

**Engineering Thermodynamics**  
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**Week-08**  
**Lecture-36**  
**Exergy**

Welcome to part 2 of exergy. In this lecture, we will learn more about exergy and especially, we will discuss exergy change. In the last lecture, we learned that exergy is the maximum useful work which can be taken out of any system under given condition and the final state of the system can be dead set or will be in that process. Exergy is a type of energy that can be converted into work. We have termed it exergy. And you can extract it through reversible work. So, let's move forward with this discussion. And especially let's discuss on this that if you have a system given to you, let's say that it is a closed system and it is fixed mass, then it has internal energy. How much of it do we have? useful work.

You can imagine that you have installed a heat engine and you have brought the process and system to the environment with the help of a heat engine. The work is converted into something else. The same work will be your maximum useful work, which will be exergy. So, if we derive this, we will do something like this. We will take a system, which is your closed system. We also believe that if you take a system with that system, let's say if you talk about piston cylinder, then you can also move its boundary. So, if we take a system, Initially, the temperature is P and T. The question is how much of the internal energy will you convert, or will you access the system? This is the piston. P will come from P to P<sub>0</sub> finally and T to T<sub>0</sub> which is the condition of your environment. So, in this process, the energy that you will change, how much heat you will convert, you will convert this heat reversibly. For this, your reversible device should be the process. So, we will take a heat engine for this, which will do some particular work. We are assuming this as differential, that in this process, P to P<sub>0</sub> and T to T<sub>0</sub>, in any moment, delta Q is the heat and heat engine is taking it and rejecting something in it which is in our environment and the rest is converted to your heat engine according to the work and in this process some work is also being done which is boundary work and we will call it useful work Now you can balance it with the energy balance.

Let's do it. So, the energy balance is applied to this system.

$$\delta E_{in} - \delta E_{out} = dE_{system}$$

$$-\delta Q - \delta W = dU$$

$$\delta W = PdV = (P - P_0)dV + P_0dV$$

$$\delta W = W_{b,useful} + P_0dV$$

$$\eta_{th,rev} = \left(1 - \frac{T_0}{T}\right) = \frac{\delta W_{HE}}{\delta Q}$$

$$dS = \frac{\delta Q}{T}$$

$$\delta W_{HE} = \left(1 - \frac{T_0}{T}\right) \delta Q$$

$$= \delta Q - \left(\frac{T_0}{T}\right) (T' dS)$$

$$\delta W_{HE} = \delta Q + T_0 dS$$

$$\delta Q = \delta W_{HE} - T_0 dS$$

So, we have called it fixed mass and closed system and we are taking this piston. It is a system and can be called a device. Now, if we consider the system in this, this is the system. So, E in is the only heat transfer from the system. So, this is minus delta Q which is going out. And what is E out? The work we are doing outside, so it is assumed that it has worked. And this is your dU and change in kinetic energy potential of the system is assumed to be negligible and zero. Now, let's discuss delta W. What will come out of this? Because we have a piston cylinder in this, so notice that we have seen two things. One is delta WBV, which is working on the surroundings. So, delta W will come out, which is delivering the piston cylinder. Let's assume that the initial pressure is PdV, But the external pressure, So this PdV is more connected to the boundary works in this piston cylinder device. There is no other work associated with this. But when P is expanding, some additional work is being done against it, because there is atmospheric pressure. So, the actual work will be like this. P minus P0 plus P0dV. Isn't it? So, this total actual work is... Some parts are going to get lost, which is happening on the surrounding. The only useful work is this, which we will call WHE Useful. So, this was your Delta W. Now let's talk about Delta Q. Since it is reversible, and in such cases, the relation of Delta Q coming in the heat engine is related to the eta reversible with Delta W. And in the reversible process, you can consider it as Carnot and this process is happening from T to T0. T is your temperature source which is T0 is your Sink. So, eta is your thermal reversible 1 minus T0 by T and this is your delta WHE divided by del Q. Because we have considered that this part is your reversible heat engine which you deque. There is no final difference in this. It is slowly getting calmed and slowly changing and reversibly happening. So, delta Q will of course keep changing, as the temperature of the system will also keep changing. But T, in any condition, the differential change that will happen, and its corresponding eta is this. And we can relate it to the differential work that is being done, when this differential Q is being received from the system. So now you can take out W like this that del W is H E which is 1 minus T0 del Q and you can convert del Q into dS because you know that in the case of reverse work D S is equal to del Q by T and D S is here so here you We can write it as 1 minus T0 by T You can also call it as del Q minus T0 by T And here you can replace it. Note the system you are converting from, how you should use it here. T will be cut here, but because it has come out of the system, and the relation is in this, when Q comes out of the system, S will be less. And when Q is added, S will increase. So, the DS of the system will be lessened. So here

the negative sign will be seen. So, this is the value of  $\delta Q$ . Because  $\delta Q$  is a value, but when we talk about  $TdS$ , it is negative because it flows. So, this  $T$  is cut, so this will be plus  $T_0$ . Consider this expression as a whole. We can also call it  $\delta Q$  is  $\delta W_{HE}$  minus  $T_0 dS$ . So, this expression is the result. Talking about the total work, the total work that we will do here, one will be your  $W_{HE}$ , which is doing the heat engine, and the other one is doing the piston.  $\delta w$  total is useful if your piston is not selected and the boundary is not changing then naturally your boundary work will be zero only the heat engine will be there so this is your  $\delta w$  plus  $\delta w$  is also useful so this is the result if we rearrange so this is 1 And if this is 2 and this is your initial, let's say, you put this in star equation, initial energy balance, so 1 and 2, if you put 1 and 2, equation 1 and 2 in star, So we put equation 1 and 2 in star, which means the expressions of  $\delta w$  and  $\delta q$  in star, which is your energy balance.

$$\delta W_{total,useful} = \delta W_{HE} + \delta W_{b,useful}$$

$$-\delta Q - \delta W = dU$$

$$-\delta W_{HE} + T_0 dS - \delta W_{b,useful} - P_0 dV = dU$$

$$\delta W_{total,useful} = \delta W_{HE} + \delta W_{b,useful} = -dU + T_0 dS - P_0 dV$$

So, you will get a total of  $w$ . So as an example, I am trying to replace this expression. So left hand side is minus  $\delta q$ . So this will be minus  $\delta W_{HE}$  plus  $dT_0$  because we said that minus  $\delta q$  minus  $\delta w$  is your  $du$  so now this is left hand side and first term, second term minus  $\delta u$  is your this is your  $\delta w$  is useful minus  $p_0 dv$  is  $du$  Now we have to add  $\delta w$  to this Let's do some useful work We will sum this We can rearrange this  $\delta w$  achieve plus  $\delta w$  be useful The boundary is useful work. This is your minus  $du$  minus  $p_0 dv$  plus  $T_0 ds$ . If you take the expression of both the right-hand side and the  $du$  here, then this expression comes out. So, this expression came out. Now if you integrate this, which is given state from  $t$  and  $p$  and from here you have to go to  $t_0$  and  $p_0$  so if we do this integral then what will come out and we have called this left hand side  $\delta w$  total useful so what will come out  $w$  total useful equal to  $u$  minus  $u_0$  plus  $p_0 v$  minus  $v_0$  or minus of  $T_0$  so we integrated this in final condition  $du$  minus  $p_0 v$  plus  $T_0 ds$  Note that when you integrate minus  $\delta u$ , you will get minus  $u_0$ . This is the final state,  $u_0$ . So, minus of this  $k$  will come out. Similarly, you can understand that the rest of the things will be the same. That's why we directly gave you this expression. Now, this expression is the total work that is useful to this system. to get from there to the dead state or environmental state. This is a totally useful work which we have reversibly extracted. And this expression is actually exergy. Because this is the definition of it. So, this is exergy and this is extracted for closed system. And this is boundary work of the glass. When there is no boundary work, you can remove it in the second term. Because if the volume has not changed, then you can remove it in the second term. only internal energy and heat extraction is the third term. So, this term is naturally due to heat interaction, and this is due to internal energy change and boundary interaction. So, we write this term in this way. If we say that kinetic energy and potential energy are also present in it, then we add that too. Because as we know kinetic energy and potential energy can be used completely. Therefore, in any change, its reference stage will always be zero. Because it can be used completely. So, to do this, we first put a definition. which is a surgery of your in general core closed system so in this term comes  $u - u_0$  plus  $v_0 v$  minus  $v_0$  minus  $t_0 x$

minus  $x_0$  apart from this if there is mass in the system then its kinetic energy plus  $mgh$  you can add this too because this is also your If the kinetic energy possesses potential energy, you can also exploit it. We can do the same in per unit mass. So, this is 5 per unit mass. Remember that symbol  $X$  or  $\Phi$  is normally used for non-flowing closed system. So  $U_0, V_0, S_0$  are the reference of dead state. That's why  $\Phi$  will be 0 in dead state. plus,  $P_0 V$  minus  $V_0$  minus  $T_0 S_0$ . So, in this case, your kinetic energy potential is embedded in this, which is in  $E$  itself,  $U$  plus  $V$  squared by 2 plus  $GH$ . When we have to make a change in the closed system, we will write it in the form of  $\Delta X$ , which is  $X_2$  minus  $X_1$ . We know that when  $X$  is 0 in dead state, because after that, if  $x_1$  is  $u_0$ , then  $x_2$  is 0. But if  $x_2$  or  $x_1$  is not a dead set, then you can also make changes in the exergy. It means that if your initial state is  $t$  and  $p$ , and you are going from  $t_1$  to  $p_1$ , then there will be a difference in the exergy. But if  $t_0$  and  $p_0$  are dead states, and you have not achieved that, then essentially you can find out how much of the exercise has changed. So, in a way, you can find out how much useful work you have achieved from this change of exercise. So, sometimes it matters how much of the exercise change is there in our system, because you can relate it to other things. And we will ask a lot of questions that will be on that. So, in any process, if the process is going on, suppose  $T$  and  $P$ , and you are coming out of here in  $T_1$  and  $P_1$ , so if the corresponding exergy is  $X_1$  and this is  $X_2$ , then the change of  $\Delta X$  will be  $X_2$  minus  $X_1$ . So, you can say it like this. And of course, if  $X_2$  is your final dead state, then the total change will be  $X_1$  only. Because in dead state, exergy will be zero. Always remember that the exergy definition will be in the reference of dead state. And when you subtract here, the reference of dead state gets cancelled. Let's try to find out the expression  $\Delta X = X_2 - X_1$ . If you take mass as per unit mass, then you get the expression  $\phi_2$  and you can also remove this key if this is the key then this will be your note what we are doing here we are removing the dead set  $S_0$  because it automatically because if we do  $\phi_2 - \phi_1$ , the reference will be cancelled so only what remains will remain and you can expand this too you can expand it in such a way that this will be your  $U_2 - U_1$  you can expand  $E$  plus  $P_0 V_2$  minus  $V_1$  minus  $T_0 S_2$  minus  $S_1$  plus mass  $V_2^2$  square is the change of kinetic energy.  $V_1^2$  square 2 by 2 plus  $m$  mass  $g z_2$  minus  $z_1$ . So, this is your  $\Delta X$ . so you can also get mass from this way. This expression. And I am summarizing this thing here. As I said, this  $W_2$  is a totally useful term. Which is nothing but exergy. And we have expressed the difference like this. We have expressed per unit mass in  $X$  and change in exergy is expressed in exergy. You can also do unit mass. If you look at  $\Delta \phi$ , then simple  $\phi$  will be 2-  $\phi_1$ . Or it will come in this form, per unit mass. So, this part will be embedded in  $E$ . Which is kinetic energy and potential energy.

$$X = (U - U_0) + P_0(V - V_0) - T_0(S - S_0) + m \left( \frac{v^2}{2} \right) + mgz$$

Per unit mass

$$\phi = (x - x_0) + P_0(V - V_0) - T_0(S - S_0) + \frac{v^2}{2} + gz$$

$$\phi = (e - e_0) + P_0(V - V_0) - T_0(S - S_0)$$

$$\Delta X = X_2 - X_1$$

$$= m(\phi_2 - \phi_1)$$

$$= (E_2 - E_1) + P_0(V_2 - V_1) - T_0(S_2 - S_1)$$

$$= (U_2 - U_1) + P_0(V_2 - V_1) - T_0(S_2 - S_1) + m \frac{(v_2^2 - v_1^2)}{2} + mg(z_2 - z_1)$$

When property of system is not uniform,

$$X_{system} = \int \phi \delta m = \int \phi \rho dV$$

let's discuss exergy further. If the property is not uniform of the system, whenever you want to change the mass in the differential form, you can integrate it as well as the rest of the properties. But, to do this, you need to have a relation, like the relation of  $\Phi$  with the volume. If you have all these relations, then you can integrate the system with the alarm. So,  $\Phi$  will be integrated with  $\Delta M$ , and you can write  $\Delta M$  as density into  $dV$ . So, if there is a change in  $\Phi$  across the cross-section in the cross-section volume, or if there is a change in the property of phi, then you can integrate it and remove it. So, this way you can solve problems. But since we are discussing this, we will not solve problems in such a way that your exergy is non-uniform. Note that exergy is a state property. As you define it, you get its value. And when exergy is in flow, at any time, like when it takes a nozzle, exergy in this nozzle The changes are negligible because the state is not changing. There is no change in the environment or the conditions. So, temperature, pressure, etc. If there is no change, the energy, the energy change will be negligible, it will be zero, if it is in a steady state. So, if there is no change in the steady state in the system, then in such cases, the change in the energy, the energy change will be zero. As we talk about your steady state. In a closed system, positive or zero will always remain, and the exergy will never be negative.

Now let's talk about the flow system. You have noticed in the closed system, when we talk about the flow system, then in this, the exergy, because it is also flowing, so the exergy already there in the system, which is non-stationary system, plus the additional exergy that will come out, which will be due to the flow. So, let's first see the flow of energy. If the flow is fluid, then we discussed the flow work before some lectures. So, when we defined the flow work, we told it to imagine that there is a piston and how the fluid is working through this piston. And we said that the PV has a flow work. So, this work is imaginary boundary work and that will be your flow work. That will be your exergy. So, the flow energy that is connected to exergy is the PV term here. And it will do against this  $P_0$ . So, it will have to be minus. So, PV minus  $P_0V$ . The total volume displaced is P minus  $P_0V$ . So, this will be the total exergy connected to this flow work. The maximum energy you can extract from this. So, the flow system of exergy will be non-flowing system plus X flow. So, we will have to add this term to this. And we are seeing the same thing here. For example, if we talk about per unit mass in a non-flowing system, So, this will be U plus  $U_0$  plus this term which we did plus this term. So, this is your non-flowing which we saw in the last slide. Now, we will add this which is happening due to flow. And the flow work has come out from here. The corresponding x has come out which is connected to the flow. Now, you rearrange this because this term is also here. So, then you can see that You can take out the PV term from this. Here, we will multiply P and V and put them in this. And if we take out the minus term, then we can take out  $U_0$  and  $P_0V$  from here. This is a very important thing. Take out  $P_0V$  from here. When you take it out, this expression can be converted into enthalpy very easily. So, if you pay attention, then I have expanded it here. And this term, I have expanded it here. If we add this, it will cancel, and these two terms will remain. This term will go to U plus PV and this term will go to  $U_0$  plus  $P_0V$  and what is this? Enthalpy. And this thing is H minus  $H_0$ . So, this comes out as H- $H_0$  minus  $T_0S$  which came out here. And the rest is kinetic energy plus potential energy. So, if you see, the only difference in the flow system is that you have written internal

energy in enthalpy. This is the only difference you have made. As you have seen earlier, when the flow system comes, we use enthalpy. And we use internal energy in the flow system. So, when it comes to exergy, we have used enthalpy instead of internal energy. But you have seen its derivation, why it is used, it has a reason. Because flow is the work with which exergy is connected, we added it with non-flowing term. So, this expression is later derived. So as we discussed about change in exergy in the flow system, similarly, you can also take out change in exergy in the flow system. The expression of this will be same only this change will come, rest will be similar. So, this term was changed, in this enthalpy will be changed. And this is your flow exergy, its symbol we have changed. We used phi in closed system, in closed system we will use psi. So, this psi was used, and the difference of psi is your exergy changed in closed system. But enthalpy will change, minus  $d_0$  is  $2$  minus  $s_1$  plus kinetic energy plus potential energy is changed. So, this represents, like in your closed system, change in exergy represents the maximum amount of useful work that we can do.

Flow exergy:

$$\Psi = (h - h_0) - T_0(s - s_0) + \frac{v^2}{2} + gz$$

Exergy change:

$$\Delta \Psi = \Psi_2 - \Psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) + \frac{(v_2^2 - v_1^2)}{2} + mg(z_2 - z_1)$$

Similarly, if your work is consuming, then you will have to do minimum work to change that system. You can think in two ways, that exergy change represents maximum or minimum work, depending on what you are talking about. You can think of the stream and exergy in the same way. This is reversible, remember that whenever we are removing it, it represents reversible, so it is W reversible. And it can never be negative in closed system. But the flow system, if the pressure, its exergy can be negative, because it is more than  $P_0$ , then notice that the flow exergy, particularly the flow, the exergy of the flowing system, can be negative. Let's try to understand the concept of exergy by using an example. In this case, we have said that this is a 200 m<sup>3</sup> rigid tank which has to be compressed at 1 MPa and 300 K. We have to find out how much work we can extract from this air if the environment condition is 100 kPa and 300 K. To do this, we need to do exergy change environment naturally is that is the maximum work you can do So we have to find  $x_1$  which is exergy which is  $m s_1$  mass x exergy per unit mass because this is a closed system Now to find  $\psi_1$  you have to find  $m u$  minus  $u_0$  plus  $p_0 v$  minus  $v_0$  minus  $T_0 s$  minus  $s_0$  plus  $V_1$  square by 2 plus  $gz_1$ . Now we assume that there are no changes in your Z and in this sub, there are no changes in kinetic energy and in this we are neglecting the system. We will consider air as ideal. So, we have considered air as ideal. And since your reference is dead state, we will also consider that these terms So this term will be zero because in reference it is also 300 Kelvin environment is also 300 Kelvin, initial is also 300 Kelvin and it is an ideal gas U is the function of temperature for ideal gas ok so only temperature only function of temperature means only temperature depends on it so it means U minus  $U_0$  will be zero because there will be no change in U So, this is the only remaining term.  $P_0 V$  minus  $V_0$ . So, your x is  $1 M P_0 V$  minus  $V_0 S_1$  minus  $S_0$  Now, we can remove the mask. pressure, volume,  $t_1$ , p, v, q is 200m<sup>3</sup> and  $t_1$  is given so in fact you have all the information so if you plug in the information 100kPa, 200m<sup>3</sup>, 0.287kPa m<sup>3</sup> per kg Kelvin and this is 300 Kelvin so you will get 2323 kg so you will get mass now the question is how to do other things to do other things you have to take out first term  $P_0 V - V_0 - T_0 S_1 + S_0$  so to take this out we will use simple information of Ideal Gas  $P_0 V_1$  minus  $V_0$  will be  $P_0 R T_1$  by  $P_1$  minus  $R T_0$  by  $P_0$  because it is a specific volume so we can write  $R T$  by  $P_1$  easily

and this will be  $R(T_0 - P_0) \ln P_0$  by  $P - 1$ . Note that the temperature is fixed.  $T_1$  is  $T_0$  only. Only the pressure changes. So  $P_0$  by  $P$  is minus 1. Now, in the same way,  $T_0 S_1 - S_2$ . Since it is an ideal gas, you can write  $S$  as  $C_p \ln T_1$  by  $T_0$ . The  $\Delta S$  will come out.  $C_p \ln T_1 - T_0 \ln P_1$  by  $P_0$  this is zero. So, it came out.  $\ln P_1$  by  $P_0$ . Now you can plug in this thing with the alarm because you have all the information.  $P_0$  is also known as your 100kPa. You have  $P_1$ ,  $P_0$  also known and  $P_1$  is your 1000kPa and this is your 100kPa. So, you know this information.  $T_0$  is 300 kPa. So, you know both these terms. So, you will get  $\phi_1$ . So, it will come out easily. It comes out to you. 120.76 kJ per kg. And  $x_1$  will come out to you.  $M_1$  into  $\phi_1$ . So, it is important to get this expression. This expression. This will also be removed because you have all the information. This will also be removed easily. If you plug in like this, then the final answer will be removed. I have avoided doing all the details, to save time, but you can do it easily. And this comes out as 281 MJ. So, this is its potential to work in this system. The one we have just removed is  $X_1$ , this is its potential. can do maximum useful work to this compressed air. Now let's move forward, let's discuss one more thing, let's try to understand through the refrigerant system. In this you have a compressor which takes air in these conditions and particularly refrigerant in this and it releases you in this condition. The surrounding environment is  $T_0$  at 20 degrees Celsius. So, the condition is 20 degrees Celsius of  $T_0$  and  $P_0$  is 95 Kilo Pascal. Now what we have to remove is that the exergy chain of the refrigerant is in this process. And since it is a compressor, something will work. So, we have to remove this minimum work. So, we will consider it as reversible for minimum work. So, this can be removed from the exergy. We have to remove the exergy  $\Delta X$ . We will do per unit mass. I will not write everything. But the first important thing is how we will do it. First thing is we will consider it as steady state. So, this is the steady state condition. And second is that the key changes in this table are negligible. The third thing is that you have a table to see. Because this is a reference, you have to see the table. So, you have to remove the table of inlet condition, of inlet and outlet. If you look here, then you will get the properties  $H_1$  and  $S_1$ . and you will also get  $S_2$  so you can use this as an inlet and outlet condition using the table data will be obtained. Now when this data is obtained, you can extract the exergy change of the refrigerant from the alarm. For that, since this is a flowing system, we will have to extract  $\Delta \phi$ . And we are talking about per unit mass, so this is  $\phi_2 - \phi_1$ , sorry, there is  $\psi$  in it, so we will have to extract  $\Delta \psi$ , we are talking about per unit mass. This is  $\psi_2 - \psi_1$ . And this is your  $S_2 - S_1 + V_2^2 - V_1^2$  divided by 2. This is the difference of kinetic energy and  $G$  is  $z_2 - z_1$ . So we assume that there is no change in this. So basically this is kinetic energy, and this is zero. We just have to see this thing  $S_2$  and  $S_1$  can be taken out of the table. Similarly,  $S_2$  and  $S_1$  can be taken out of the table.  $T_0$  is the temperature of the environment, which is given as 20 degrees. Which is 293 Kelvin. So, in this way, you can take out  $\Delta \psi$ . Which comes out of this particular example. So, what I mean to say is that when the refrigerant's exposure increases in this particular system, this is the minimum work required to get this refrigerant system from here to here. So, this is the minimum work exergy had to do. I hope you understood how exergy is calculated for a flowing and non-flowing system. And how to use it to understand how to get maximum work output or minimum work in. We will continue this discussion in the next lecture. We will talk about the destruction of the body and how the entropy generation and acceleration destruction is connected. So, we will meet again in the next lecture.