Basics of Mechanical Engineering-1

Prof. J. Ramkumar

Dr. Amandeep Singh

Department of Mechanical Engineering

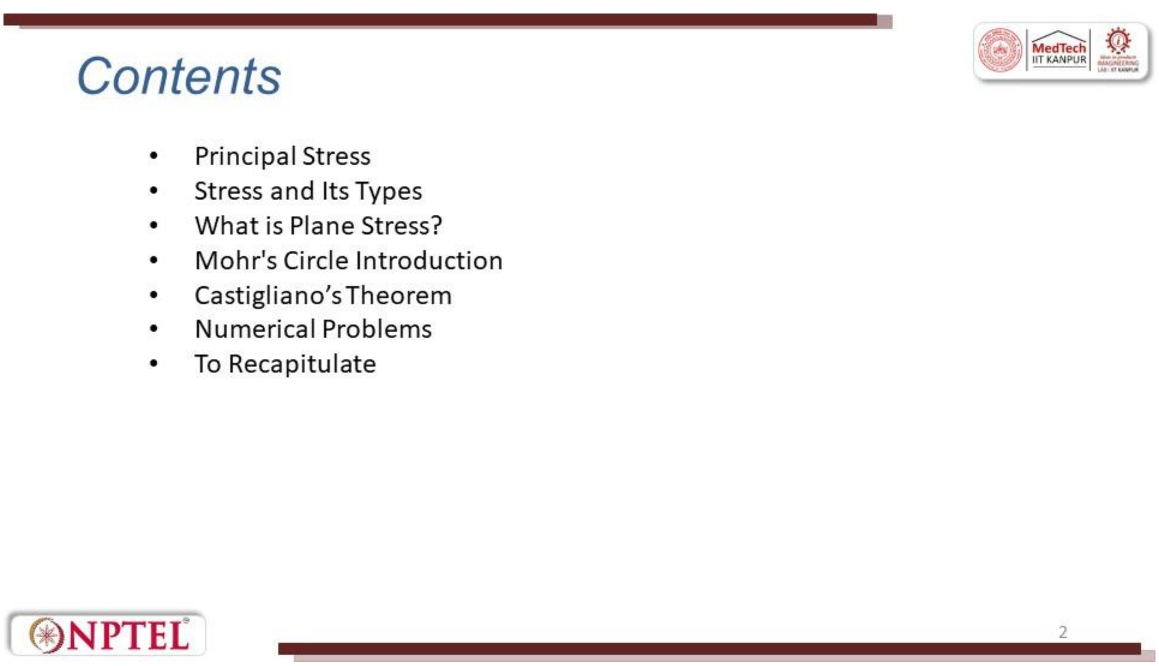
Indian Institute of Technology, Kanpur

Week 03

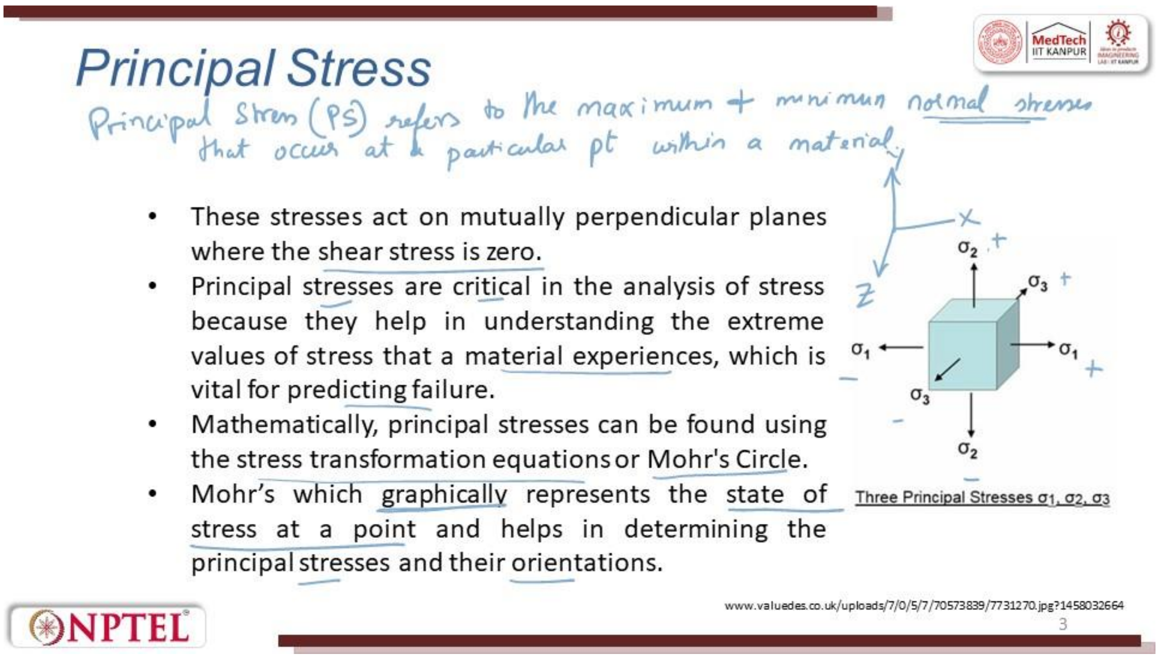
Lecture10

Principal Stress and Castigliano's Theorem

Welcome to the new lecture on Principal Stress and Castiglione's Theory. Till now in the course, we have been studying about forces, units, then we moved into stress-strain relationship. From there we moved towards material. Homogeneous, heterogeneous and under heterogeneous, we saw anisotropic material.



In this lecture, we will predominantly focus on Principal stress. Why is this important? When we are trying to manufacture a part to meet out to the requirements, we have to do some stress analysis. In order to make our stress analysis more closer towards the real application, we have to have some brief knowledge about principal stresses and this theory. In this lecture, the content we will see principal stress, then Stress and its type, what is Plane Stress Condition, then introduction to Mohr's Circle diagram, then Castigliano's theory, one or two numerical problems, and finally, we will have a recap.



So, principal stress. Principal stress is henceforth I will call it as PS refers to the maximum and minimum normal stresses that occur at a particular point within a material. Why is this important? Because at a particular point the stress levels can be very high.

At particular point the thermal stresses can be high because of the friction whatever happens. So, it is always a good idea to find out the principal stress. Principal stress refers to maximum and minimum normal stress. I hope you will remember what is normal stress that occurs at a particular point within a given material. These stresses act on mutually perpendicular plane where the shear stress is zero.

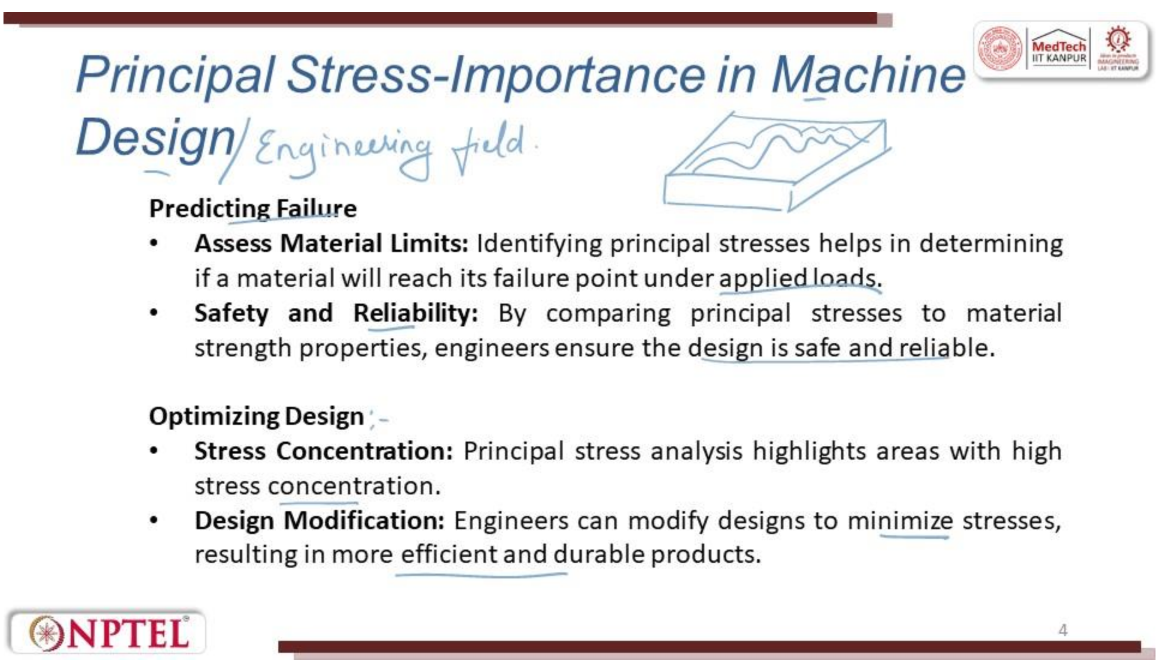
This is very important. So, these stresses act on mutually perpendicular. So, if you try to take this as x, y and z, they are mutually perpendicular. So, that is what you see here. Σ 11, this is positive, negative, this is positive, negative, this is positive, you can have negative.

Σ 11, σ 22, σ 33. The principal stresses are critical in the analysis of stress because they help in understanding the extreme values of stress that a material experiences which is vital for predicting failure. In the entire plane, there can be few points where there is maximum stress. Because of this maximum stress, the failure can start.

The rest of the plane will be as normal as possible, leaving that three points. That's what is very important. Principle stresses are critical in analysis because it tries to understand the stresses that material experience. So that, it can try to predict the failure. Mathematically, principal stresses can be found out by a tool called Mohr's circle. Or it can be done through stress transformation equation.

That means to say, σ 1 1 to transform to σ 2 2 or σ 1 1 to shear whatever it is. We will see some problem in that direction. We will try to solve some problems in the tutorial. So mathematically, principal stress can be found using stress transformation equation or Mohr's circle. Mohr's circle is a graphical representation of the states of stress at a point and helps in determining the principal stresses and their orientation.

Mohr's circle is easy. It is a graphical representation. And in the graphical representation, we try to represent the state of stress at a point. So, when we stress at a point which helps in determining the principal stress and their orientation.



Why is principal stress important in machine design or part design? Whatever it is, let it be a camera stand, let it be a chair, let it be a laptop, let it be engine shaft.

For everything, we do a design and if you do it for a machine, it is called as Machine Design. So why is it very important? It helps us to predict failure, assesses material limits. It identifies the principal stresses which helps in determining if a material will reach its failure point under applied loads. When we talk about safety and reliability, we sit in a chair with the confidence that the chair will withstand our load.

We sit in a car hoping that the car has all safety measures and whenever I drive at 100 kilometers per hour, all the mechanical parts has a reliability that it will not fail. Think of it, the flight tries to fly at a speed of 400 kilometers per hour or 800 kilometers per hour. A typical car in an express highway road travels at a speed of 100 kilometers per hour. How are we able to reach this speed? Some cars even go 160 kilometers per hour.

How are they able to drive this? The driver has a confidence that the system will not fail. That is nothing but reliability. You have to inbuilt safety and reliability into your system. So, safety and reliability by comparing principal stresses to material strength properties, engineers ensure that the design is safe and reliable.

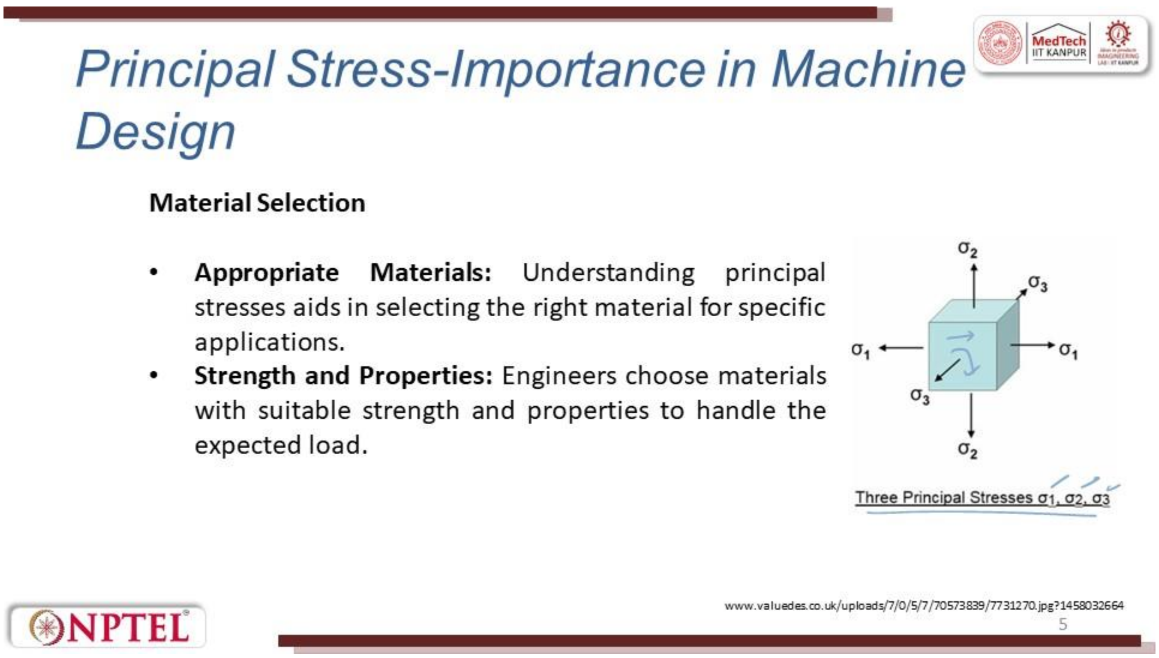
All mechanical components will be designed for safety and reliability. A process will be developed for its repeatability. Please try to distinguish the difference. A chair, a spoon, a keyboard, a TV, everything will be designed for its reliability and safety, here in which our prime focus is only on mechanical loading. So, why is it important in machine design or in engineering field?

Our focus here is more towards mechanical, in engineering field is because it helps in predicting failure. Can the same theory be used for electromagnetism? Yes. When we try to build a circuit or a small device which is used in aerospace industry, automobile industry or biomedical, there is a EMI and EMC test which is done. So, where in which the safety and reliability of the circuit will be predicted, right.

And more than predicting the failure, it helps today in optimizing the design. So, here the stress concentration points are figured out. And accordingly, the part geometry are nowadays made complex. For example, earlier we used to have flat plates for resting objects. Today, we have started making grooves in the flat plate through which the stress concentration points are eliminated and through which the lubricants can easily flow.

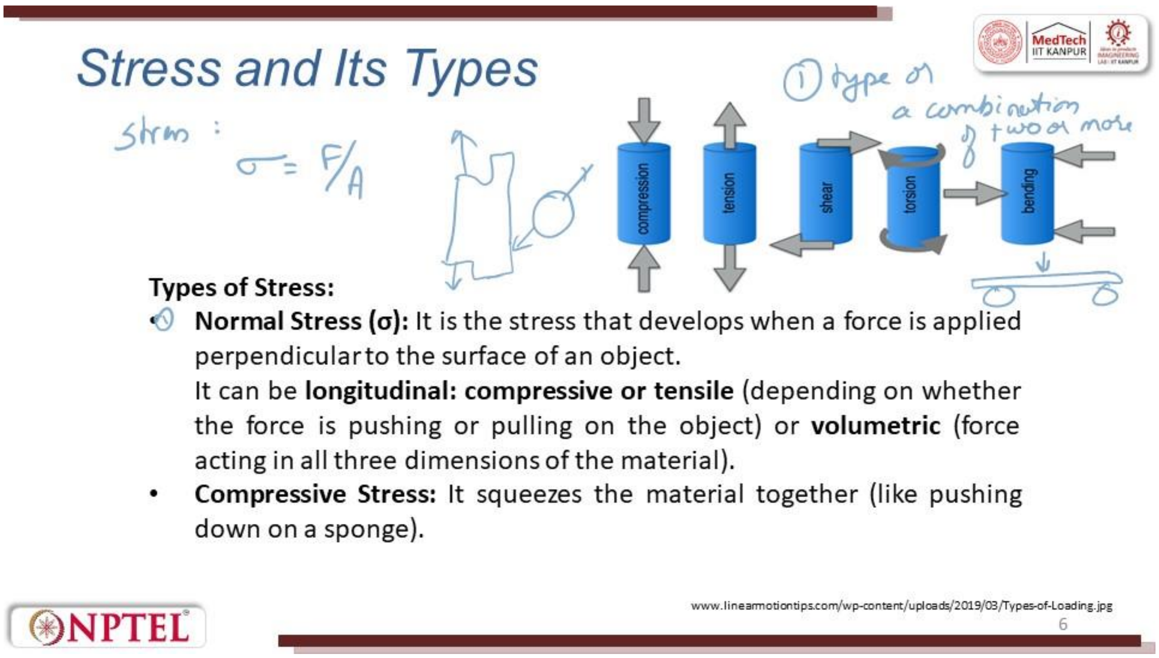
By doing so, the friction is reduced. So, once the friction is reduced, the longevity of the component is enhanced. So, optimizing design, it tries to find out the stress concentration and helps in changing the design. So, Stress Concentration, principal stress analysis highlights the areas where high stress concentration points are there. Next, Design Modification.

Engineers can modify design to minimize stresses resulting in a more efficient and a durable product. So, according to the prediction of failure and optimization, the material choice is made.



Appropriate materials are chosen for a given application. Appropriate materials, understanding the principle stresses aids in selecting the right material for specific application. The next one is strength and properties.

Engineers choose materials with suitable strength and properties to handle the expected load. So as I told you in the beginning, there are three principal stresses, σ 1, σ 2 and σ 3. All the three are principal stresses acting mutually perpendicular to each other. Moment there is a shear load, you have to resolve the σ 1, σ 2, σ 3 accordingly to get the shear or the torsional load.



Stress and its type. So, we will try to write down the stress as a small recap.

$$\sigma = \frac{F}{A}$$

There are several different types of stresses. Predominantly, these are the five different types of stresses which act on any given material. It can be one type or a combination of one or I can say two or more.

More types can happen. Typically, in real-time application, no single load acts. Typically, in a real-time application. There will always be a combination. It can be compression stress and torsion.

It can be tensile and shear. It can be compression and bending. So, all the five are important. We will always try to keep these five different types of forces to solve any problem. So, compression, sometimes once you compress it leads to bending also or it can be exclusive bending.

For example, if you sit on a plank, you sit then it is bending. If you compress a spring, compression happens leading to bending. Tensile, of course, yes. Then shear, when we are trying to apply a torsional load, we try to push it and then try to do a torsional load. So, there can be a shear component, there can be a torsional component.

Shearing is almost slipping of cards which we saw and application for shearing is all the metal cutting operations undergo shearing force. Torsion is drilling. When you do a drilling operation, when you do a screw, a turning operation to fasten, it is torsion. So, you will have five different types of stresses which act. So, stresses means five different types of loads.

Types of stresses. Normal stress. So, if you go back, I have defined the normal stress here. So, what is a Normal stress? Normal stress is a stress that develops when a force is applied perpendicular to the surface of the object. You should also understand many a times the object might have a profile like this.

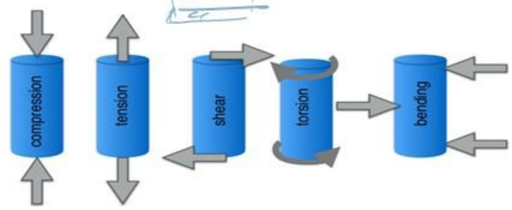
When you apply tension here, it is not perpendicular to the surface of the object. Or we can try to take a ball. You try to take a load like this. So in real time it can happen. So that is why we are defining normal stress very clearly.

Normal stress is the stress that develops when a force is applied perpendicular to the surface of the object. It can be longitudinal stress. It is compressive or tensile depending on whether the force is pushing or pulling on the object. It can also be volumetric where the force acting in all the three dimensions of the material. For example, hydrostatic pressure.

You are trying to take an object inside water. Compressive stress, so this is one type. Compressive stress, it squeezes the material together like pushing down on a sponge.

Stress and Its Types

- **Tensile Stress:** It is the stress, that tends to pull the material apart (like stretching a rubber band).
- **Volumetric Stress (Bulk Stress):** It takes place when the deforming force affects an object in all three dimensions, changing its volume.
- **Shear Stress (τ):** It acts when an object is subjected to opposing forces that act parallel to each other, but in opposite directions.
- **Torsional Stress:** It occurs when a material is subjected to a twisting or rotational force, the individual layers of the material deform and get twisted relative to one another, leading to shear stresses within.



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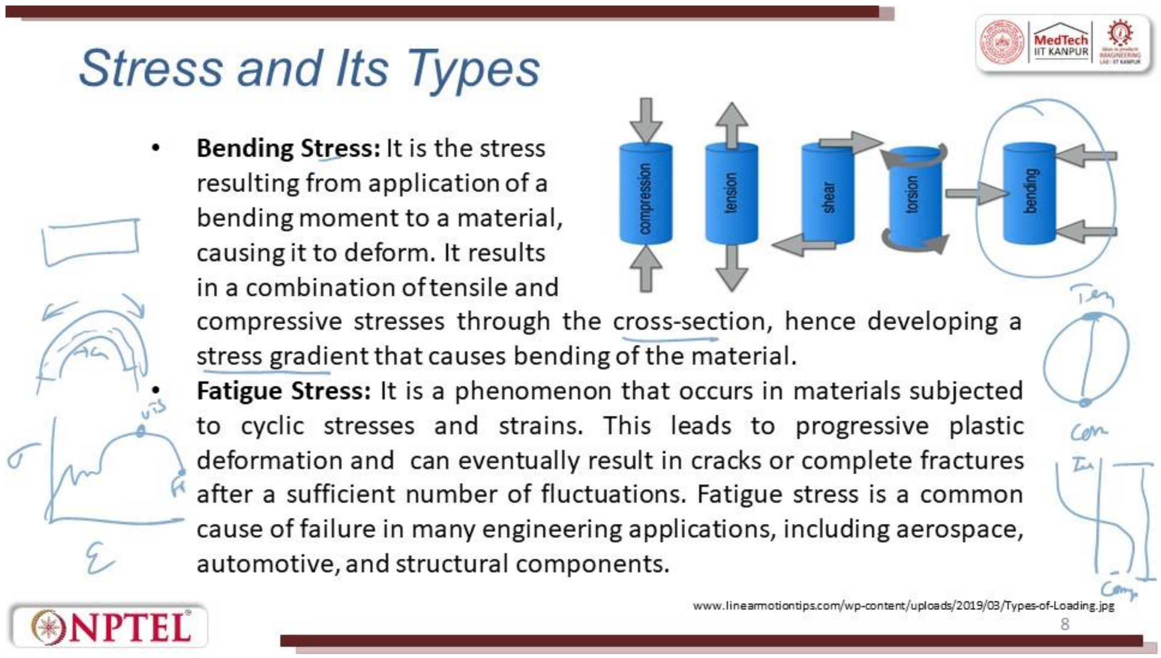
The third one is Tensile stress. It is the stress that tends to pull the material apart. We saw an example of rubber band. Volumetric stress, it takes place when the deformation force affects an object in all the three directions, changing its volume, is volumetric stress. Shear stress τ (τ), it acts when the object is subjected to opposing forces that act parallel to each other, but in opposite direction I said, you have liquid which is flowing, right, liquid which is flowing, this is a plate, there is a liquid which flows through that.

So, now, if the top surface is moving and bottom surface is stationary, right, so then it is one face moving in the opposite direction of the other. So it acts when the object is subjected to opposing force that acts parallel to each other but in opposite direction. Torsional stress, it occurs when a material is subjected to twisting or rotational force. The individual layers of the material deforms and get twisted relative to one another.



There can be a bending stress. This is bending, right. So, when you are trying to take a rod and bend it to make into a hook, that is bending, right. When I take this pen and I try to bend it. So, it is the stress resulting from application of bending moment to a material causing it to deform. When I bend it, you see it is interesting, when I bend it, I put a load here.

So, this is compression, this is tensile. So, there is a combination of Tensile stress and Compressive stress. It results in a combination of tensile and compression through its cross section. Hence, developing a stress gradient because here on the top there is a tensile, bottom there is compression. Somewhere the tensile should come and get converted into compression.

So, for example, if you have an object like this, when you try to bend them, you have tensile and then you have compression. So, you have the axis. Somewhere from here to here, there is a transition happening. That is what is stress represented as or told as stress gradient. The stress changes across. So you take a cross section.

Here it is tensile. Here it is compression, from here to here, there is a change. For example, if I say this is tensile, so this is tensile, I say tensile stress along the direction, this is compression. So, here this is a gradient. So, a stress gradient that causes bending to the material.

The other one is Fatigue stress. All these things what we saw are all just based on loads. Normal stress, compressive stress, tensile, volumetric, shear, torsion, bending, all these things are force which is related. What is Fatigue stress? Fatigue stress is nothing but a cyclic loading.

The cyclic loading, like for example, I try to bend, relax. So, that is cyclic loading. I drive a cycle wherein which the pedal is resting on an axle and that axle from top to bottom I move, it undergoes a cyclic load, right. A switch on-off is a cyclic load. A button which you press in your smartphone, on-off is a cyclic load.

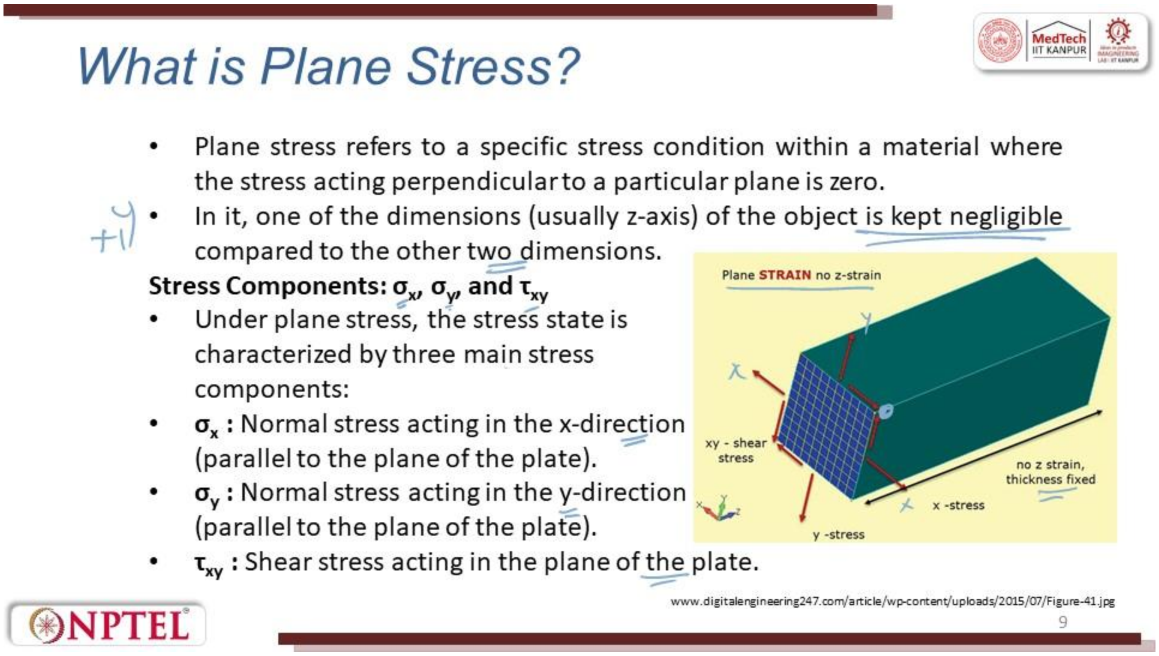
Opening and closing of door, cyclic load. The damper which is kept on the top of a door, cyclic load. Opening and closing of a door when you go to the airport, cyclic load. The sensor undergoes a cyclic load. In an engine, when there is an engine, there are cylinders.

In each of the cylinders, there is a piston which moves up and down. This piston which moves up and down are connected with a crankshaft. So that is again the crankshaft undergoes a cyclic load. Fatigue stress, it is a phenomena that occurs in materials subjected to cyclic stress and strain. This leads to progressive plastic deformation and can eventually result in cracks or complete fracture after a sufficient number of fluctuations.

I keep on doing on and off like this. After some point of time, you will see that this material gives off. So, it can give, when you go back to your stress-strain graph, you can see ultimate strength and then slowly it goes towards, it goes down. So, you have stress. You have strain.

You can see here it goes like this, right. And after ultimate, slowly it falls down. The yield strength falls down towards a fracture. This is UTS, right. So, that is what we are saying.

This leads to a progressive plastic deformation and eventually results in a crack or a complete failure. So, ultimate when it is crossed, then the crack keeps growing and then a fracture happens. In many engineering applications, including aerospace, automotive and structural components.



What is Plane stress? Plane stress refers to a specific stress condition within a material where the stress acting perpendicular to a particular plane is zero.

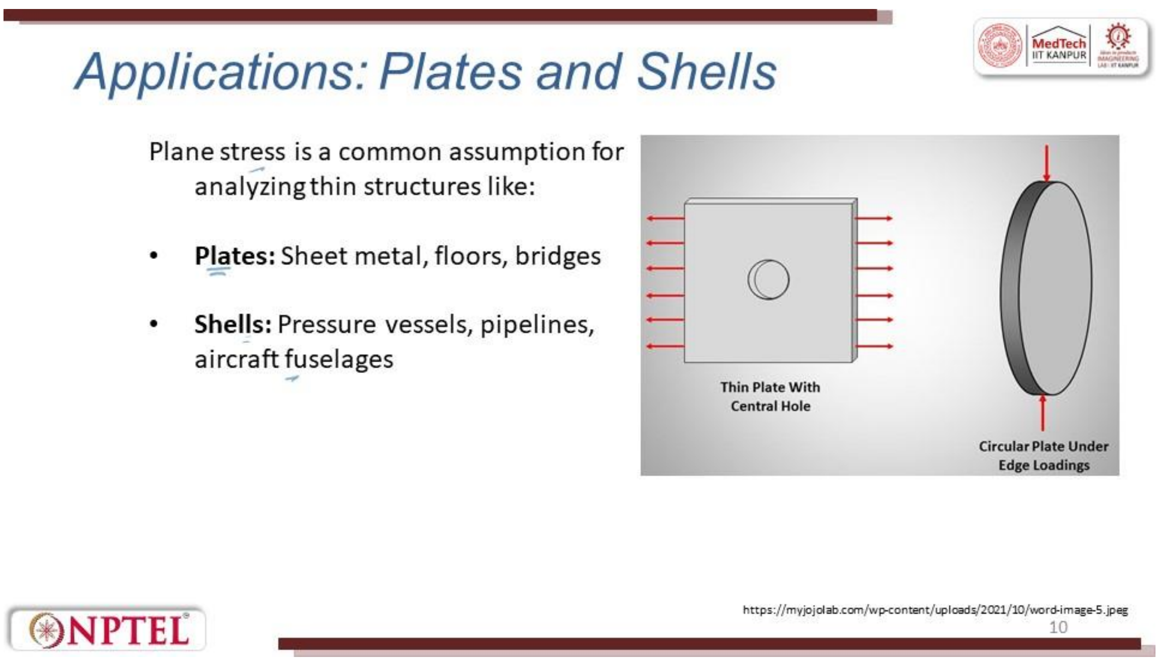
For example, you take an object. Here it is called as plane strain, no in the z direction. This is the X stress, this is the Y stress and when there is a shear action, X and Y, there is a, what happens here is called as a shear stress. So, this also happens here, right. So, this is Y, this is X.

X stress, Y stress and which happens perpendicular to this two is XY shear and no strain thickness is fixed along the Z direction. This is the Z direction. So if you rotate it and keep Z direction is on the top. So in it one of the dimension usually z axis of the object is kept negligible. The z whatever is there it is kept negligible.

For example I have here an object. This object I completely reduce it in the z direction. In the Z direction, okay, let us take it like this. I have X and Y, whatever it is, and then I have a Z direction. Now, what I assume is, whatever I do in the front plane, I will assume that everything happens in the subsequent planes.

So, I assume that no Z strain thickness is fixed, right. So, it is one of the dimension of the object is kept negligible compared to other two dimensions. So, here what are the two dimensions? X and Y. The stress components are going to be σ_{xx}, σ_{yy}. So, when I have an object and when I am trying to pull the object, there will be shear also coming into existence, right.

So, τ_{xy}. Under plane stress, the stress state is characterized by three main stress components, σ_x, σ_y and τ_{xy}. Σ x is normal stress acting in the x direction. Σ y is the normal stress acting in the y direction. So, x direction is parallel to the plane of the plates. Y direction is parallel to the plane of the plates. τ_{xy} is the shear stress acting in a plane of the plate.

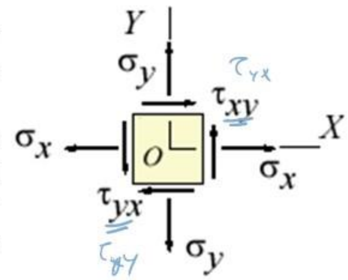


So, plate and shell at both places we use the plane stress condition. Plane stress is a common assumption for analyzing thin structures like plates and shells. This is a plate.

Plate means sheet metal, floor, bridges are taken as plate. Shell means it is pressure vessel, pipelines, aircraft fuselage are taken as shell. Thin plate with a center hole. Cylindrical plate under edge loading. So, plane stress is a common assumption for analyzing thin structures like plates and shells.

Mohr's Circle - Introduction

- Mohr's circle is a cornerstone in the analysis of plane stress problems.
- Mohr's circle is the 2-D representation of transformation equations for plane stress problems.
- It is a powerful graphical tool that allows you to visualize the relationship between the normal stresses (σ) and shear stresses (τ) acting on a point within a material under plane stress conditions.



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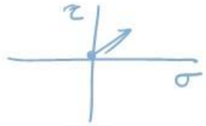
As I told you earlier, Mohr's circle is a graphical representation. It is very easy to represent it in a graphical form. So, now let us understand Mohr's circle. Mohr's circle is a cornerstone in the analysis of plane stress problems. So the stresses which we already saw in the previous slide, square.

It is a cube. From a cube, we have said it has made into a square. Square, you have x direction, y direction. Σ along the x direction are σ_x. Along the y direction are σ y. And then perpendicular or I would say shear comes as τ_{xy} and τ_{yx}. Please note down the notation.

So, otherwise what will happen if you try to write it in a inverse, here if you try to write it τ_{xy} and τ_{yx}, instead of yx, you write xy, then it is not the correct convention. Mohr circle is a cornerstone in the analysis of plane stress problems. It is a 2D graphical representation. It is a 2D representation of transforming equations from plane stress problems. Mohr circle is a 2D representation of transformation equations for plane stress problems.

It is a powerful graphical tool that allows you to visualize the relationship between normal stress σ and shear stress acting on a point within a material under plane stress condition.

Mohr's Circle



How it works:

1. **Axes:** Mohr's circle is plotted with normal stress (**σ**) on the horizontal axis and shear stress (**τ**) on the vertical axis.
2. **Plotting the In-plane Stresses:** The known in-plane stresses, σ_x and σ_y, are plotted on the horizontal axis at their respective values.
   - The shear stress, τ_{xy}, is not directly plotted on the circle itself.
3. **Center and Radius:** The center of the circle lies exactly halfway between σ_x and σ_y on the horizontal axis.
   - The radius of the circle represents the magnitude of the maximum shear stress (τ_{max}) within the material.

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How does it work? The first thing is axis. Mohr circle is plotted with normal stress σ on the horizontal axis and shear stress τ on the vertical axis. Horizontal axis, this is horizontal axis, I represent σ and this is vertical axis, I represent τ.

Mohr circle is plotted with normal stress on the horizontal axis and shear stress τ on the vertical axis. Plotting the in-plane stresses, the known in-plane stresses σ x, σ y are plotted on the horizontal axis. This is the horizontal axis. The shear stress τ xy is not directly plotted on the circle itself. Center and radius, moment you represent, you have to represent, I said more circle. So there has to be a circle, right.

So, circle should have a center and radius. The center of the circle lies exactly halfway between σ x and σ y on the horizontal axis, right. The radius of the circle represents the magnitude of the maximum shear stress within the material. So, you should understand, more circle will have axis, then it will have axis, will have stress, it will have vertical axis τ. Then, since I said circle will have a center point and a radius.

So, now we are trying to understand where will the centre point be and what will be the radius. The radius of the circle represents the magnitude of the maximum shear stress τ max within the given material.

Mohr's Circle

To draw a Mohr's circle for a typical 2-D element, we can use the following procedure to determine the principal stress.

Define the Shear Stress Coordinate System:

- Define the coordinate system for the normal and shear axes.
- Tensile normal stress components are plotted on the horizontal axis and are considered positive.
- Compressive normal stress components are also plotted on the horizontal axis and are negative.

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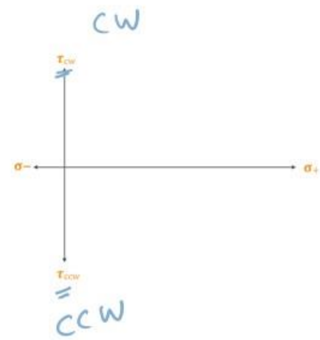
To draw a Mohr's circle for a typical 2D element, we can use the following procedure to determine the principal. So, define the Shear Stress Coordinate System. Define the Coordinate system for the normal and the shear stress.

So, this is minus σ_x and this is τ. Tensile normal stress components are plotted on the horizontal axis and are considered positive. Compressive normal stress components are also plotted on the horizontal axis and are assumed to be in the negative direction. So, positive, negative.

Mohr's Circle

Define the Torsional Coordinate System:

- For the construction of a Mohr's circle, shearing stresses are plotted ABOVE the normal stress axis when the pair of shearing stresses, acting on opposite and parallel faces of an element, form a CLOCKWISE (cw) couple.
- Shearing stresses are plotted BELOW the normal axis when the shear stresses form a COUNTERCLOCKWISE (ccw) couple.



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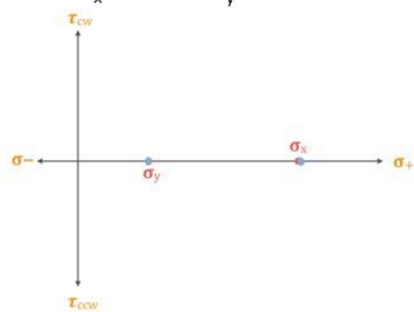
For the construction of a Mohr circle, shear stress are plotted above the normal stress axis when the pair of shearing stresses acting on opposite and parallel faces of an element forms a CLOCKWISE (cw) couple.

So, you will have τ kept on the top as well as τ here and here. So, this is clockwise, this is COUNTERCLOCKWISE (ccw). Shear stress are plotted below the normal axis when the shear stress form a counterclockwise couple. So, this is CLOCKWISE couple, this is COUNTERCLOCKWISE couple.

Mohr's Circle

Plot the Shear Stress Values:

3. Plot the shear stress values given in the problem statement, or plot generic points on the σ_x axis and σ_y axis as shown:



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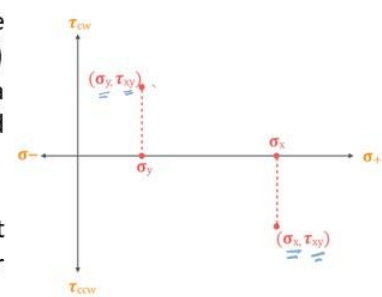
Plot the shear stress value. For plotting the shear stress value given in the problem statement or plot generic points on the σ_x axis and σ_y axis are shown here. Σx and σ_y, right. Plot the magnitude of the couple. So, you can see σ_x and τ_{xy}. Σ y and τ_{xy}.

Mohr's Circle

Plot the Magnitude of the Couple:

4. Plot the magnitude of the couple given in the problem statement with a clockwise (cw) couple being plotted above the $\boldsymbol{\sigma_x}$ axis, and a counterclockwise (ccw) couple being plotted below the $\boldsymbol{\sigma_x}$ axis.

- If no values are provided for the moment, plot generic points above and below the **σ** axis for $\boldsymbol{\tau_{xy}}$ as shown:



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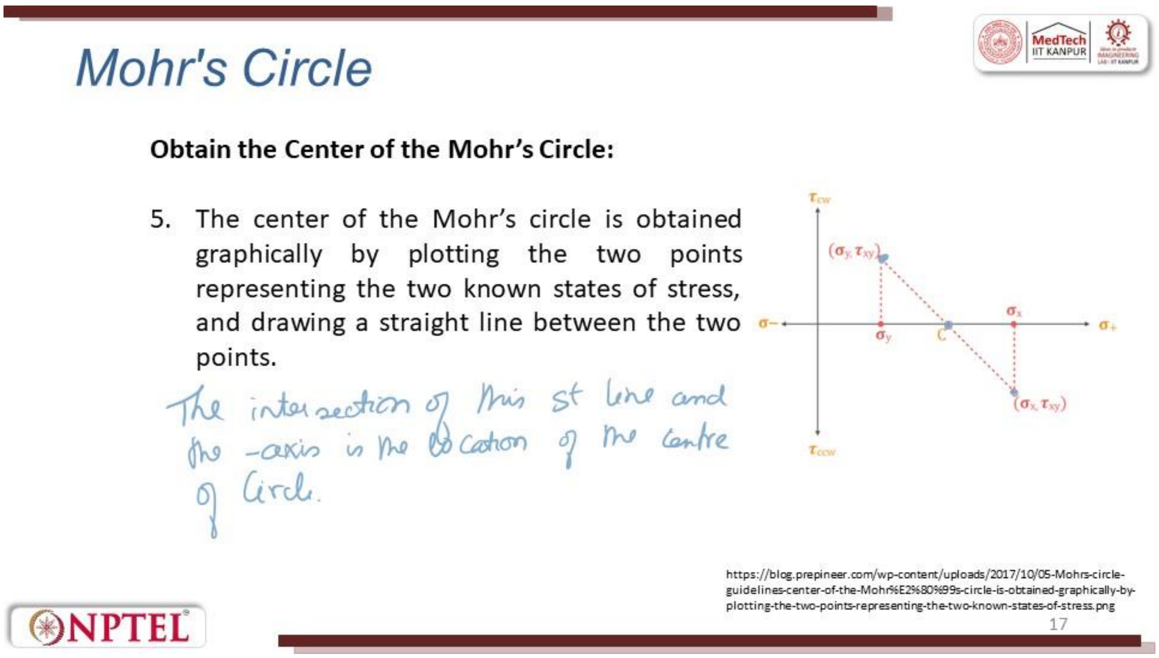
Plot the magnitude of the couple given in the problem statement with a clockwise couple being plotted above the x-axis and a counterclockwise being plotted below the x-axis.

If no values are provided for the moment, plot generic points above and below the x-axis. Σ axis for τ_{xy} as shown in the figure.



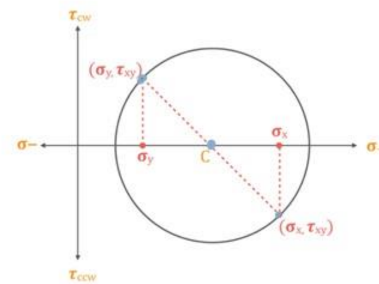
Now, to obtain the center of the Mohr circle, what we do is, we try to connect this point and this point passing through the x-axis. The center of the Mohr circle is obtained graphically by plotting the two points representing the two known states of stress and drawing a straight line between the two points. So here the intersecting point, the intersection of this straight line and the axis is the location of the center of circle.

So, from here, it here passes through the axis that becomes the center of circle. Now what we do is we draw a circle keeping this point as center.

Mohr's Circle

Draw The Mohr's Circle:

6. Draw the Mohr's circle assuming the connection line as the diameter of the circle, using the intersection of the diagonal straight line and the σ-axis as the center of the circle.



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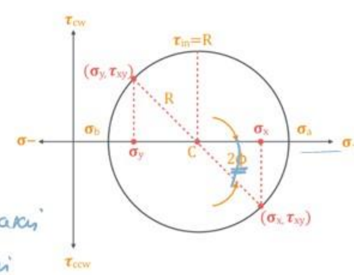
Draw the Mohr circle assuming the connecting lines are the diameter of the circle. Using the intersection of the diagonal straight lines and the σ axis as the center of the circle.

Mohr's Circle

Stress Analysis With The Mohr's Circle:

7. Stress Analysis on Mohr's circle – To get normal and shear stress values at any plane theta, take angle 2ϕ in the Mohr's circle starting from diagonal of the circle and locate a peripheral point as as shown.

Shear stress value will be on the y-axis & normal stress will be in the x-axis



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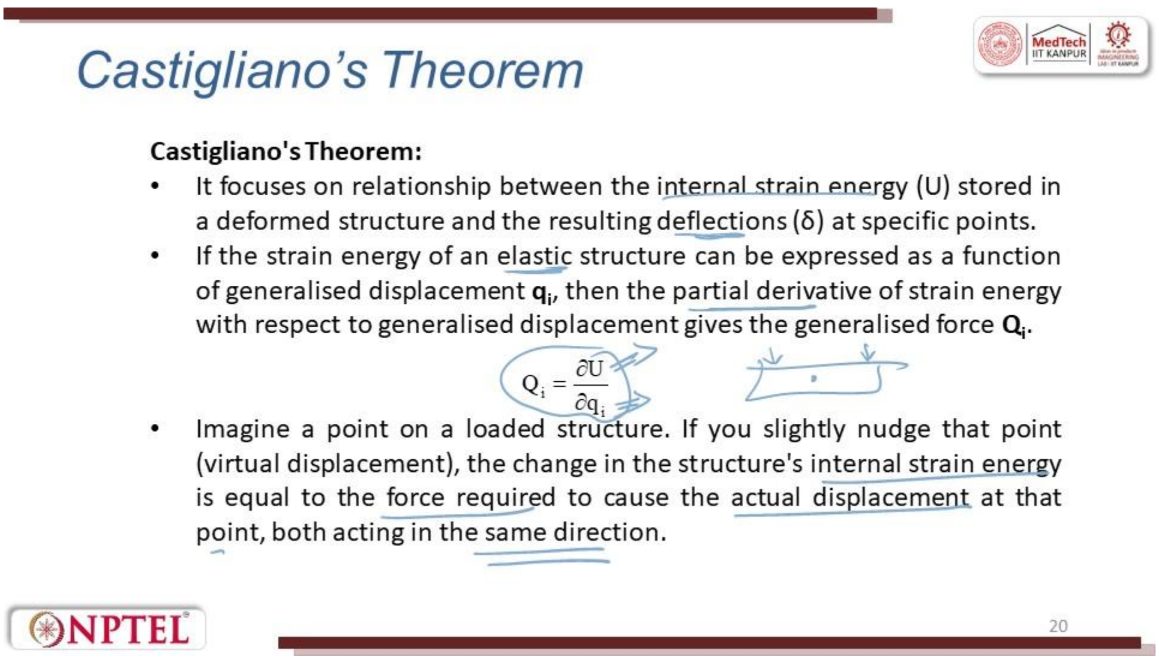
The Stress Analysis with the Mohr circle. The stress analysis on the Mohr circle, how do you find out? It tries to tell to get the normal and the shear stress values at any plane theta. Take angle 2 theta in the Mohr circle starting with the diagonal of the circle and locate a peripheral point as shown. So what we are saying is to get the normal and the shear stress values at any plane theta. This is there at any plane theta take 2 theta.

So, at any plane theta, take 2, this is 2 theta right this is 2 theta. Take angle 2 theta in a Mohr circle starting from the diagonal of the circle and locate a peripheral point as shown. So, now the shear stress value will be on the y-axis and normal stress will be in the x-axis. Clear? This is your y-axis, this is your x-axis.

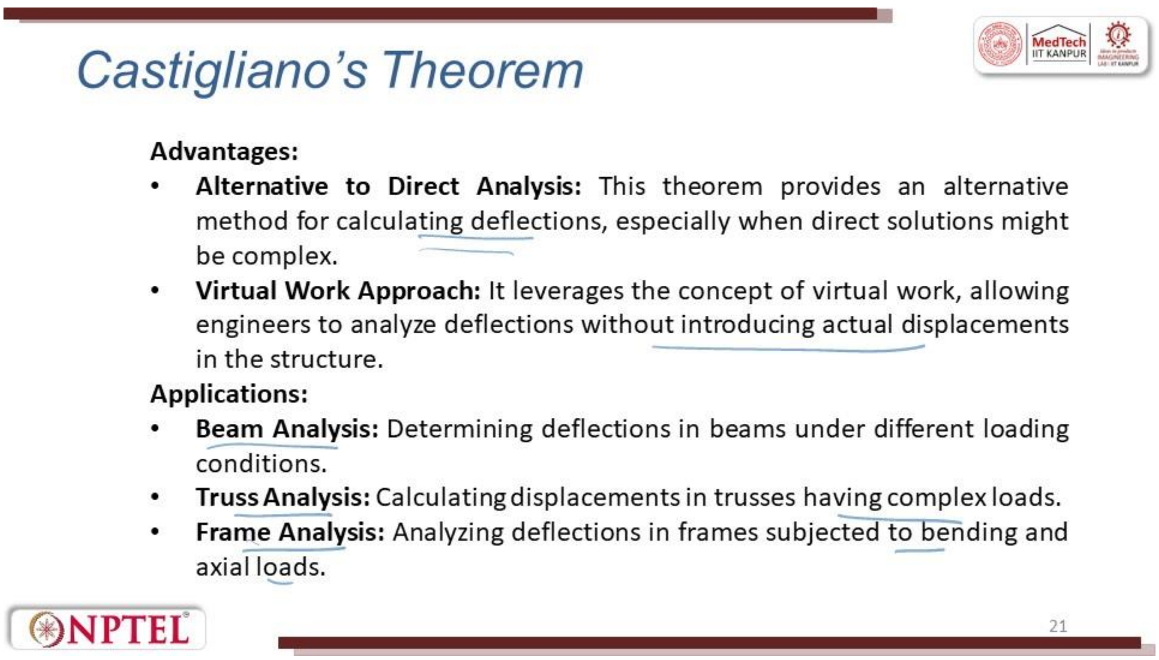


Castigliano's theorem. What is Castigliano's theorem? This theorem states that it focuses on relationship between the internal strain energy U stored in a deformed structure and the resulting deflection del at a specific point. So, it talks about two things. One is internal strain energy U stored in a deformed structure. So when you are trying to deform, apply load, you try to deform.

The relationship between the internal strain energy U stored in a deformed bend structure and a resulting deflection del at a specific point. If the strain energy of an elastic structure can be expressed as a function of generalized displacement Q_i, then the partial derivative of strain energy with respect to a generalized displacement gives the generalized force, which is nothing but

$$Q_i = \frac{\partial U}{\partial q_i}$$

If the strain energy of an elastic structure can be expressed as a function of generalized displacement. When you are elastically doing it in a generalized displacement of q_i, then the partial derivative of the strain energy U with respect to the generalized displacement gives the generalized force Q_i. Imagine a point on a loaded structure, right. If you slightly nudge the point, the change in the structure's internal strain energy is equal to the force required to cause the actual displacement at the point, both acting in the same direction. If you see very closely, if you slightly nudge that point, right. The change in the structure's internal strain energy is equal to the force required to cause the actual displacement at the point both acting in the same direction.

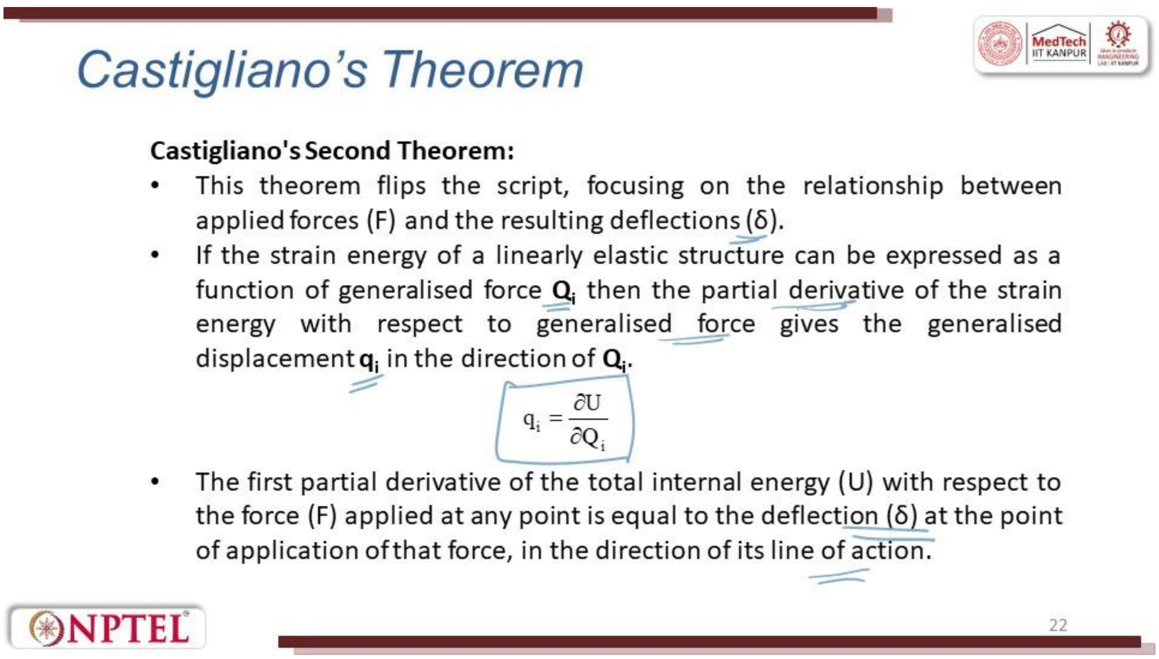


Why is this theorem very important? It has advantages. Alternative to direct analysis, this theorem provides an alternative method for calculating deflections. Especially when direct solutions might be complex. Virtual work approach can be seen.

It leverages the concept of virtual work, allowing the engineers to analyze deflections without introducing actual displacement in the structure. It is used in Beam Analysis, Truss Analysis and in Frame Analysis. Beam analysis, determining deflection in beams under different loading conditions. You have pillar, you will have a beam. This is a beam, right.

So, which is used in bridges or it is used in construction. Determining deflection in beam under different loading conditions, we can use this theorem. Then Truss analysis, truss calculating displacement in truss having complex loading. Then Frame analysis like a wooden frame, aluminium frame for window, analysing deflection in frame subjected to bending and axial load. At all these places, we use this theorem.

So, this theorem basically goes around this equation. In this equation, you will have energy which is stored and you will have Q_i which is nothing but displacement.



Castigliano's second theorem, this theorem flips the script, focusing on the relationship between the applied force and the resulting deflection, del. If the strain energy of a linearly elastic structure can be expressed as a function of a generalized force Q_i, then the partial derivative of the strain energy with respect to generalized force gives the generalized displacement q_i in the direction Q_i. The first partial derivative of the total internal energy (U) with respect to force (F) applied to a point is equal to the deflection del at a point of application of the force in the direction of line of action.

Castigliano's Theorem

Applications:

While less commonly used compared to the first theorem, the second theorem can be helpful in specific scenarios:

- **Compatibility Checks:** Used to verify the compatibility of displacements in structures where deflections are predetermined due to other constraints.
- **Residual Force Calculations:** In some cases, the second theorem can be used to calculate residual forces arising from temperature changes or other effects that induce small deformations.

Both Castigliano's theorems have limitations:

- **Linearity:** They are primarily applicable to linearly elastic materials and structures with small deformations.
- **Strain Energy Complexity:** Calculating the strain energy can sometimes be complex for intricate structures.

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So the second law finds its application while less commonly used compared to the first theorem. The second theorem can help in the specific scenario like compatibility check used to verify the compatibility of displacement in structure when deflections are predetermined due to other constraints. Compatible of displacement in structures when deflections are predetermined due to other constraints. Suppose you have a rod or let us take a shock absorber. The shock absorber spring is there.

It is placed and then it is kept inside a cylinder. So the deflections are predetermined due to other constraints. Residual force calculation. In some cases, the second theorem can be used in calculating the residual forces arise from temperature change and other effect that induce small deflections. Both the theorems have its own limitations.

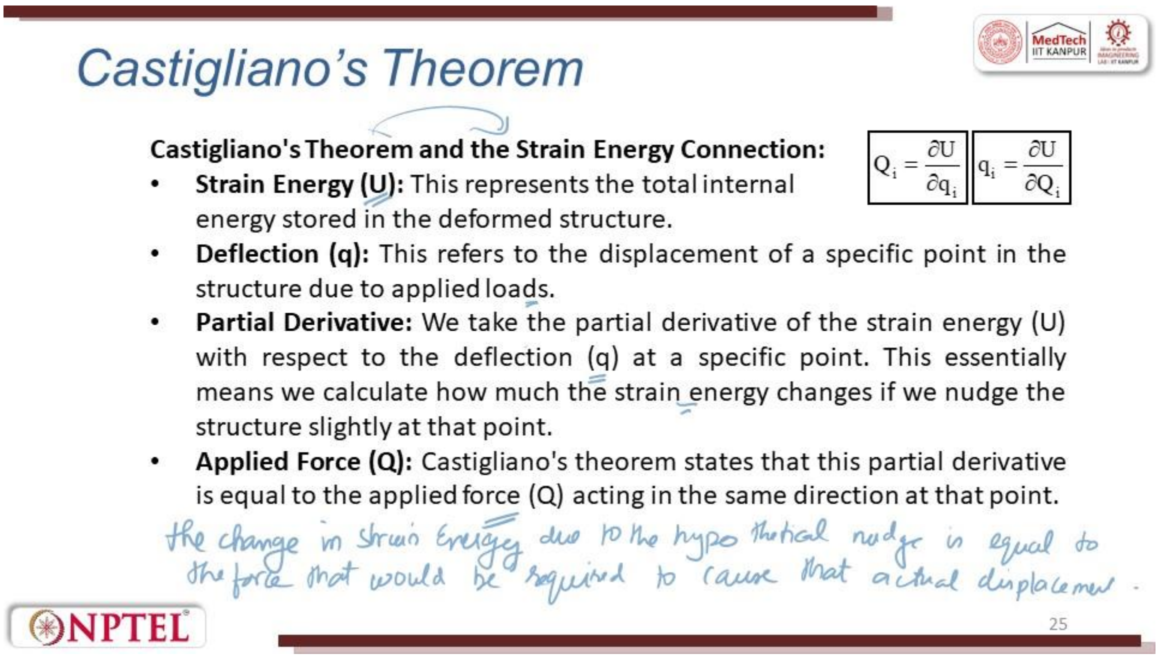
It takes in only linearity and the strain energy complexity. Linearity, they are primarily applicable to linear elastic material and structures with small deflection. Large deflection it cannot, like a cantilever swinging it cannot. Strain energy complexity, calculating the strain energy can sometimes be complex for intricate structures.

Castigliano's Theorem

Strain Energy: The Fuel for Deflection Analysis (Castigliano's Theorem)

- Strain energy is a fundamental concept in structural mechanics.
- It represents the internal energy stored within a deformed elastic material due to applied loads.
- Imagine bending a beam; the strain energy captures the invisible "work done" on the material to cause that bend.

The strain energy theorem, the strain energy, the fuel of deflection analysis. Strain energy is a fundamental concept in structural mechanics. Strain energy, it represents energy. The internal energy stored within a deformed elastic material due to applied load. So now you will be very well versed with elastic material, deflection, applied load, everything. Imagine bending a beam, the strain energy captures the invisible work done. There is a work done, right. Invisible work done on the material to cause that bending.



So, Castigliano's theorem and the strain energy connection. What is the connection between these two what we studied? Strain energy is U. This represents the total internal energy stored in the deformed structure.

Deflection is Q. This refers to the displacement of specific point in the structure due to the applied load. Partial Derivative. We take the partial derivative of the strain energy (U) with respect to the deflection Q at a specific point. This essentially means we calculate how much strain energy changes if we nudge the structure slightly at the point. Nudge is press. Applied force (Q). The applied force in the theorem states that this partial derivative is equal to the applied force (Q) acting in the same direction at a point.

In other words, the change in strain energy due to the hypothetical nudging. Nudge is equal to is equal to the force that would that would be required to cause that actual displacement. In other words, the change in strain energy due to the hypothetical nudge is equal to the force that would require to cause the actual displacement.

Castigliano's Theorem

Why is it Important?

By understanding how strain energy changes with deflections, engineers can use Castigliano's theorem to:

- **Calculate Deflections:** They can indirectly determine the deflection of a structure at a specific point by analyzing the change in strain energy due to a hypothetical "virtual" displacement.
- **Optimize Designs:** Knowing how strain energy relates to deflections allows engineers to design structures that can handle loads without excessive bending or deformation.

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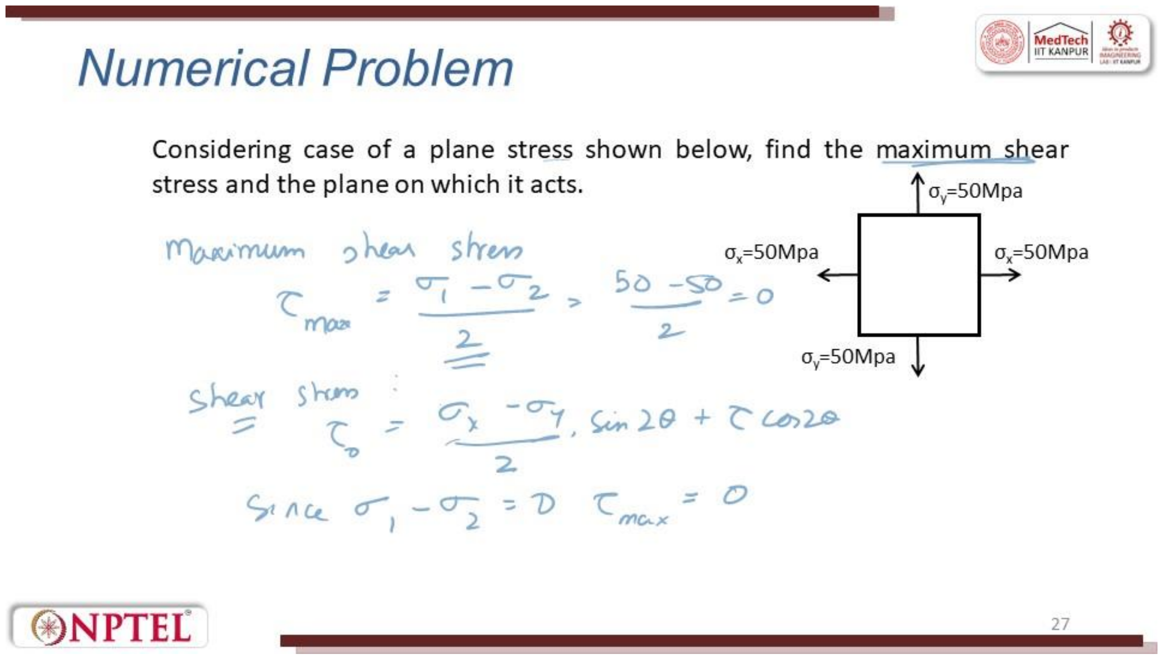
Why is it important? By understanding how strain energy changes with deflection, engineers can use

Castigliano's theorem to calculate deflection optimized design. This can indirectly determine the deflection of a structure at a point by analyzing the change in the strain energy due to hypothetical virtual displacement. It also can use in optimized design knowing how strain energy relates to deflection allows the engineer to design structures that can handle load without excessive bending or deflection. Let us try to solve a numerical problem.



Consider case of a plane stress shown below. Find the maximum shear stress and the plane on which it acts.

Maximum shear stress,

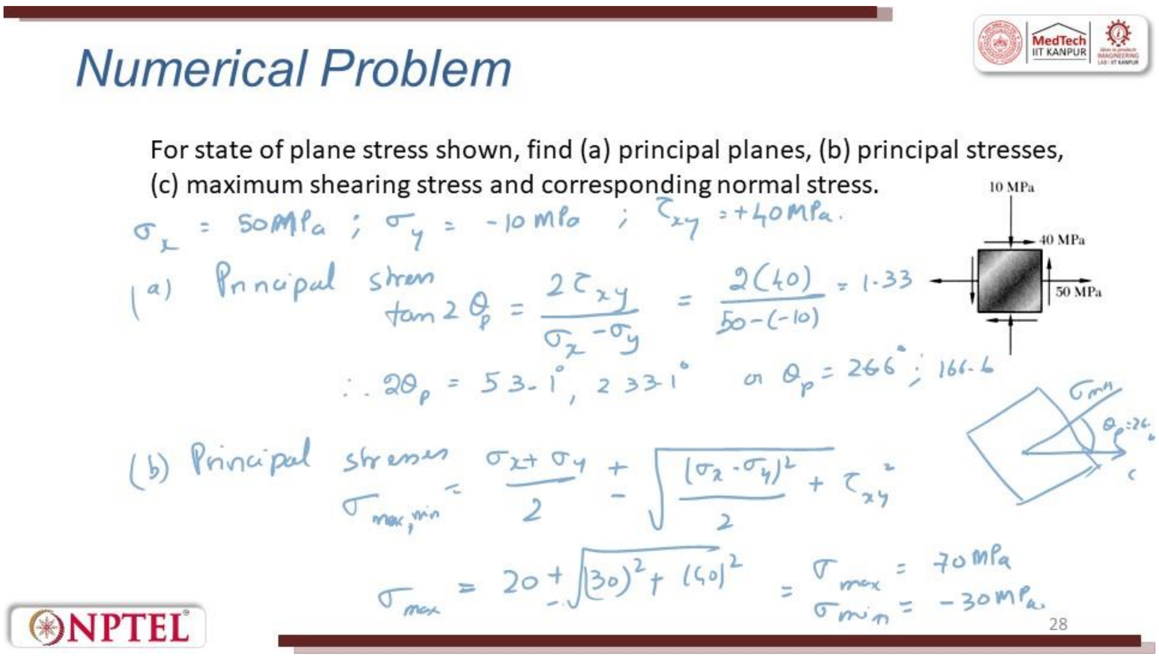
$$\tau_{max} = \frac{\sigma 1 - \sigma 2}{2} = \frac{50 - 50}{2} = 0$$

Shear stress,

$$\tau_0 = \frac{\sigma x - \sigma y}{2} . sin2\theta + \tau cos2\theta$$

$$Since, \sigma 1 - \sigma 2 = 0, \tau_{max} = 0$$

So, plane strain condition show the maximum shear stress. So, we will try to find out the shear stress through this formula acting on it. So, the shear stress is going to be defined like this.



Let us try to solve one more problem for the plane stress shown. Find A, principal plane, B, principal stress, and C will be maximum shear stress and corresponding normal stress, where σ_x = 50 MPa, σ_y = -10 MPa and τ_{xy} = +40 MPa.

a) Principal Stress

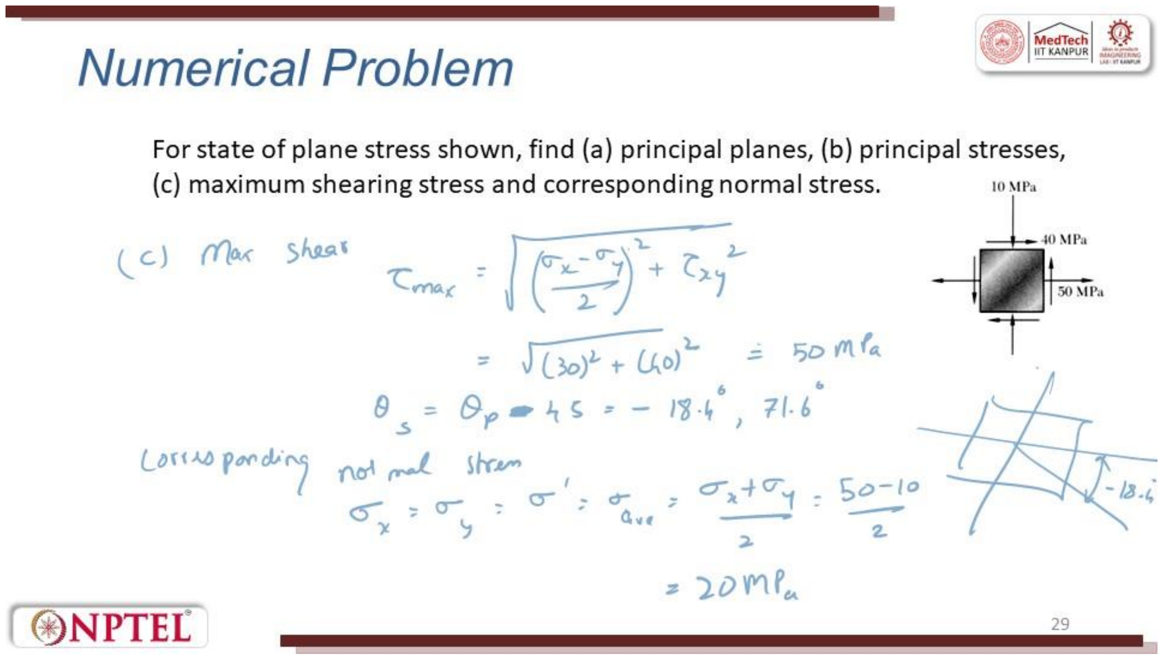
$$tan2\theta_p = \frac{2\tau\tau_{xy}}{\sigma 1 - \sigma 2} = \frac{2(40)}{50-(-10)} = 1.33$$

$$\therefore 2\theta_p = 53.1°, 233.1° \; or \; \theta_p = 26.6°, 166.6°$$

b) Principal Stresses

$$\tau_n = \frac{\sigma 2 + \sigma 4}{2} \pm \sqrt{\frac{(\sigma 2 - \sigma 4)^2}{2} + \tau_{xy}^2}$$

$$\tau_{max} = 20 \pm \sqrt{(30)^2 + (40)^2} = \tau_{max} = 70MPa \; and \; \tau_{min} = -30MPa$$



c) Max shear

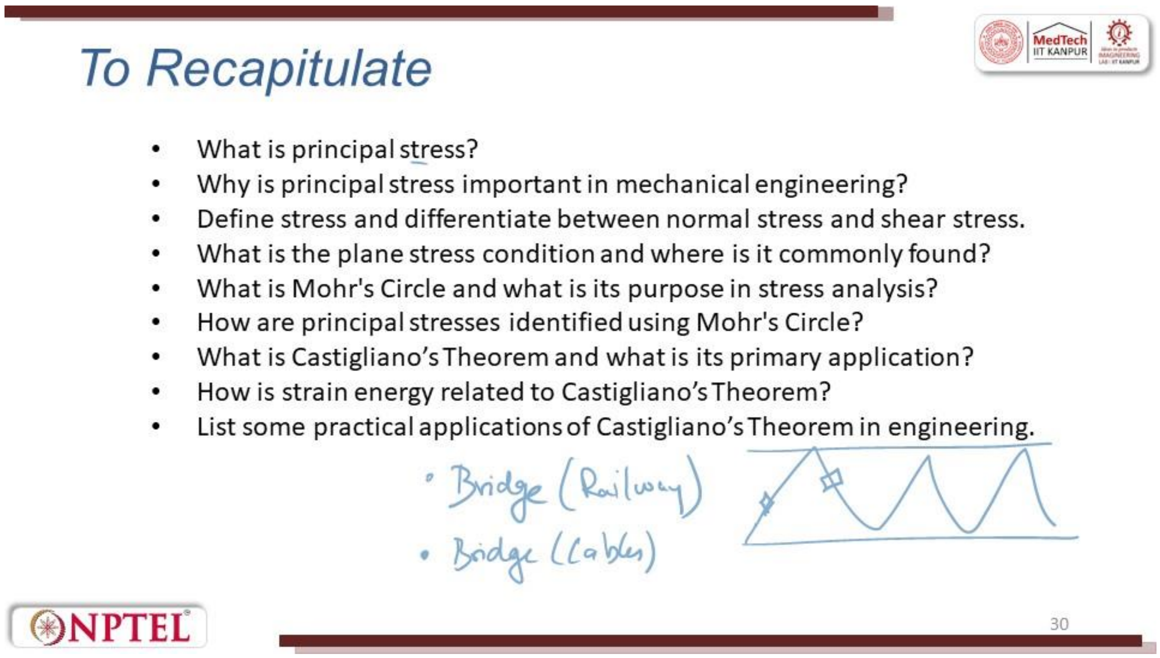
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \tau_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \sqrt{(30)^2 + (40)^2} = 50MPa$$

$$\theta_s = \theta_p = 4s = -18.4°, 71.6°$$

Corresponding normal stresses

$$\sigma_x = \sigma_y = \sigma^1 = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 - 10}{2} = 20MPa$$



So, what we went through in this lecture is going to be we have gone through principle stress, principle stress it is important in mechanical engineering define stress and differentiate between normal and shear.

What is a plane stress condition? Why is it or where is it commonly found? What is Mohr circle? What is its purpose in stress analysis? How are principal stress identified using Mohr circle?

What is Castigliano theorem? And what is its primary application, how is strain energy related to the theorem, list some practical applications to this theorem in engineering. Now what I would request you is, you try to take any bridge structure, right, railway bridge, where a train crosses, right. So you will always see a structure like this, right and what I want is try to find out how many members are there, what are the stresses it undergoes and if I try to take an element here.

Or here, try to draw the plane stress condition and assume some value, try to draw a Mohr circle. And next, try to take a bridge which is supported by cables. Again, try to do this analysis. Try to see what are the difference in the values and how complex it is to solve. These are the references which we have used in making this lecture.

Thank you very much.