Engineering Thermodynamics

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Week-04
Lecture-19
Mass Energy Analysis of Control Volume

Welcome to this new lecture which will be on a new topic. This will be Mass Energy Analysis of Control Volume. In this lecture, we will study the conservation of mass principle. So, this is conservation of mass principle. And we will apply this conservation of mass principle specifically for some particular systems. So, let's start. So, if you consider any control volume This is a non-uniform control volume. If we look at it practically, some things will come inside, like mass, energy, and some will go in the output, in the outflow. So, this will go through the surface. That's why we call it a control surface. This is the control volume. And this boundary is the control surface. And we have represented in this form. And the flow rate of the mass is called a dot. If we call this mass flow rate So this is the amount of mass that is more than the cross-sectional area per unit time. This means that if you take this cross-sectional area of this surface and the differential of this cross-sectional area is called dAc, so if this is your cross-sectional area, then the area that will go through this is also a very small amount, we will call it differential mass flow rate. We can connect this to the density of the fluid which is more than here. If the density is supposing rho and since mass is a rho into volume and since this is mass rate so we can represent volume as area multiplied by This is the velocity component. Because it is a velocity component, it is flowing. So, we can multiply it by velocity. So, this is your volume rate. If we call the volume this, then this will be your cross-sectional area. And in that, the rate at which the fluid is flowing, we will multiply it and get the volume rate. So, density Multiplied by volume rate will be your differential mass flow rate. So, this is your velocity. Now we will take normal velocity in this. This is your normal component 2, this is differential. So, this is your normal component 2. And this is the corresponding velocity. So, this is velocity, this is normal velocity. So, this part is your volumetric flow rate. Volumetric flow rate. Because if you multiply the velocity by area, then you get volume rate. Because the velocity is dx by dt, and the distance is per unit time. So, when you multiply it by area, then you get volume per unit time. So, this is your flow rate, volumetric flow rate. You multiplied this by density.

$$\delta m = differential\, mass\, flowrate = \rho v_x dA_c$$

Rho is density and vxdAc is volumetric flowrate.

$$m = \int dm = \int \rho v_x dA_c$$

Since this is a differential element, a small surface So this amount will be your delta m dot So we will call this differential mass flow rate so again Let's talk again, we took control volume We saw its surface, the surface became your control surface If it is flowing in it, then the amount that is going out to analyze, we took a differential area. And here we said that the amount of mass that comes out of here, we will call it differential mass flow rate. Now the value that will come out will be its density multiplied by volumetric flow rate. And to take out the volumetric flow rate, we took its differential area. Then multiply the normal velocity of the cross-sectional surface. This is the differential mass flow rate. Now you will integrate this and add it to the entire surface. Then you will get the total mass flow rate. This is one way to get the mass flow rate. So, m dot is now integral of differential flow rate, but this is on the entire surface area. So, we will take the entire cross section. cross-sectional area. So, whatever surface you are making, you will integrate it, you will sum the elements of the differential element, you will sum it, means you will integrate it, then your mass flow rate will come. Sometimes the surface of the control volume is not uniform. Sometimes it is difficult to integrate. For example, if you take a pipe or a duct, it is not necessary that the pipe is uniform. The diameter of the cross section can increase or decrease. This is the same as Doug's cross section. The second thing is that if you take a cross section of Doug, then it is not necessary that the velocity across the cross section is the same. It can vary, it can change. So, these two things are important. That is why sometimes it is not practical to integrate. Sometimes you have to take an average. You can take a duct or pipe and draw a velocity profile. Let's assume that it is like this. So, your velocity will be more in the center and if 0 doesn't slip on the surface, then it will be around 0. If we represent the average velocity, then this value is the total normal velocity and the cross-sectional area of this value. Suppose we are going to take this. and integrate it but then multiply it from its total cross section area and divide it divide it from its total cross section area so this value we will call it V average sometimes it is written like this sometimes we can write it like this that it is V average weighted average so if you know this and if we use it then your value m dot What I am writing here means that you can add or integrate the differential area across the surface. This means that your row Vavg is like this. and its unit is kg per second. Now you can connect M dot and volumetric flow rate. As I said, what is volumetric flow rate? Velocity multiplied by the area. If you take differential area, you will get dA. Velocity is known. This means that mass flow rate and volumetric flow rate are related. and that is by simple multiplication of density so this is your definition that density multiplied by volume which is mass if you put dots, it means rate volumetric flow rate and mass rate here so this is your mass flow rate so their relation is very simple you can also add volumetric flow rate to volume So in such cases As we said earlier, mass flow rate is related to volumetric flow rate. Volumetric flow rate is the volume of the fluid through a cross sectional per unit time. How the volume is moving per unit time, which is a cross sectional. So, if you believe that this is a cross sectional, then how is it moving? So again, we can write this in an average book, like the volume we talked about. We can take it as average velocity or integral So we have written,

$$v_{avg} = \left(\frac{1}{A_c}\right) \int v_x dA_c = < v_x >$$

$$m = \int \rho v_x dA_c$$

$$m = \rho \int v_x dA_c = \rho v_{avg} dA_c$$

$$m = \rho v$$

Let's move ahead and discuss the conservation of mass principle for control volume. In this case, we have seen a generic control volume surface in which you have a tank and a flow through pipe, and you have PITI specific In this way, you have an inlet condition and an outlet condition. It also shows that an external force has been applied to it. And an external force has also been applied to it. So, if you apply a net mass transfer to such a system, then we can simply say that The total mass entered at any time, which is called delta t, in a time frame, that total mass entering, that is, the amount entered, minus the mass that has come out, and this should be equal to the mass changing in our control volume. So, the amount that came in, the amount that went out, the amount that came in here, and the amount that went out, and subtract it, it should be equal to control volume which is changing inside the control volume which is dmCv d by dt So this is a basic conservation mass principle. It means that the total mass in, the mass that entered, let's say at delta t time, and delta t means in a time, t2-t1, minus the amount that entered, minus the amount that came out, this should be equal to the amount that finally the control volume has been changed in mass. This is the simple principle of mass conservation. You can also write this in the form of rate. When you do the rate, you do d by dt. So, in this way, if you do delta t, then you will get a dot. Here the dot comes, and you can write this as mCv. You can also call it dmCv by dt. So, this is basically your definition of the top. Now let's move this topic a little bit forward and try to understand it. I have drawn this with the help of simplified control volume and surface. This is your dash line that you can see. This is your control surface. This is your volume. If we want to get the mass of the mass, we will take the mCv as the differential volume And dm will be the mass of the differential volume, which is the density of the row multiplied by the differential volume Now integrate this into the control volume, and you will get the mass of the mass So this is the integral of dm Control volume So this is the integral of rho dV controller. So, this is your mCv. Now you can calculate the rate of this. This is your dm by dt Cv. This will come out here. This will be d by dt integral Cv rho dv. So, this is a simple mathematical framework. It is a simple explanation. Let's move forward and try to find out how to get the differential condition. If you consider that the mass flow is in and out from the control surface, and to represent that, we have drawn a differential element on the surface, which we call dA. This is the dashed I am doing. So, to get the differential mass flow rate, which we tried earlier to explain, we know what we did earlier, we took the rho, and we have to calculate volumetric flow rate so we have to multiply volumetric flow rate with dA and then V is the normal velocity in this differential element if this velocity is equal to normal velocity then what will be the normal velocity of this velocity multiplied by cos theta so this is your rho V n dot n So this will be your dot product, dA, which means rho V cos theta.

Now you integrate this into your Net. Right, we have already read this. We are writing it back in a different way. So, this is the control surface. So, this is your net flow. Now notice that in the net edge, both of your net can come. It can also come as much as it goes inside. Because it is not only coming out, but it can also come inside from this surface. So, it will effectively capture both directions. The velocity is opposite to the normal, the perpendicular direction is different So it will capture this direction from this particular vector So that's why this is effectively capturing these changes.

Now let's see it carefully Because we said that Total mass entering the control volume during delta t Minus total mass entering the control volume during delta t So leaving and entering are written entering and leaving control and what I just drew this effectively captures the control surface from where the mass is coming and going it is representing it, so this part is somewhere related to this. and this part is related to this part which is change in control volume so we will write it in simplified way as we said that dmCv by dt is the total mass entering this is total mass in minus total mass out is coming as a little bit later. As we have written in the left-hand side, mass entering minus mass leaving is equal to net change in mass within the control volume. So, I have written the same way, mass in minus mass out is equal to dmCv. Now, we can use the definition written in differential form. Conservation of masses. I can do it in two ways. First, I will rearrange this expression. Now, this is the d by dT. Then we can segregate this part m out and m in as specifically taking the area which is corresponding to the out. I will say it again. And this part, the out and in part, you can segregate it. You can also write the mass out as well. Rho which is normal is out and area corresponding to the out part is the cross-sectional part. The area in this is related to the inlet and the area in this is related to the Outlet. So, you understood how to take a conservative mass. Now let's see some specific examples. If the process is steady state, then there will be no change in the control volume mass. It will always remain the same. In such a case, the rate of change of mass with time, which means that if the steady state process is there, then dmCv by dt is zero. This means that conservation mass dmCv by dt is equal to m in minus m out. So, if this becomes zero, then these two are bars, which means that this relation comes. The total amount of mass that enters the control volume will be the same mass that will come out of the control volume. This is your steady flow process. There are multiple inlets, many inlets and many exits. That doesn't mean that we are doing summation. It may be that you have many inlets. But the mass that goes inside the control volume, the same mass comes out of the control volume. so, we'll move ahead. Let's move ahead and try to understand this steady flow process through the exam. Here you have a device with two inlets and one outlet. So, in this particular example, if you want to balance, then naturally if it is in steady state, then the amount that goes inside will be the same amount that will come out. This means that the sum of M3 that is coming out will be the same. There are many similar engineering devices like nozzles, diffusers, turbines, compressors, pumps. Normally, there is only one stream in this. like if you have a pump, you have only one fluid and you are pumping or compressing it or using it with a compressor in turbine also, if the fluid comes, you have only one inlet and outlet so the stream is one so if the stream is one, then in the steady flow process, your simple m1 is equal to m2 which means the one that came in, it went out you can do this with density and volume too that the density that has gone into the fluid inside, according to m1 and the cross sectional inlet of the velocity and area and similarly you can write m2 as roh2 v2 A2 in single stream so this is one way to write so now let's talk about incompressible fluid in general, there are many devices in which there is a compressor in which single stream will be used If it is single, then mass flow rate will be same. But volume flow rate can change. And this can be due to two reasons. Your main density can also change. Like in volume flow rate, rho, mass is into flow rate Volume is flow rate Velocity into area Velocity can be changed or area can be changed So in such a case, when there is a single stream here, then this condition will definitely match. This will be correct because this is your steady flow process. In this case, your m1m2 will be there. if density Like in this case, it's not necessary that the density is fixed. Because if the density is compressed, then the density can change. But it's acceptable that it's incompressible, so in such cases, your density is If it is incompressible, you can cut the density. It will be removed. This condition will be valid only in steady flow and incompressible. If it is single, then V in volumetric flow rate multiplied by V out will be. You can also take it from velocity. What is V in? This is your condition. Steady incompressible flow. You can simplify this in such situations and solve many examples.

So, in this particular lecture we studied the conservation of mass principle and applied it on a steady flow rate. In the next lecture we will move forward and also talk about the first law of thermodynamics and the particular energy principle. But mainly we will introduce some more new terms. That's it. I hope you will join us in the next lecture. Till then, bye-bye.