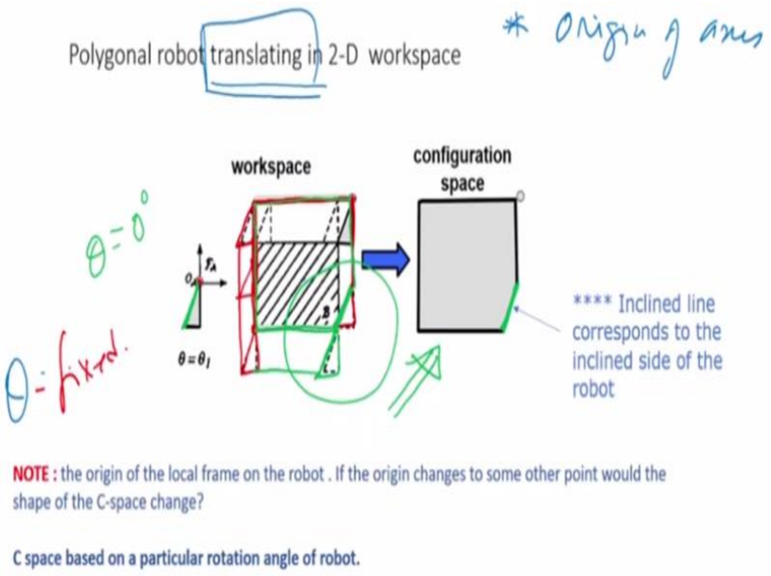

So, next, let us look at a triangular robot. I hope you understand this how the shape we got this shape? It is very easy. At the points, you have, at the corner points, you have to be careful because at that point the robot can actually rotate about that point and that is how you get these curves. Otherwise it is a straight line. Now, let us look at a triangular robot. Now, this is the triangular robot which is shown here. And, it is right angled.

Now, something to note very carefully, please note, now, you are supposed to note the number 1 is the origin of axis. Where is the origin of the frame? The origin of the frame is here. Please note that very carefully because the shape will depend on that origin wherever it is. If you change the origin, the shape will change. Now, let us imagine, this robot can translate in 2D. That means the triangle we start off by saying that $\theta = \theta_1$.

And, we keep this θ is fixed. It does not rotate, something very important. So, we start off from here. My origin point is here. So, I can translate it by going forward like this. So, it can go right till the top. So, the robot has come here. I hope you can imagine. It is translating and it is going up. So, this is my robot at that position. This is my robot at this position. What am I tracing? I am tracing the origin of that local frame there which is given by O.

So, what is the line trace by the origin? It is tracing like this going till here. Now, it can go. So, it has gone right on top there. Now, I push it this side. So, it is going here. Then, it has gone here. And, it has gone here. It has gone here. So, what do I get? The origin is tracing again this line. But, at this point, it cannot go down. So, it has to go forward little bit more. Now, you can see, it has gone forward till there.

Only then it can come down because it is a physical body. It cannot go inside the obstacle. So, this is my obstacle. Now, it has gone towards the right. Now, it can slide down. It is sliding down. It has come till here. Now, something to be careful about, once it is here at this location, this is an inclined edge. So, it can slide along the inclined edge and come here. So, what do you get here is this shape. The locus now becomes like this.

That means I started from here, I went there. Then, I went there. I went down. But, here, it is sliding down. So, the origin is moving along this line. Please note that carefully the origin is moving along this line. And, it comes till here. Now, this is my origin there. Again, the origin can go left like this. And then, I trace. So, what did I trace? I trace this, this, this, this and that. That is my shape of the C-space obstacle.

So, I will repeat this again because it is very important that you should know how to do this by yourself. It just takes a bit of practice and that is all. So, I will repeat this again so that everything becomes clear from here. So, in the previous case, what did we do? I took the circle. So, let me come here. So, what did I do is I took the circle and I placed the circle on the boundary.

And then, I trace the cg of the circle at the center of the circle. What did I get here? I get a curve. I get a straight line here. I get a curve here, straight line, curve and that is my shape. So, the shape of the obstacle becomes this. Now, it is like saying I took the circle and I moved the circle along the boundary like this. Now, similarly, here, the robot is not a circle but it is a triangle.

And, the origin is fixed at this point because that means I am tracking the origin basically. In the previous case, I was tracking the CG. So, this was my origin in the previous case. That is my x y. So, I was tracking the center point there. Now, I am tracking the origin which is the tip of the robot. θ is fixed. It does not rotate. It only translates. So, I start off from the corner here. I place my robot there. And, it can only translate. So, it can go up like this.

So, these points you can see that it is going up. And, which point is being tracked? The origin is being tracked. So, it is a straight line. So, it is going right up till here because until it goes up till there you cannot move it towards the right. So, which line did it trace it trace this line? Now, I take it on the right side, it goes still there. And then, I bring it down. So, now, I bring it down. It comes till here. So, this point is being traced.

Now, at this location because there is a curve, there is a slant edge there. This is a slant edge. This can come down like this. It comes down like this. Robot comes to this configuration. So, it has come down like this. Then, it has gone like this again. So, what was the trajectory of that origin? It is like this, like this, like this, like this. Please note that curve there. So, my configuration space becomes has the shape now.

Please note that there is one incline here. The other places, it is rectangular. It is 90^0 only. Whereas here, there is an incline, this incline corresponds to what? This incline corresponds to this side. So, now, so, this is for a fixed orientation. Let us say this is $\theta = 0^0$. Let us simply say it is equal to 0^0. Now, suppose I change the θ.

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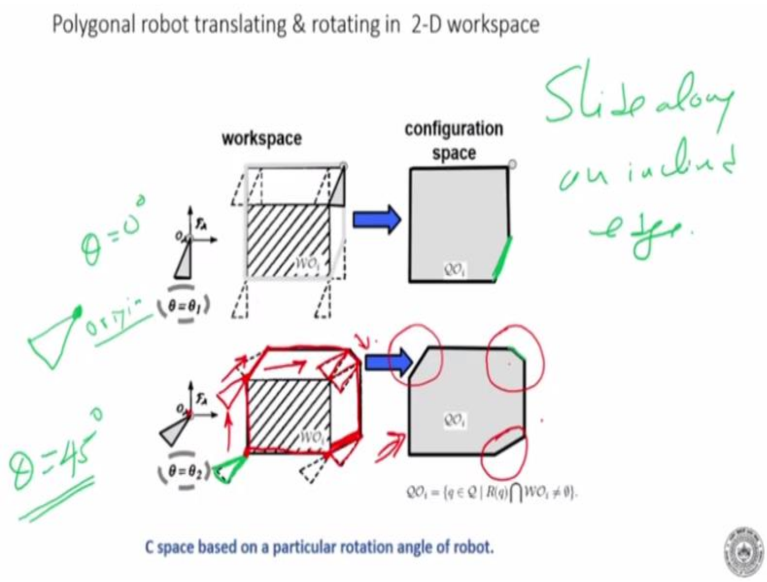

So, this was $\theta = 0^0$. Something we did just now. Suppose I change the θ. Suppose I make $\theta=45^0$ now. 45^0 means and this fellow is bent now let us say that is 45^0. Now, you see. Again, please note, where is the origin? The origin is here only. Now, will the shape be the same as before of the C-space obstacle? No, it will not. Why? Because this fellow is rotated now.

Now, you also appreciate for in the case of a circle is much easier. That is why mobile robots are all circular. You can draw this C space very easily. Now, let us start drawing this from here. So, I place it here. This is my origin. The tip of that robot is my origin. Now, I move it upward like that. The robot moves up. It is moving up and it goes right till there. It goes till here there. So, I get this surface.

Now, at this location, I can slide the robot forward. It is sliding forward like this and going here. That means it can always slide along an inclined edge. So, now, it has slid like this and gone here. Now, you move it right it has gone till here. Now, there is another edge here. So, what it can do is it can slide like this. So, we are marking the tip. We are taking the locus of the tip. So, this part comes here. Then, it goes down. Just come to this location.

Now, it can slide again and come down. So, it is sliding down. So, we are getting this surface. And, once you get that surface, then you slide it leftward and it comes here. So, what is the shape is this, this, this, this? So, this is my shape. Now, please note, in the previous case, when the robot $\theta = 0^0$, you had one inclined face only, the C obstacle space, in the C obstacle, not C space.

In this particular case, we are seeing that there are 3 inclinations. That is one here, one here and one there. And, you should exactly figure out how they have come. I will repeat it again because this is something very important. Please do it by yourself once. It will become very clear. If it is still not clear to you, what I would suggest is that cut a paper in this shape in the shape of a triangle and then you have an obstacle which is rectangular.

You move this paper triangular robot around and see, if, which is the locus of the points track by the origin of the robot. It becomes very easy then. So, I will start again. We have this robot which is inclined at a particular angle. Let us say 45^0. So, I start drawing. This I start from here always. There it is. Where is the origin? The origin is at the tip here. So, I am going to track the locus of the motion of the origin. So, what do I do here?

I move it upward. So, I move it up till where. I move it up to here. So, it has gone up till there. Now, once it has gone up here, you can slide it along one inclined edge because you can slide this. So, it goes up like that. Then, you move it rightwards. It goes till the end there.

Now, here you can slide it again in this direction because of the inclined edge. So, it comes down like this still here. Move it down. It comes to this point.

And, once it comes to this point then what we can do is we can slide it around this edge along this edge. And, it comes here. And then, I move it left. It comes here. So, what is the shape? Shape is like this, like this, like this, like this which is this one. The C-space obstacle shape is given here. Now, there were 3 inclined edges. So, on this case also, you are getting 3 inclined surfaces here, 1 here, 2 here and third here.

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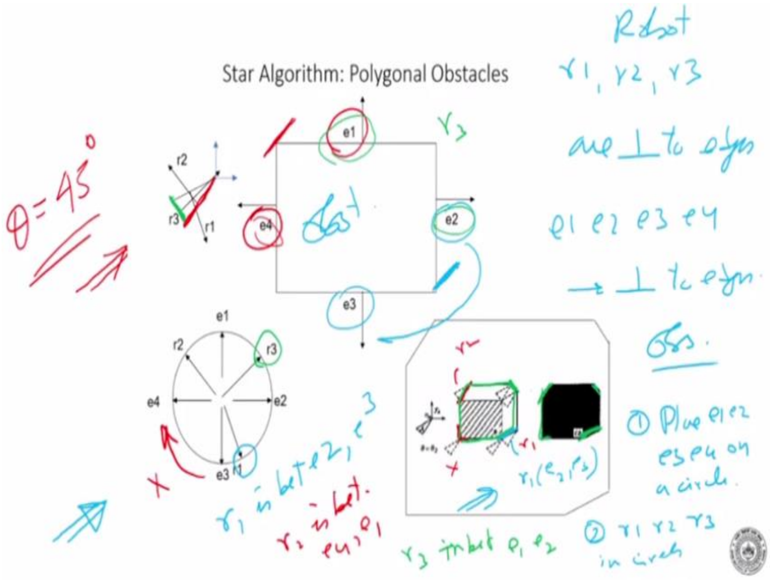

So, there is some correlation between the inclined surfaces of the robot and in the C-space obstacle or the C obstacle where the inclined faces are going to come. And, there is an algorithm which is called the star algorithm because we have to which can automatically find out which is an indicator and can help you how to get the actual shape of the C-space obstacle.

So, let us take the example of this robot here which is at let us say θ is equal to 450. The robot has rotated by 45^0. Now, in this robot, what we are doing is we see there are 3 inclined faces. Each of the inclined faces we draw this perpendicular. This is r_1, r_2 and r_3. So, r_1, r_2 and r_3 are perpendicular to edges or sides. So, you have r_1, r_2 and r_3. Now, on the obstacle, this is my obstacle.

In the obstacle also, on each of the edge, I draw perpendicular e_1, e_2, e_3. So, e_1, e_2, e_3, e_4 perpendicular to edges again of obstacle. This is case of robot. This is case of obstacle. Then, I make this diagram. What is this diagram? I start e_1, e_2, e_3, e_4. I arrange it like this. This is e_1. This is e_2, clockwise, e_2, e_3 and e_4. So, this is going clockwise. So, I put my e_1, e_2, e_3, e4. Then, I take my r_1, r_2, r_3, r_4. So, first, what do I do? Place e_1, e_2, e_3, e_4 on a circle.

Then, what do I do? I take my r_1, r_2, r_3. Then, I take r_1, r_2, r_3. The directions are already given. Look at the direction of e_1. e_1 is in this direction. This e_1 is also in that direction. So, it maintains the same direction. Then, what do I take r_1, r_2. I maintain this distance and these directions. r_1 is this side. That is my r_1. r_2 is that side. This is my r_2. And, this is my r_3 which is here.

So, I place my r_1, r_2, r_3 in circle along the same directions in which they originally are there. Now, I start drawing my C-space obstacle. This is my C-space obstacle. So, this gives us an indicator of where the inclined surface will be. So, let us look at r_1. r_1 is between e_2 and e_3. Where is e_2 and e_3? So, e_2 is here and e_3 is there. So, r_1 is here in between here. So, and, r_1 corresponds to which side? r_1 corresponds to this side.

So, this incline will come where? It will come between e_2 and e_3. So, where is e_2 and e_3? e_2 is here. e_3 is here. So, it will come here now. So, it will, so, on my C-space obstacle, this will come here. So, that is because of r_1 between e_2 and e_3 that inclined surface. So, we will get this inclined face there. So, this is my inclined face. This corresponding to which side of the robot. That r_1 side. Next, let me catch the next one.

Let me say, let me, sorry, we are going clockwise. So, in between 3 and 4, there is nothing. So, there is no inclined face between 3 and 4. There is 3 and 4 is here. 3 is here. 4 is here. So, here, there is no inclined face. You can see that. The next one is r_2. r_2 is between e_4 and e_1. So, where is e_4? e_4 is here. And, e_1 is there. And, r_2 is coming there. Where is r_2? r_2 is this face. That means it will come there. So, you can see it is coming there.

So, that one is because of r_2. So, this is because of r_2. There is nothing here this because of r_1. And now, let us look at we are left between e_1 and e_2. So, e_1 and e_2, we are left. And, there is r_3. So, r_3 comes between e_1 and e_2. So, r_3 is in between e_1 and e_2. So, where is e_1? e_1 is here. e_2 is here. So, r_3 is coming there. And, r_3 corresponds to which edge? It corresponds to this edge. So, my angle will come there. That much will come there.

So, what is the shape I get? I get this shape, this shape, this, this, this, this and that. That is this one. So, I get 1 here. I get 1 here. I get 1 there. I get nothing here. That is the shape. So, this basically gave us an idea of how to draw the shape and where those inclines will come? To be done automatically.

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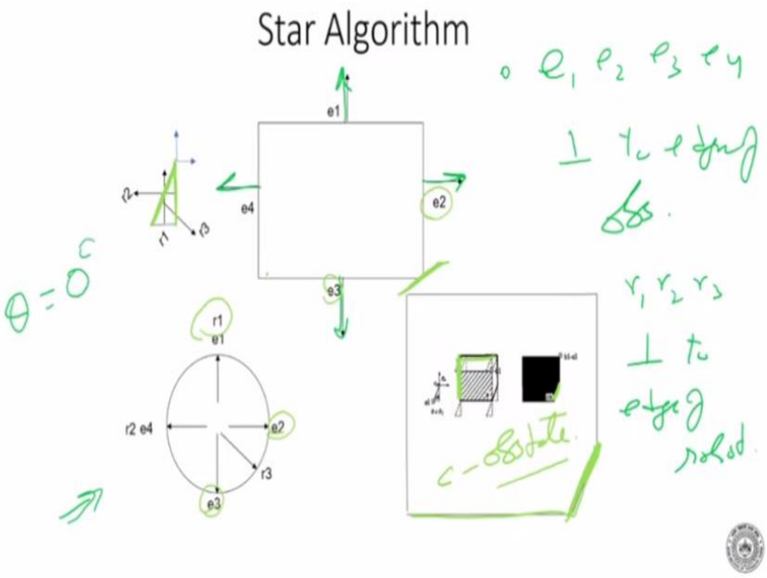

This is another example. In this particular case, the robot we can say is not rotated. So, $\theta = 0^0$. I follow the same procedure. I start with e_1, e_2, e_3, e_4. These are perpendicular to the, so, I first mark e_1, e_2, e_3 and e_4 they are perpendicular to edges of obstacle. Next, what we do is I take the robot and I mark r_1, r_2, r_3, r_4 which are perpendicular to edges of the robot r_2, r_3 perpendicular to edge of robot.

Then, I draw my circle which is here. So, I mark e_1, e_2, e_3 and e_4. And, I place my r_1, r_2, r_3, r_4. So, this is my r_1. Please note r_1 is not here now. r_1, r_2 because r_1, r_2 are perpendicular to the edges. And, the edges are perpendicular to xy axis. r_3 is here only. Please note r_1, r_2 and r_3 were there. Now, we start drawing the inclined surfaces. So, I start off between e_2 and e_3. So, e_2 and e_3. So, there is something there. What is there? r_3 is there.

What is that angle here? The inclined surface is there. So, I get this surface there, so, which is here. Now, next, what do we do? We go to e_3 and e_4. So, where is e_3 and e_4? e_3 and e_4, r_2 is along with e_4. So, r_2 is along e_4 which basically means that it is not coming in between e_3 and e_4. It is along e_4 only along way as e_4. So, this will not cause any inclined. It is like saying it is a straight line here. It is not an inclined line.

Similarly, r_1 is along e_1. So, it is going to be this one. It is not going to cause any angle there. So, if there is an angle, it will cause an inclined surface on the C obstacle only if the edge is inclined. In the previous case, if you see, this edge r_1, r_2, r_3, they were all inclined. If you can imagine, this is my x axis. That is my y axis. So, if an edge is perpendicular to this or perpendicular to this, it will not cause any angle on the other side.

Only if it is inclined, then it will cause an angle. So, this is the shape. There is only one inclination. r_1 is along e_1 and r_2 is along e_4. So, there is only one inclination, so, my C-space obstacle. So, this is my C obstacle. So, this is the shape of my C obstacle. I hope it is clear how things are done. Now, this will give you an idea that for us for a circular robot, it is pretty easy because the circular robot there is no orientation.

Even if the robot is rotating, it is still a circle. So, there is no change in the C-space obstacle. But, in the case of a triangular robot or any other shape, if it starts rotating, then what will happen? This shape of the configuration space obstacle will change. That means for every rotation the shape of the obstacle will be different. So, what we can do is we can start off by saying that this robot starts with $\theta = 0^0$ is changed by 1^0 up to 360^0.

And, for each of this particular degrees, you will get a particular shape of the obstacle C-space obstacle. Now, you combine all of those and you get the total shape of what the total C C-space obstacle will look like for the translating and rotating system.

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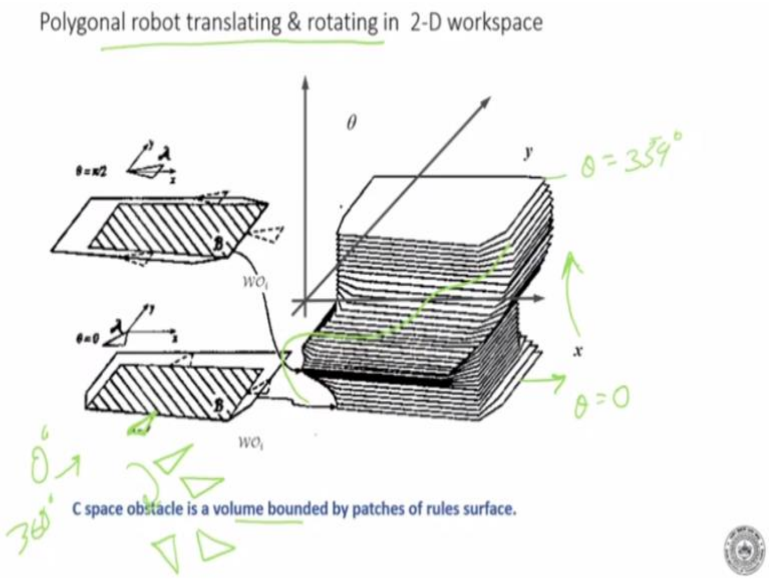

So, in this particular case, the robot is faced like this. Then, we rotate the robot a little bit maybe we make it like this. Then, we rotate a little bit more make it like this. Then, we make it like this. Then, we make it downwards. So, we rotate from 0^0 to 360^0. And, for each of those, you get a particular shape. So, you stack those shapes say for example this is $\theta = 0$. That is $\theta = 360^0$, it's in shape 359^0.

So, you stack them all together. And, this is what we call a volume. So, this is a volume bounded by patches of rule surfaces. So, polygonal robot translating and rotating in 2D space is actually you will be getting a volume. A C-space obstacle is a volume bounded by patched rule surfaces. So, if you are doing motion planning where the robot is also rotating and translating, it becomes pretty complicated because it will start moving here now.

So, it is not planar anymore. But, I hope you understand, how this, how do we get this volume.

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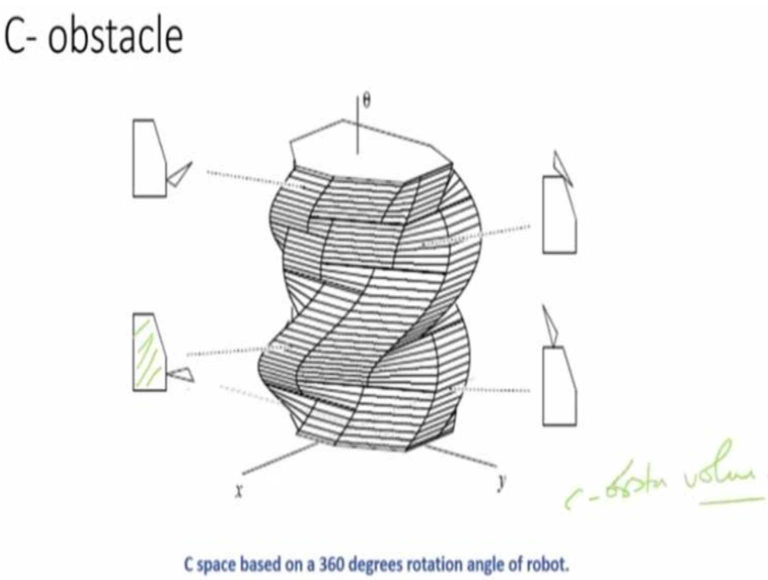

C space based on a 360 degrees rotation angle of robot.

So, this also gives us an idea of, you have the obstacle which is shown here. And, you have the robot which has a particular angle. And, when the robot is rotating, for each of those rotations, you get a particular shape. And then, you stack all of them together. And, you get your C obstacle volume. So, it is a volume now.

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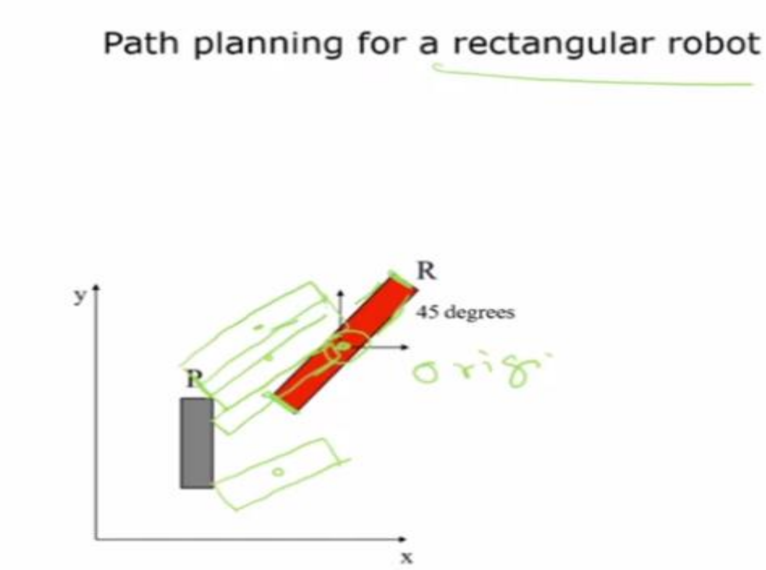

Now, path planning for a rectangular robot, so, when we are doing path planning, we always have to work we always have to plan in the configuration space. So, we have to increase or enlarge the size of the obstacle and decrease the size of the robot. Now, let us look at this particular example where you have a rectangular robot. And, this is rotated at 45^0. So, you can imagine that we place the robot here. We go in like this.

So, it can rotate. It can slip around this point. So, the robot is here. Then, the robot is there. So, it can come down in this configuration. So, the robot can move, what are we tracking? Please note, where is the origin? The origin is in the center of the robot there. So, we are tracking the center of the robot. Now, you can imagine that there is an incline here. There is an incline there. There is incline there. There is an incline there.

So, this is inclined by 45^0. So, what will be the shape of this fellow? The shape of this, the shape would be something like this.

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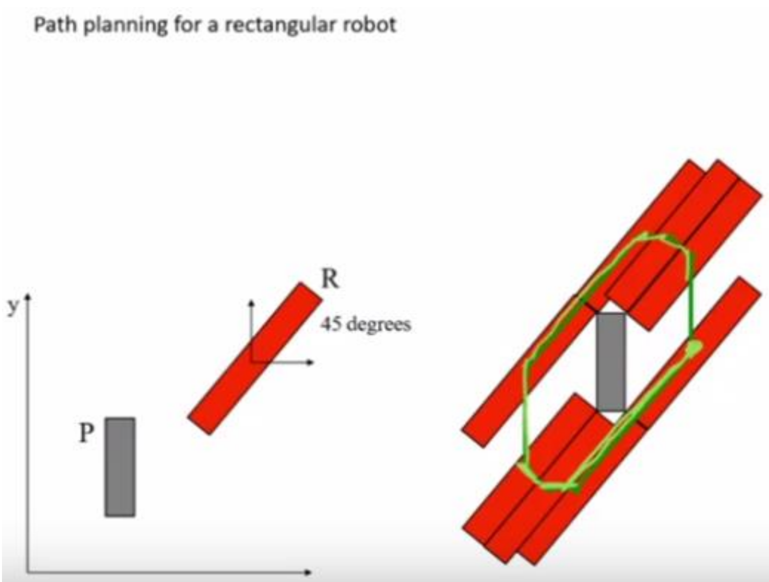

So, you can imagine that on this side, it can slide. So, what you are getting? The CG is sliding like this. Then, it is moving slightly here. You can move that side. Then, again, it is sliding. Again, sliding here, moving here, moving here. So, we are basically tracking the CG of the robot now. And, as the red robot is moving around the obstacle, you can see that we are tracking the CG and this is the shape that we get.

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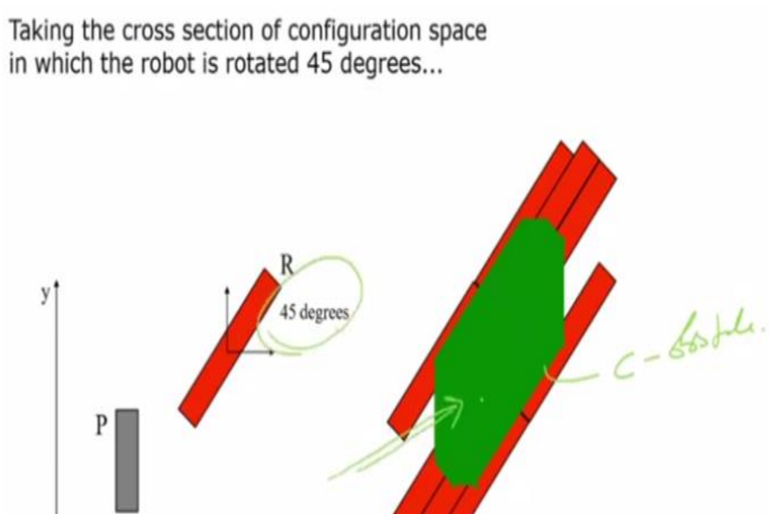

So, what is the shape? The shape will be like this. So, this is C obstacle. So, if I change this 45^0 angle, this shape will change again. That is logical.

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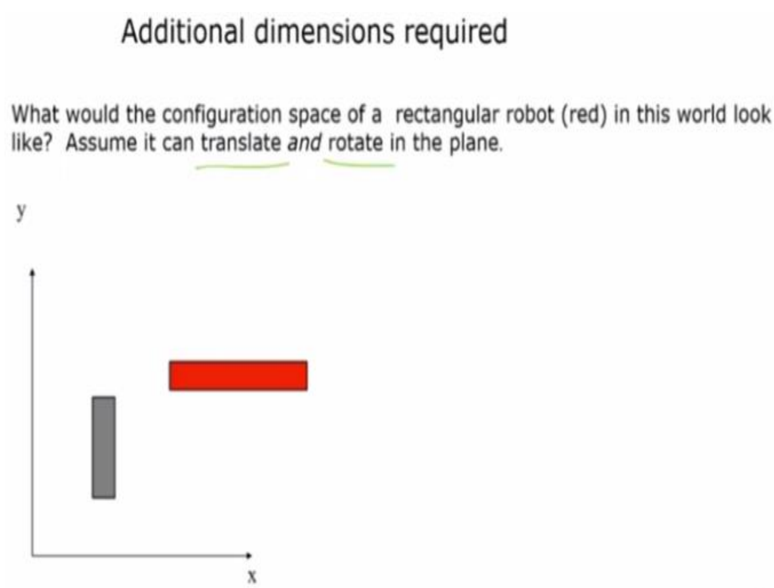

Is not it. So, for example, in this case, it is like this which is facing this side. And, suppose it can translate and rotate, it can rotate obviously. For every rotation, there will be a different C space.

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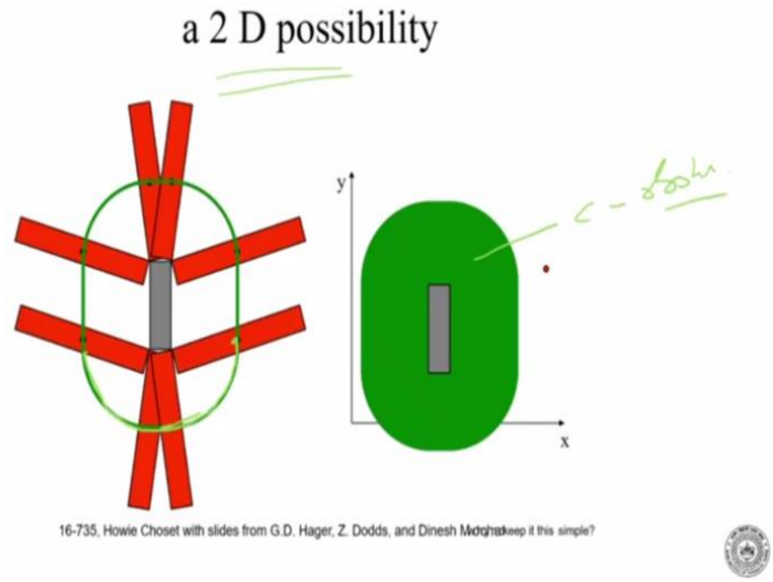

So, what we are doing is essentially in a case of 2D possibility. It can rotate and translate. Then, in that case, what we do when we track? What we are getting? Because it can rotate also now and translate. What will be getting is this is my C obstacle.

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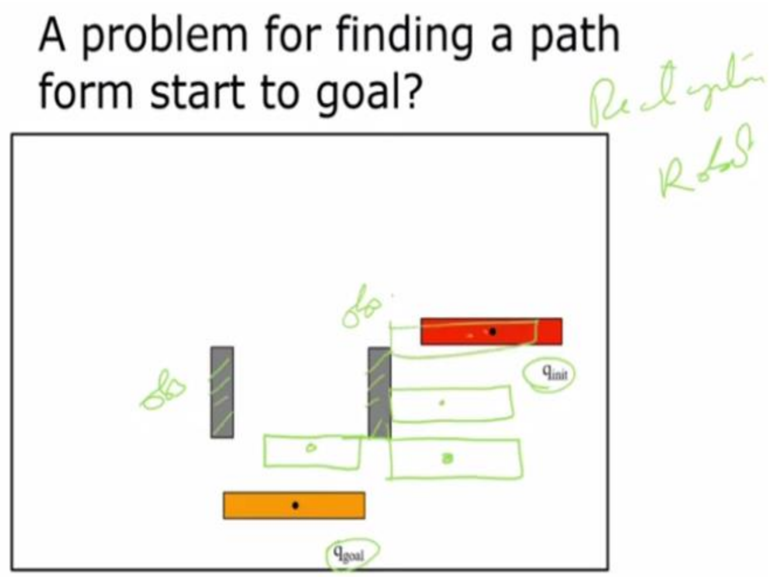

Now, a problem for finding a path from start to goal point, suppose, this is my goal point, this is a rectangular robot. And, this is my initial point and this is my goal point and these are obstacles. So, this is my obstacle. That is also obstacle. So, what we can do is I can assume this robot to be translating only around the obstacles and then I can make my C obstacle. So, what I am doing is basically I am making this fellow go and sit like this, go and sit here.

I can make it sit like this. Then, it can translate this side, come and sit till here. What am I doing? I am tracking the CG of this obstacle. Sorry, CG of the robot when it is moving. So, when I track the CG of the robot, what I do get is this is my C obstacle the shape of the C obstacle.

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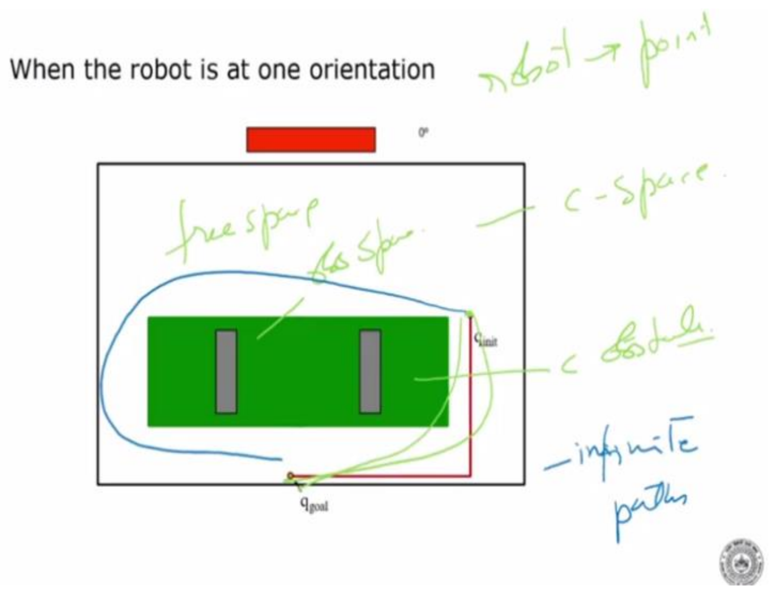

In the center, please note that when it is going and sitting in there, the whole space is going which basically means that is why it is. So, in the center also, the robot is unable to go through the center. Now, we have made our C obstacle. So, this is my C obstacle. Now, the robot has become a point. The obstacle has enlarged. So, this is basically my C space also with the C obstacle. The robot has become a point.

Now, when I want to go from q_{goal} to from initial point to final point, it is very easy to see that there is free space. This is my free space. And, this is my obstacle space. So, wherever there is free space, it can very easily connect the free space to the goal. And, there are that many parts. In this particular case, it can go like this. It can go like this. You can also go from the other side. So, it can also go from this side and go like this also.

So, there are large number of paths. So, there are infinite paths. Then, the question comes, which path is faster? Which is shorter length? But, something to note here is that suppose we are trying to solve the problem in this space. Now, if I asked you, can the robot go from here? It is very difficult to say. Because we are relying on our visual data and we are guessing. So, for example, this is the robot, you will have to see whether it will fit in.

It does not look like it will fit this and this. It does not look like it will fit there. So, it can go. But, here, it is very clear. In this particular case, it is extremely clear which is free space and which is obstacle space. So, very easily it can go in the free space only and connect the initial point to the goal point. So, this gives you the advantage of using C space where the robot has been converted into a point. And, the obstacles have been enlarged.

And, they have become the C obstacles. So, path planning is done in configuration space or C space.

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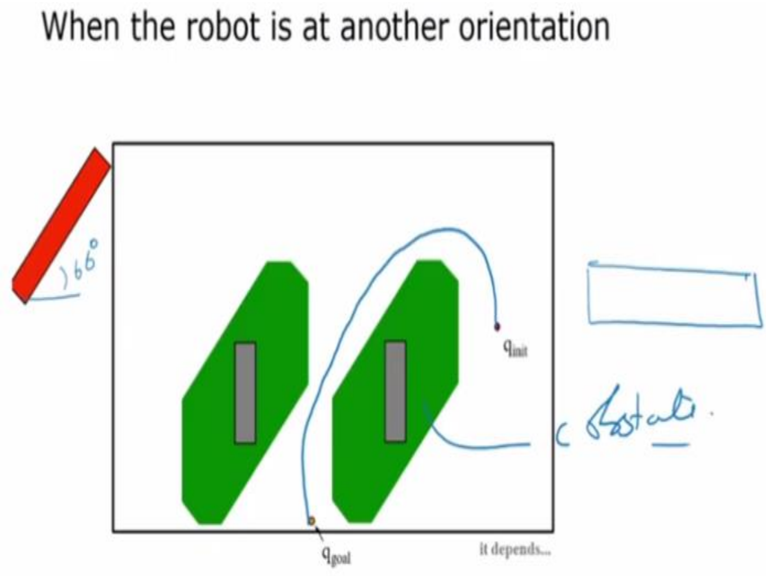

Now, in the previous case, the robot was like this. Now, in the second case, suppose I rotate the robot by say 60^0, in that case, my C-space obstacle, C obstacles become like this. Now, in this configuration, can the robot go through here? Yes, it can go. It can go from here. It can go like this. It can go here. Now, also, it is very clear that what are the various parts possible?

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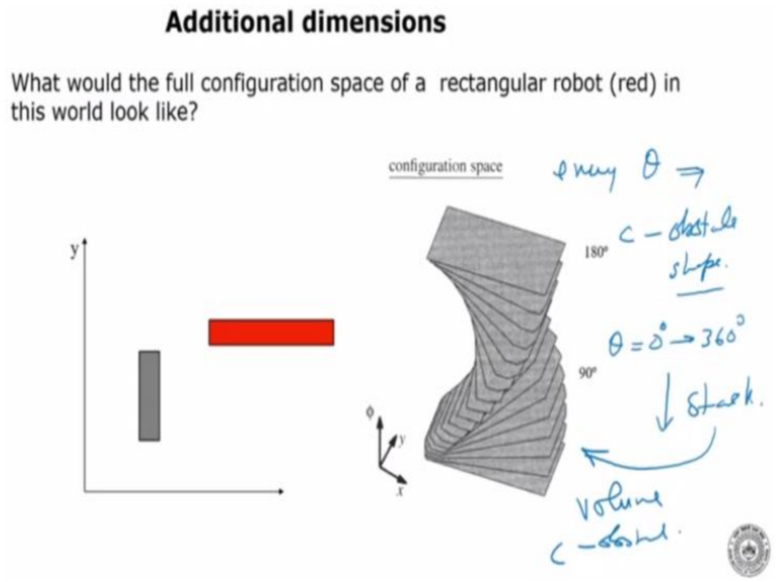

Now, this also indicates that for every angle, you will get a particular shape. So, for every θ, you will get a C obstacle shape. So, we can vary from $\theta = 0^0$ to 360^0, get different shapes and then stack them up like this and this. We get a volume. So, this is my C obstacle volume. So, this gives us an idea of the C obstacle and also the C obstacle volume when the robot can rotate and also translate.

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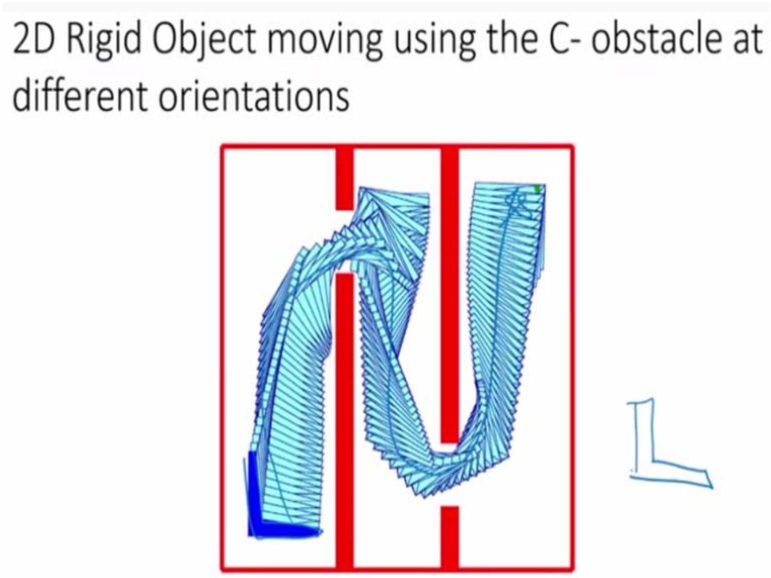

Now, doing this, we can have rigid bodies moving along the C obstacle at different orientations. So, for example, this is the shape. So, it is shaped something like this. And, I want to move it from here through this through this to there. So, this is what we do that we can move this rigid body with different orientations and take it from the initial point to this goal point.

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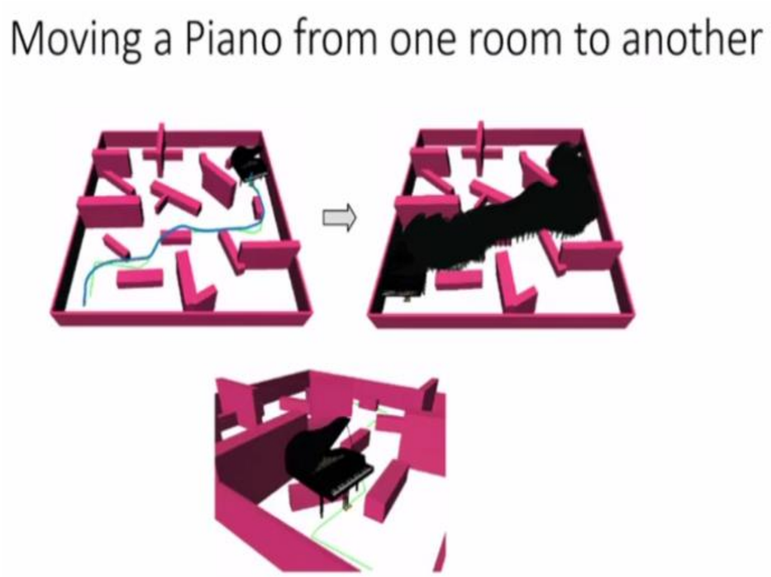

Similarly, moving a piano which we said that the original motion planning problem, started with, moving the piano from one room to another. So, in this case also, what we are doing is we can take the piano like this, like this, like this, like this and take it there. That was the original motion planning problem.

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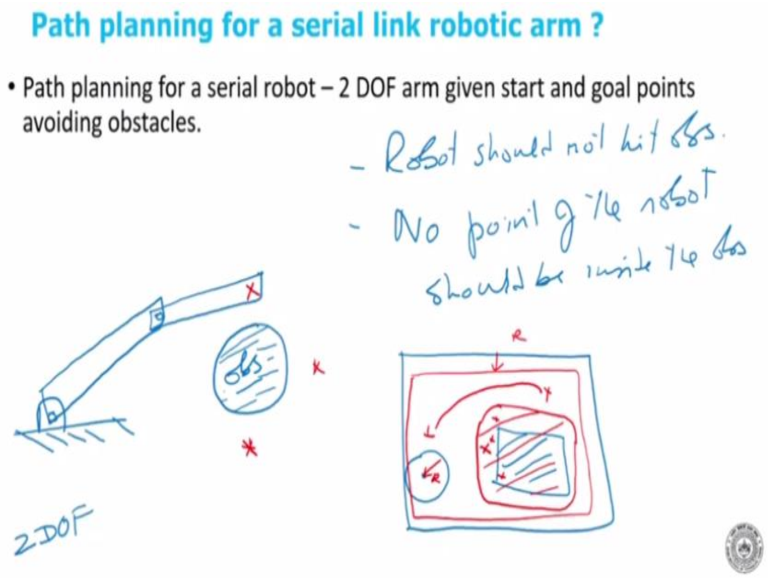

Now, let us move on from here to the next part which is the configuration space or C space for a serial arm. So far, we have been talking about mobile robots. Now, I am sure you can also guess why we are talking about mobile robots. First of all, they are circular mobile robots because it is easy very easy to see these C space. Now, path planning also for a circular robot is easy. Why?

Because it is circle, so, we can simply check where the circle is circle can go and where the circle cannot go. That is it. Even if it is rotating, it is still a circle. The moment you make it a triangular then the problem of rotation starts coming inside. But, for a serial arm, it is more difficult. Why? Because we said that the obstacle or the robot not hitting the obstacle means that no point of the robot should be inside the obstacle. Let us take an example here.

So, here, we have a mobile robot. So, we have a serial arm here Now, we are talking about serial arms, very simple 2 degree of freedom serial arm. There is one arm here. And, there is one another arm there, 2 arms. This is 2 degree of freedom planar. Let us say we have an obstacle here. That is my obstacle. That is an obstacle. Now, we say that robot should not hit the obstacle. What does it mean?

It basically means no robot no point of the robot should be inside the obstacle. This is what it means in the case of mobile robot was very easy. You could understand. Shrink it to a point and then you can see. That is the obstacle space. That is the C-space obstacle. And then, no point is inside there. So, it is very easy. So, all I have to do in terms of geometry is if I have a mobile circular robot and I have a obstacle like this.

All I need to do is shrink this fellow to a point reduce by R decrease that one by R. So, this has come down by R. I inflate my obstacle by R. This I make it the obstacle. Now, I have free space. And, I have obstacle space. So, if I want to go from this point to that point, I can simply go like this. Geometrically, just moving a point around and the point should not go inside here which is very easy also.

You know, what are the vertices of this C obstacle? So, this point should not go inside there. Now, how do we do that in the case of a serial arm? Now, in the case of a serial arm, suppose I want to go from this point to this point, how do I go? And, how do I specify that no point of the robot is inside the obstacle now? So, suppose the point is here, can I go there? Or, I can go there with hurting the obstacle. How do you figure that out?

So, when we are saying path planning for serial robot arms, we basically mean the robot no point of the robot link no point on the robot link should be inside the obstacle. That is what we mean. So, how can I figure that out is essentially by doing my forward kinematics.

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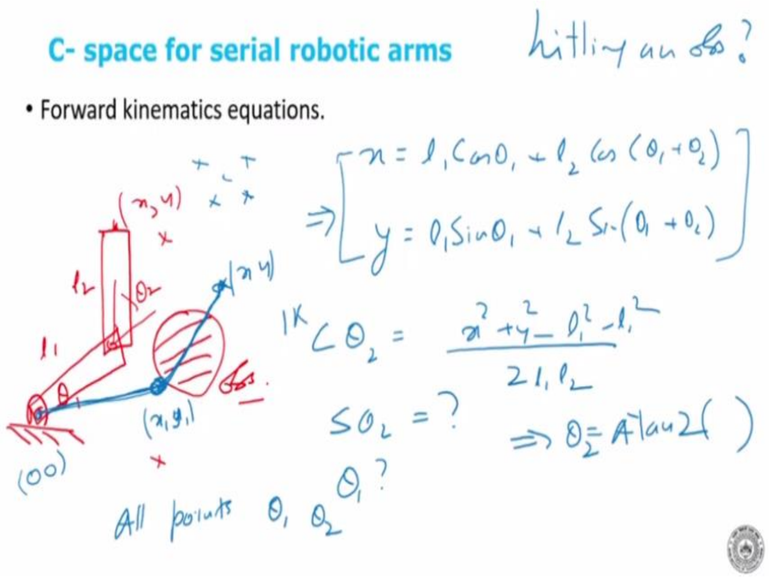

For example, let us look at this my robot arm. This is my arm, 2 degree of freedom. I just drew. So, this is l_1. This is l_2. This is θ_1. That is θ_2. This is my θ_2. This is my point (x, y). This is my obstacle. Also, this is my obstacle. I want to go from this point to this point now. So, I should be able to figure out. Now, I have my point (x, y). Now, I know that $x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$ and $y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$.

So, at any point in here, I can do forward kinematics. This is my forward kinematics. I can give different values of θ_1 and θ_2. And then, see where the end effector is. If I know this is my (x, y), I also know from this 2 equations I also know that my position is probably here like this. This is one solution. So, for every θ_1 and θ_2, I exactly know what is the end effector? And, where is this fellow also?

So, let me call this x_1 y_1, this joint. Where is this joint? And, where is this joint? I can find that from these 2 equations for different values of θ_1 θ_2. And then, I can see by connecting with a straight line. If this line is inside that obstacle that would mean that it is hitting the obstacle. So, I am trying to explain here, what is hitting an obstacle? Mean here. So, I can get different values of θ_1 and θ_2.

I can find where is (x, y) and where is $x'y'$. That is the position. 0 0 does not move. It is always there. And then, I connect by straight lines and say that this straight line is not intersecting with that obstacle. Now, I can go in the reverse direction. For every point, I know, what is θ_2? From inverse kinematics, I have done $\theta_2 = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2}$, I get θ_2. So, I get cos θ_2.

I convert that to sine θ_2. I convert it and then I can find, what is θ_2 by means of A tan θ_2 ? This we have done in inverse kinematics. Once I get θ_2, I can get what is θ_1 from geometry. That means for every point in this workspace I can get θ_1 and θ_2. So, for all points, I can get θ_1 θ_2. That will take me to that point. And then, what I can do is I can check if this link is intersecting the obstacle. And then, what I can do is I can draw minus C-space obstacle.

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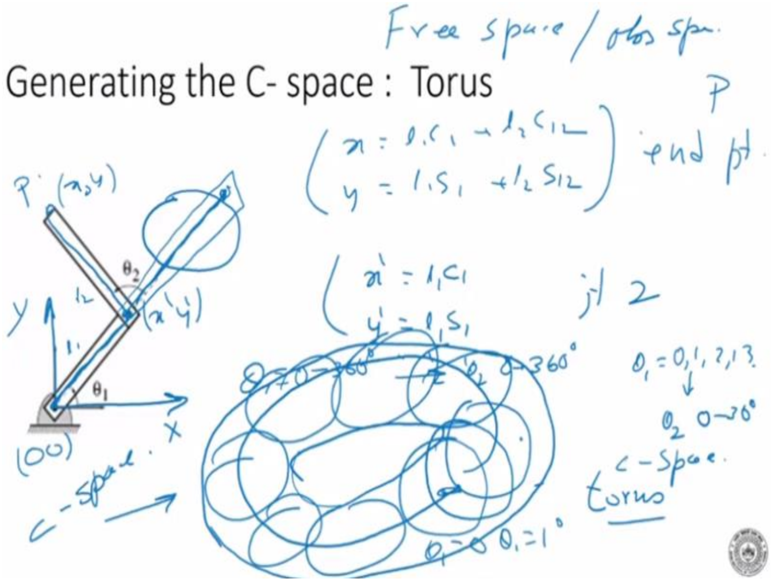

So, let me explain how we are going to go about it. So, I have θ_1 and θ_2 here as shown here. This is my l_1 l_2. Now, from that equation $x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$ and $y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$, I can get the point x y. Now, this point is $x'y'$. How do I get that? I get $x' = l_1 \cos\theta_1$ and $y' = l_1 \sin\theta_1$. So, I have got these 2 points. This is joint or let me, this is not joint 2. This is joint 2.

This is end effector point end point. Let us call it P, this point P. So, I have got this. And, I have got this. I have got these 2 points. Then, what I can do is I can join with a straight line. I have got points. And, 0 0 is always there. So, I join with straight lines. And, I check if this is intersecting with an obstacle. Suppose, there is an obstacle here and my link has become like this. Now, what is happening? This is intersecting with this now.

So, there is intersection now. So, the objective is to find free space and obstacle space. Now, please note here that generating the C space now is I am generating a circle. This is my θ_2. So, this is my θ_1. It can go from 0 to 180^0. So, θ_1 can go from 0^0 to 360^0. So, it goes from 0 to 360. It is a circle. What about θ_2? θ_2 can also go from 0 to 360^0. So, it is like saying that when $\theta = 0$, it is here.

This is my θ_1. Now, θ_2 can go from 0 to 360. So, I get a circle. Then, I move θ_1 by next angle which is here. Then, I move it by some more angle. So, θ_1 moves from 0, 1, 2 , 3 and each of this, θ_2 can go from 0 to 360. So, it is like saying this circle is actually going around like that.

So, every point here, so, this is my θ_2 has gone from 0 to 360^0. And, this my θ_1 has gone from 0 to 360^0.

So, on this circle, it is θ_1. And, for every θ_1, there is a θ_2 which is going around. So, what we are getting is a shape like this. This has become very clumsy. So, let me draw it again. This is something you need to understand very clearly that you can imagine that θ_1 is going to go from 0 to 360^0. And, for each θ_1, θ_2 can go from 0 to 360^0 also. So, we start off by saying θ_1 is at 0.

So, θ_2 can go round like that. It takes a circle. Then, θ_1 goes to the next point which is $\theta_1 = 1^0$. Now, this θ_2 can take a circle again there. And similarly, it is taking circles like this, like this. And, it is going around like this. Now, if I join these circles, what do we get? We get the shape which is basically called a torus. And, this is my C-space obstacle. Sorry, this is my C space.

So, basically, what we are seeing here is that the shape that we are getting is that of a torus. And, the C space here and the Cartesian space they look completely different. The Cartesian space is my x coordinate here and my y coordinate there whereas the torus is looking completely different from my configuration space now. This is my C space. So, this C space and the Cartesian space, they look completely different. In the case of the mobile robot, they look same. But, here, they look completely different.

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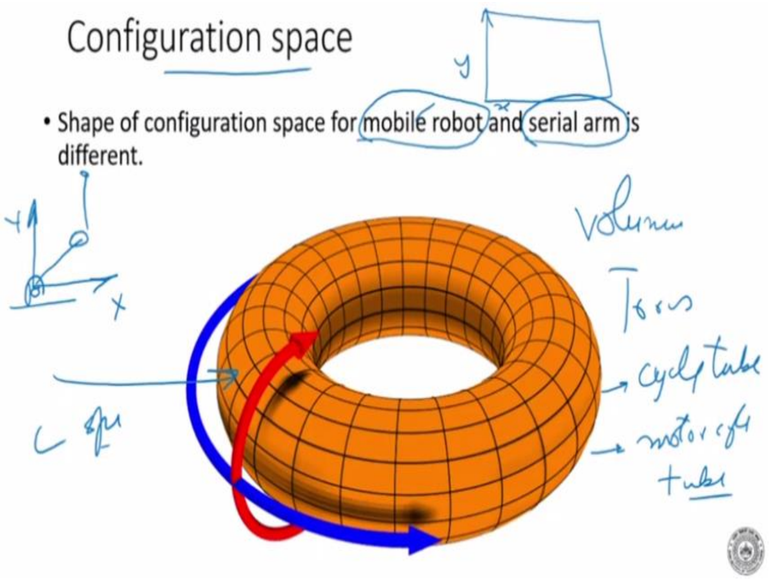

Now, this is the shape of configuration space for a serial arm. For that of a mobile robot, it is like this only. This is my x and this is my y. It looks like the Cartesian space whereas for a serial arm what we are seeing is that we have this going this side. This is my θ_2 and this is my θ_1. So, this is my θ_1 and this is my θ_2. So, it is like saying this. θ_2 is a circle which is going round like this.

And, sorry, θ_1 is going round like this. And, for every θ_1, there is θ_2 which is like this. So, if I take the complete volume, it becomes torus. This is something like a cycle tube. You must have seen the tube of a cycle, it is inflated or a motorcycle tube which is inflated. It looks like this. This is the shape of the configuration space of a serial arm which looks very different from that of the mobile robot.

So, our Cartesian space of the mobile robot looks like the Cartesian space is this is my x. That is my y. So, this is one link and that is another link. So, this is my x and y. So, it looks like the Cartesian space whereas the C space looks like this. And, it looks very different. So, please note that in the case of the mobile robot also, they are different actually. But, they look same. Here, they look different. And, they are different of course.

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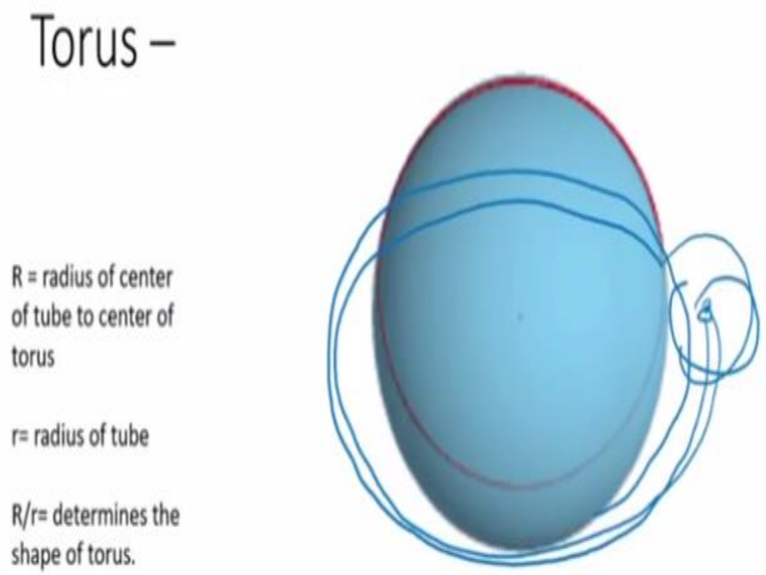

Now, this is an example of the torus. So, we start off from let me have a look at that again. So, here, we start off at a point θ equal to it is a circle which is going around this. So, it is becoming a torus. Let me look at it again. So, first of all, at a point, $\theta = 0$, we have a circle. Now, the circle as θ_1 is changing, this circle is also moving like that. So, what we are getting is this kind of a structure which is basically a torus structure.

So, please note that in the case of a serial arm, the shape of the C space is that of a torus. So, in the case of a mobile robot, they look the same C space and Cartesian space. But, they are different. In the case of the serial arm, they are different. And, they look different. So, in the next, we will stop here today. And, in the next class, what we will do is we will look into look in more details as to this torus structure. Why?

Because, when we are doing path planning, we are finding the path on the surface of this torus now. And, when we talk about an obstacle, we have to plot the obstacle on this torus only because that is the C space of the serial arm is a torus. So, we will stop here today and we will continue in the next class regarding drawing the C obstacle for a serial arm. And then, we look at planning.

How do we plan? What does it look like? And, how do we do the path planning for a serial arm?