Basics of Mechanical Engineering-1

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Week 04

Lecture15

Tutorial - 1 (Part 1 of 2)

Welcome back to the next lecture in the course Basics of Mechanical Engineering-1. Professor Ramkumar has discussed multiple facets of the basics of mechanical engineering in engineering mechanics in the last three weeks.

Now, hopefully you have submitted the assignments 1 and 2. Now, I would like to take Tutorial session on the topics which we discussed in the week 1, 2 and 3. Regarding week 1, the topics which were discussed were units and Dimensions, Scalars and Vectors, Statics, Kinetics and Kinematics and I will also have some numerical problems on Laws of Motion, Inertia, Moment of Inertia and Momentum.

Units and Dimensions

Basic Formulas and Relationships

S.No.	Physical quantity	Unit
	Fundamental units	
1.	Length (l)	Metre (m)
2.	Mass (m)	Kilogram (kg)
3.	Time (t)	Second (s)
4.	Temperature (T)	Kelvin (K)
5.	Electric current (I)	Ampere (A)
6.	Luminous intensity(Iv)	Candela (cd)
7.	Amount of substance (n)	Mole (mol)
	Supplementary units	
1.	Plane angle (α, β, θ, ϕ)	Radian (rad)
2.	Solid angle (Ω)	Steradian (sr)

Let us have a quick look on the Units and Dimensions, basic formulas and relationships that you discussed in the first week. There are multiple units, seven fundamental units were discussed like length, mass, time, temperature, electric current, luminous intensity and amount of substance and is also a fundamental unit now. With all of them having their respective units.

For example, length as a meter or mass in kilogram, time in seconds or so. Supplementary units were also discussed like plane angles α, β, θ, ϕ which are represented in radian. Solid angle Ω is steradian.

Units and Dimensions

Derived Units and Their Dimensions

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Sr. No.	Physical Quantity	Formula	Dimensions	Name of S.I unit
1	Force	Mass × acceleration	$[M^1L^1T^{-2}]$	Newton (N)
2	Work	Force × distance	$[M^1L^2T^{-2}]$	Joule (J)
3	Power	Work / time	$[M^1L^2T^{-3}]$	Watt (W)
4	Energy (all form)	Stored work	$[M^1L^2T^{-2}]$	Joule (J)
5	Pressure, Stress	Force/area	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
6	Momentum	Mass × velocity	$[M^1L^1T^{-1}]$	$Kgms^{-1}$
7	Moment of force	Force × distance	$[M^1L^2T^{-2}]$	Nm
8	Impulse	Force × time	$[M^1L^1T^{-1}]$	Ns
9	Strain	Change in dimension / Original dimension	$[M^0L^0T^0]$	No unit
10	Modulus of elasticity	Stress / Strain	$[M^1L^{-1}T^{-2}]$	Nm^{-2}

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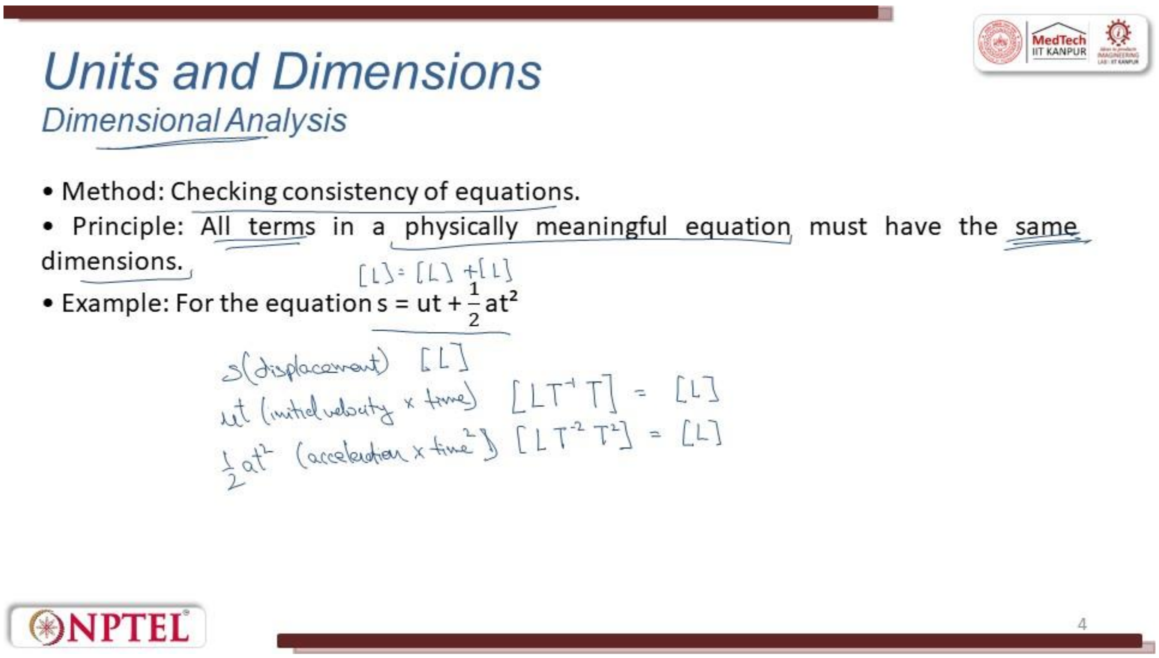
This is also one of the tables which was given in the weak one itself, certain relationships between certain physical quantities and their formulas and their dimensions which are represented in M, L and T forms. For example, force = mass × acceleration for which SI units are Newton and its dimensional formula is $[M^1\ L^1\ T^{-2}]$. And for work which is force × distance, its dimension formula is $[M^1L^2T^{-2}]$and the extra units are joules. These are all discussed energy, power, pressure, stress, momentum, moment of force, impulse which is force × time, impulse is also change in momentum.

And impulse is having SI units as Ns. Strain, change in dimension, this was discussed in the further coming weeks. It has no units because this is a change in the length. Modulus of elasticity, stress per unit strain, which is $[M^1 L^{-1}T^{-2}]$ which is Newton per meter square, which is equivalent to force.



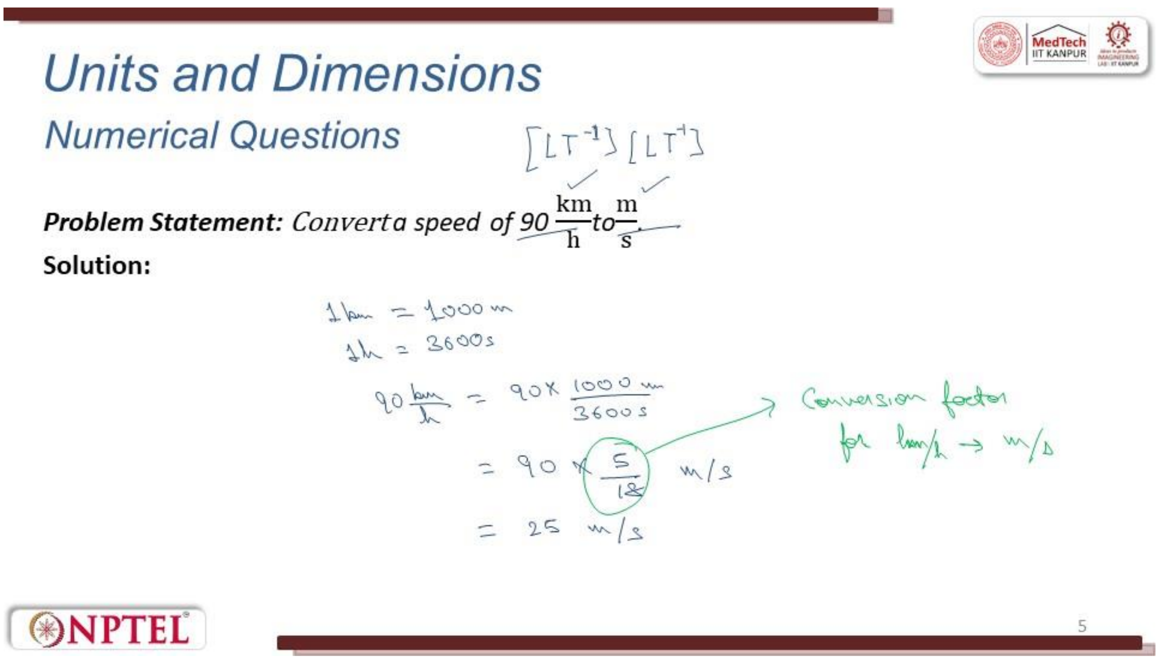
Now let me come to the certain small problems, for example, for equation

$s = ut + \frac{1}{2}at^2$ which is an equation of the motion. The dimensional analysis is method of checking consistency of equations. All the terms in a physical meaningful equation must have the same dimensions. This is the principle of dimensional analysis. For this example, if we say s is displacement which is having dimensional formula as only $[L^{1}]$.

Then ut (initial velocity × time) $[LT^{-1}\,T = L]$

And $\frac{1}{2}$ at^2 (acceleration × Time^2) $[LT^{-2}T^2 = L]$ which means this is [L] = [L] + [L] in the dimensional form in the equation. That means all terms here have same dimensions.

This is the crux of having a dimensional analysis and how do we develop the dimensional analysis? How do we learn it more? There were questions in the forum as well that how do we learn it more? This tutorial session would have some numerical problems which would be having some solutions to the different dimensional relationships. Let us go through them.



Now, I have a problem statement. Convert a speed of 90 $\frac{\text{km}}{\text{h}}$ *to* $\frac{\text{m}}{\text{s}}$.

Given: 1 km = 1000 m and

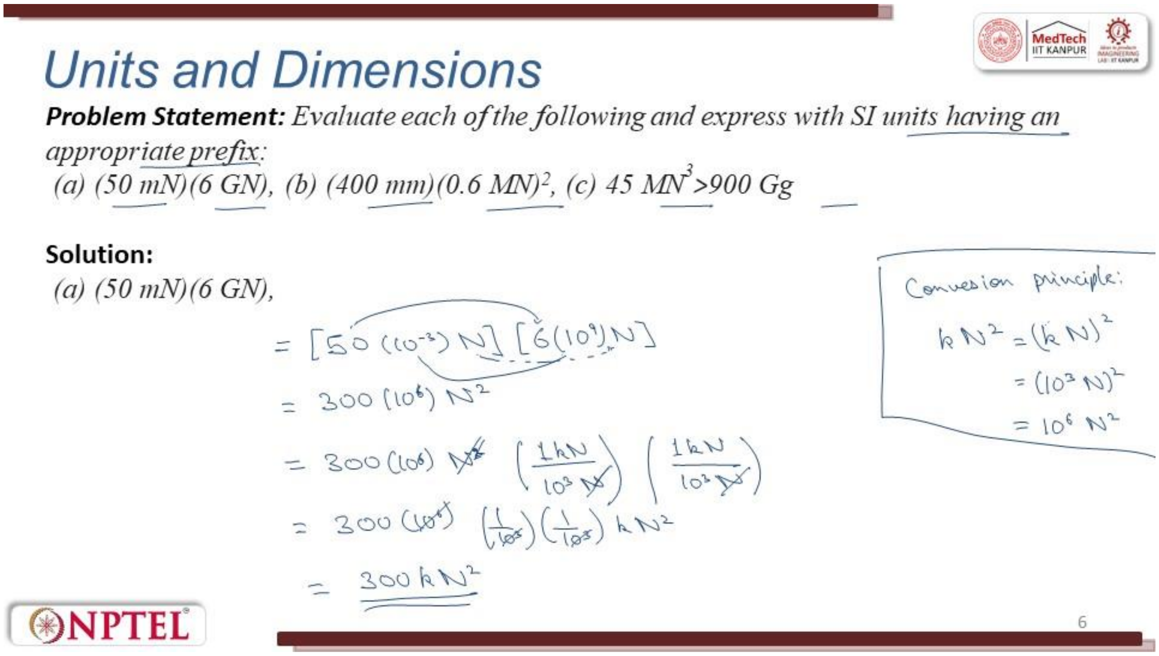
1 h = 3600s

$$90 \frac{\text{km}}{\text{h}} = 90 \times \frac{1000 \text{ m}}{3600 \text{ s}}$$

$$= 25\text{m/s}$$

So, this only means that if you convert the units. The dimension formula still remains same. So, dimensions for this kilometer per hour is again length [T^{-1}] and this is also length [T^{-1}]. The dimension formula remains same. So, there could be questions in your final exam that what is the dimensional difference or what is the dimensional formula if we change kilometer per hour to meter per second or if we change units from SI to CGS or so.

So, dimensional formula should remain same it is a major understanding here.



Now, there is another problem statement where we evaluate each of the following and express with SI units having an appropriate prefix. Because we are discussing units and dimension, so this is regarding D. SI units conversion. So, we need to have SI formulas or put them into SI form.

a) (50mN) (6GN) = $[50(10^{3})\text{ N}]\,[6(10^{9})]$

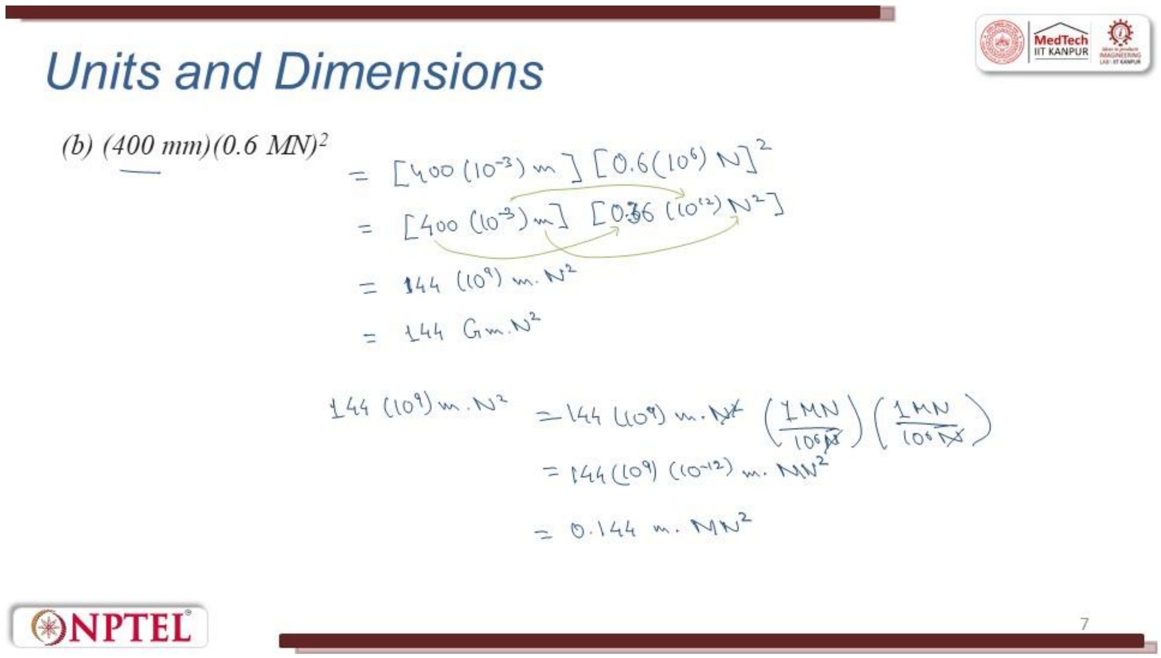
$$= 300(10^6)\ \text{N}^2$$

$$= 300(10^6)\ \text{N}^2 \left(\frac{1\ \text{kN}}{10^3\text{N}}\right)\left(\frac{1\ \text{kN}}{10^3\ N}\right)$$

$$= 300\ \text{kN}^2$$

So, we need to only remember one thing. If it is written kN^2, this always means $\text{k} \times \text{N}^2$. That means, if it is this k is $10^3 = 10^3\ \text{N}^2$. So, this turns to $10^6\ \text{N}^2$. We will use this principle. Note that the convention $\text{kN}^2 = (\text{kN})^2 = 10^6\text{N}^2$

So, this is into the SI units.



Similarly, let me see the second term which was given which also had two factors.

b) $(400\text{mm})\ (0.6\ \text{MN})^2 = [400(10^{-3})\ \text{m}]\ [0.6\ (10^6)\ \text{N}]^2$

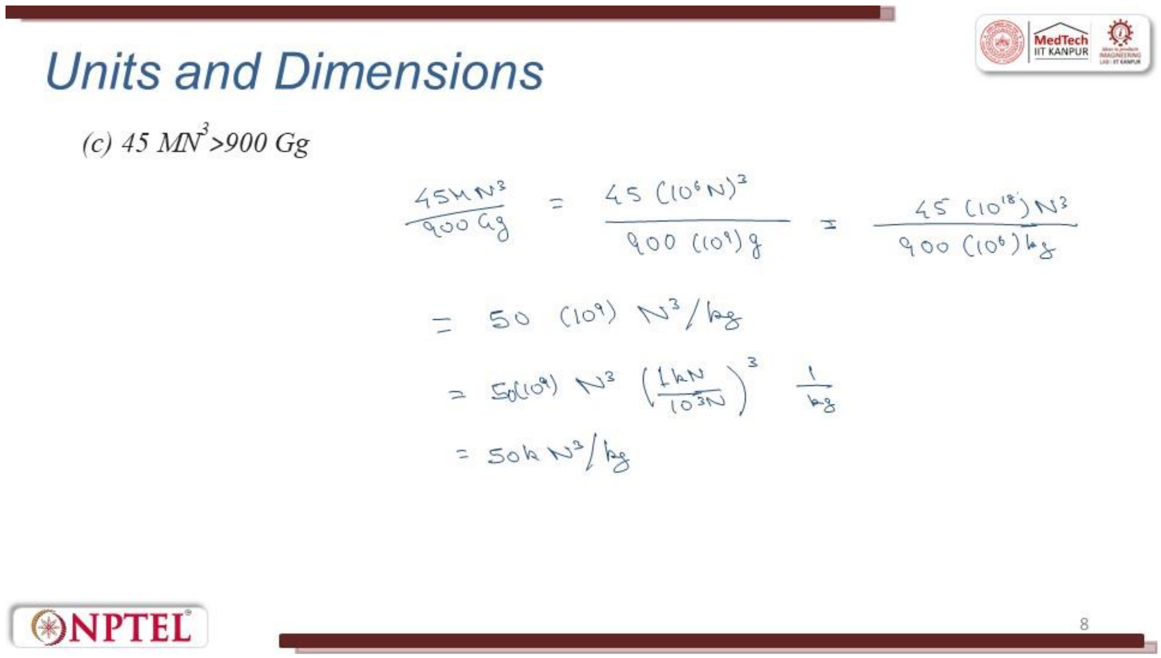
$= [400(10^{-3})\ \text{m}]\ [0.36(10^{12})\ \text{N}^2]$

$= 144(10^9)\ \text{m. N}^2$

$= 144\ \text{Gm. N}^2$

We can also write

$$144(10^9)\ \text{m. N}^2 = 144(10^9)\ \text{m. N}^2 \left(\frac{1\ \text{MN}}{10^6\text{N}}\right)\left(\frac{1\ \text{MN}}{10^6 N}\right) = 0.144\text{m.MN}^2$$



So, this is again an SI unit that I have gotten for the third expression which is given,

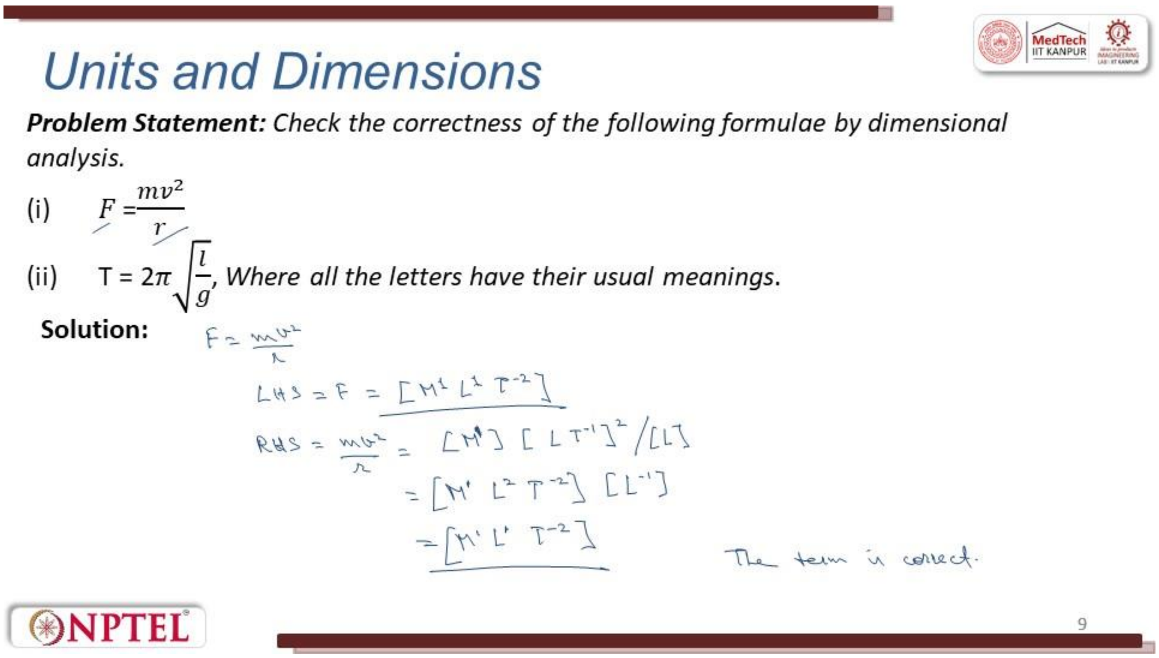
c) $\frac{45\text{MN}^3}{900Gg} = \frac{45(10^6\text{N})^3}{900(10^6)kg}$

$$= 50(10^9)\ \text{N}^3/\text{kg}$$

$$= 50(10^9)\ \text{N}^3 \left(\frac{1\ \text{kN}}{10^3\text{N}}\right)^3 \frac{1}{kg}$$

$$= 50\text{kN}^3\ /\text{kg}$$

So, this is SI units conversion only let us now try to see some problems on the dimensional analysis as I mentioned dimensional analysis practices are to be taken.



So, let us see this problem statement check the correctness of the following formula by dimensional analysis.

i) Dimensions of the term on L.H.S

Force, $F = [M^1L^1T^{-2}]$

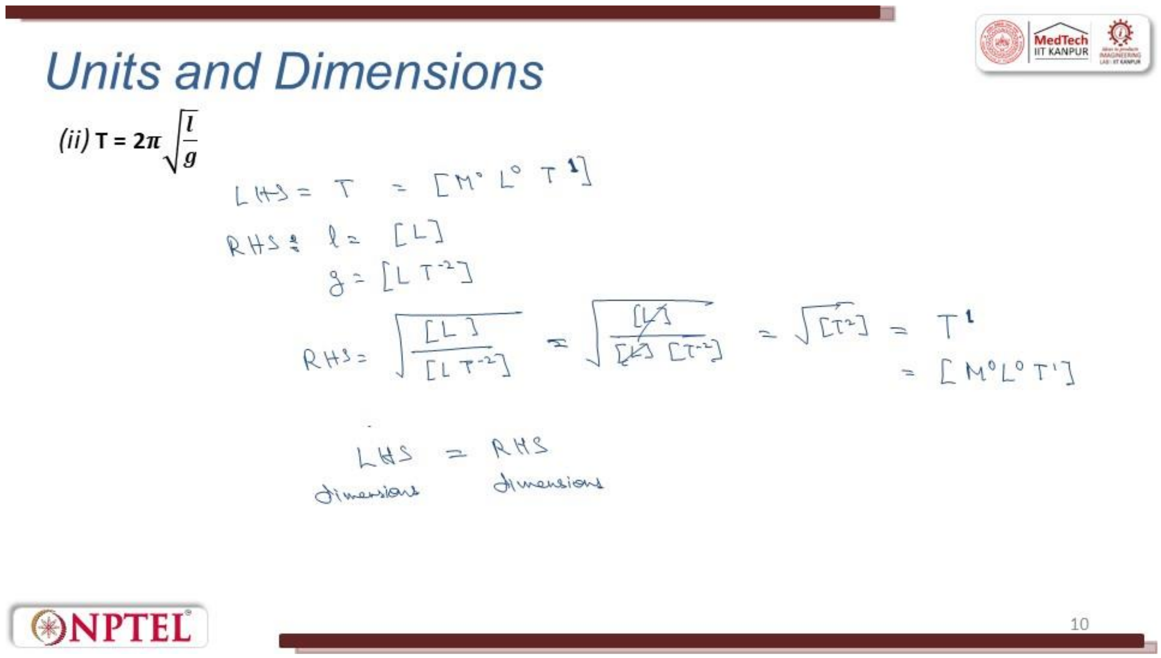
Dimensions of the term on R.H.S

$\frac{mv^2}{r} = [M^1]\,[L^1T^{-1}]^2 / [L]$

$=[M^1L^2T^{-2}]/ [L]$

$=[M^1L^1T^{-2}]$

The dimensions of the term on the L.H.S are equal to the dimensions of the term on R.H.S. Therefore, the relation is correct.



Let us see the second $T = 2\pi\sqrt{\frac{l}{g}}$relation

ii) Here, Dimensions of L.H.S,

t = [T] = $[M^0L^0T^1]$

Dimensions of the terms on R.H.S

Dimensions of (length) = [L]

Dimensions of g (acc. due to gravity) = $[LT^{-2}]$

2π being constant have no dimensions.

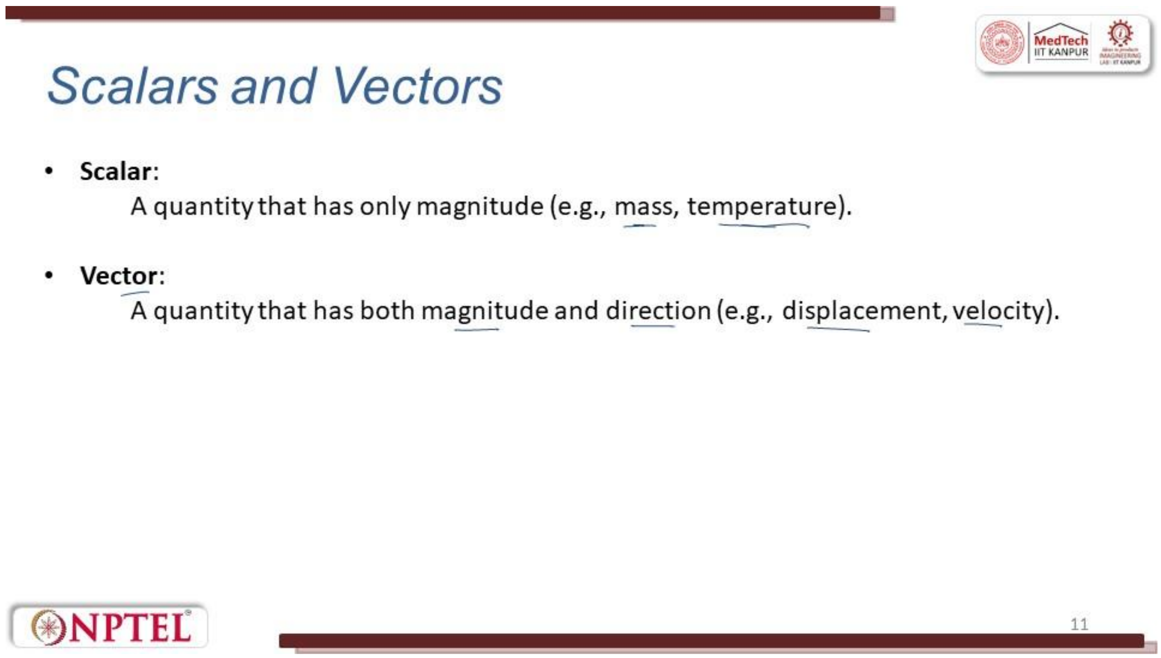
Hence, the dimensions of terms $T = 2\pi\sqrt{\frac{l}{g}}$ on R.H.S

= $\sqrt{\frac{L}{LT^{-2}}}$ = [T] = $[M^0L^0T^1]$

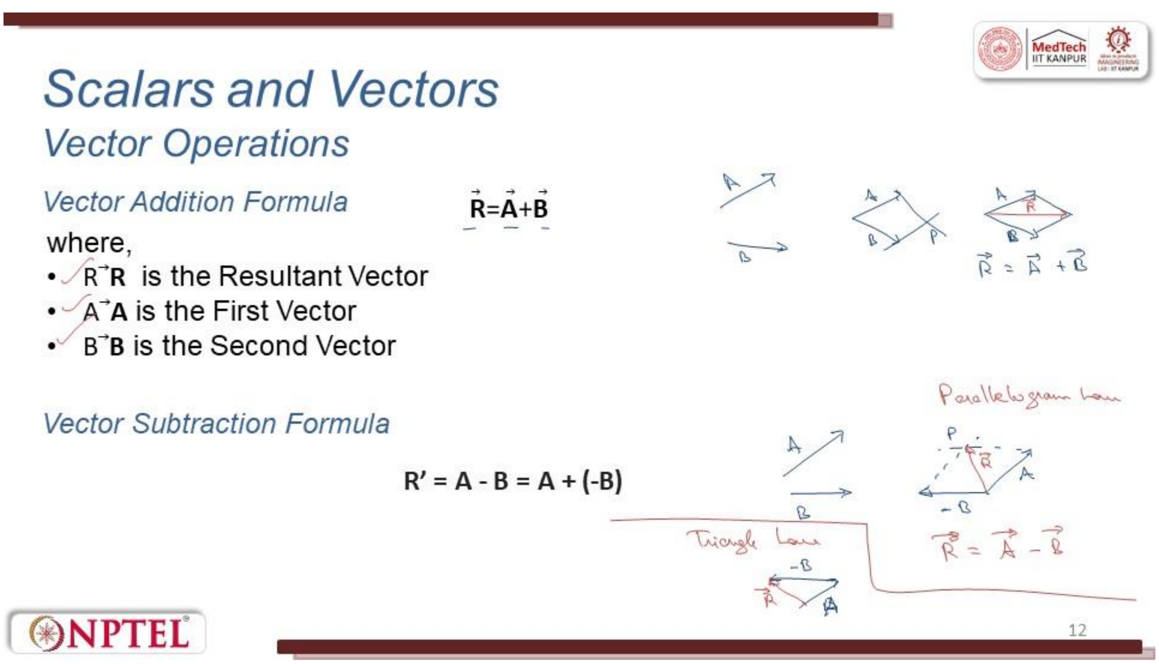
Thus, the dimensions of the terms on both sides of the relation are the same i.e., $[M^0L^0T^1]$.

Therefore, the relation is correct.

Let me come to the next topic. So, with this some practice in the dimensional analysis I have taken and the conversion to SI units these numericals are taken here.



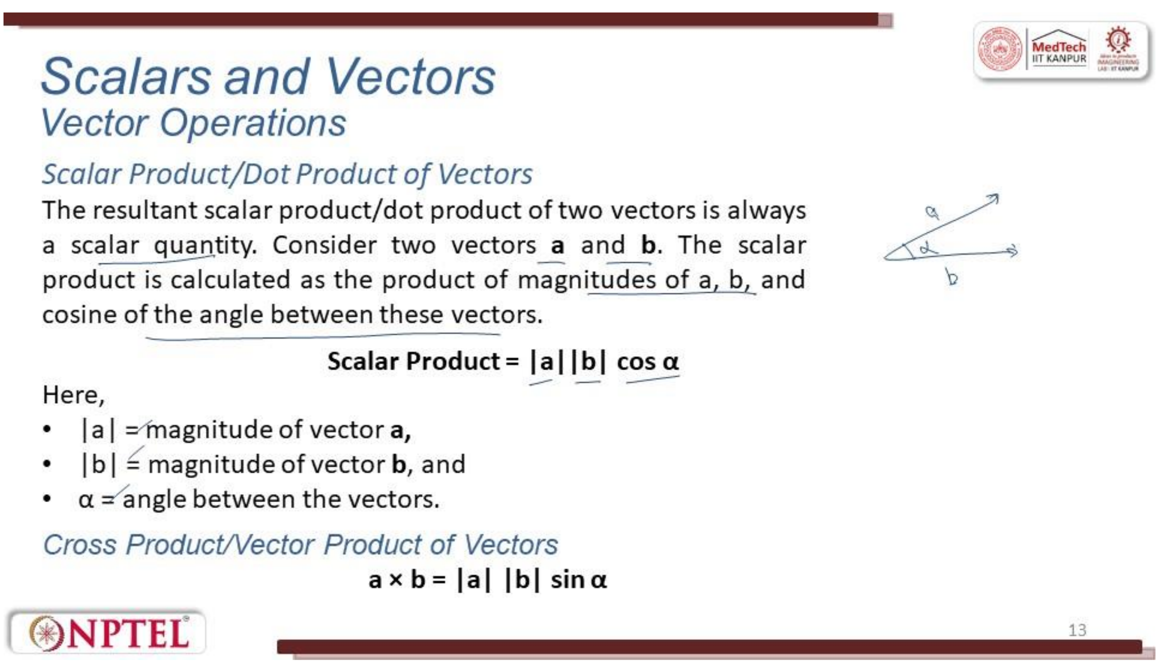
I will also now take the scalars and vector which was discussed in the week 1 in the coming numericals. A quantity that has only magnitude such as mass or temperature that is scalar this is discussed in detail week 1. A quantity that has both magnitude and direction for example, displacement velocity is a vector these definitions were taken.



Let us try to understand it further vector addition formula, we have the resultant vector for a plus b for example, two vectors are there, A and B in their different directions. This is A, this is B. So, I can maybe join them and try to complete a parallelogram, a line parallel to A and parallel to B. So, this is my point of contact P. So, this resultant could be if I try to draw A and B here. This is A, B and I have completed this parallelogram.

Here I could have my relation R = A + B, where is R here? R is given here, this is R which is the diagonal, this is A + B. So, R is the relative vector, A is the first vector, B is second vector. Similarly, for subtraction suppose we have again the two vectors as may be A and B because I am going to subtract it, I will take the B in the negative direction. This is -B and A in the positive direction only a. Now, I will complete my parallelogram. This is my point B. The resultant vector would be from the starting point of the vector to the point P. So, I will have my resultant here.

This is R. So, this R = A - B. This is parallelogram law, we can also have a triangle law, triangle law is nothing but we only draw a triangle in place of a parallelogram. We draw a triangle, we draw A like this, this is A in its direction and B is drawn in its negative direction and here we will have resultant R. So, this one is parallelogram, this is parallelogram law and this is triangle law.



So, let us see this scalar in the vector product as we discussed in the week 1 itself. The scalar or the dot product of vectors, a resultant scalar product or dot product of two vectors is always a scalar quantity.

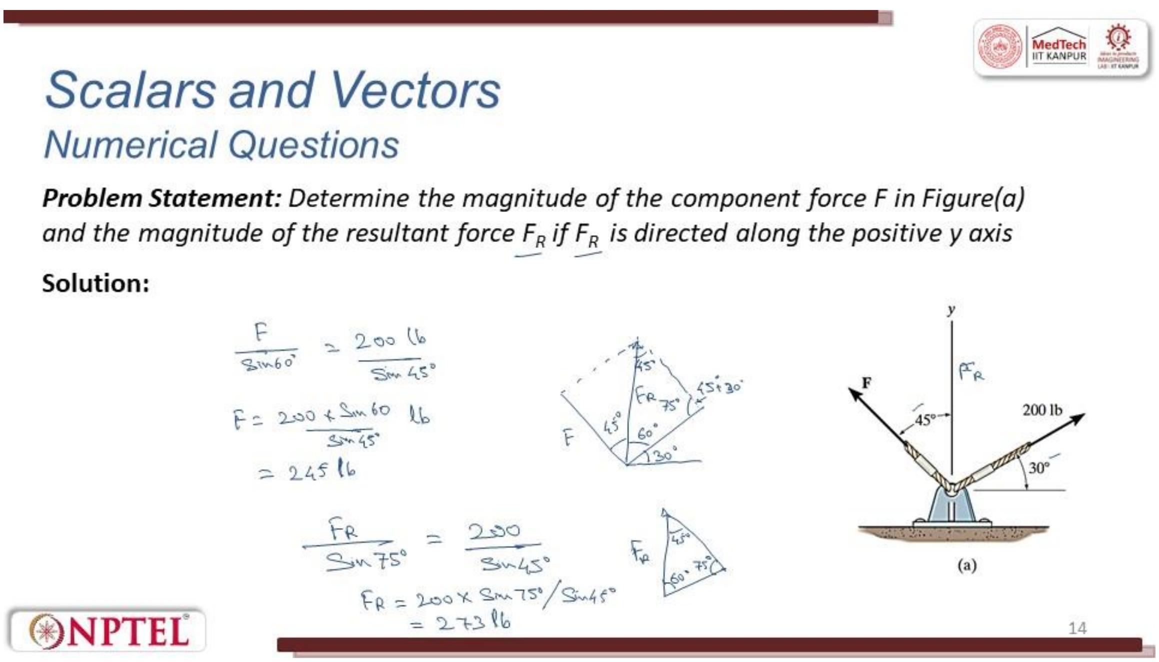
Consider two vectors a and b,

$$Scalar\ Product = |a||b|\cos\alpha$$

α is the angle between two vectors. If I am having two vectors, I am only putting magnitude here, this small a and b, so this is my α here, it is given magnitude α is angle.

$$a \times b = |a|\,|b| \sin\alpha$$

Here what we do we try to get the sin of the angle between them. So, this is the cross product or vector product.



Let us try to solve a few numericals to understand this further. Determine the magnitude of the component force F in Figure (a) and the magnitude of the resultant force F_R if F_R is directed along the positive y axis.

So, we will try to apply the parallelogram law or way or method to add the vectors here.

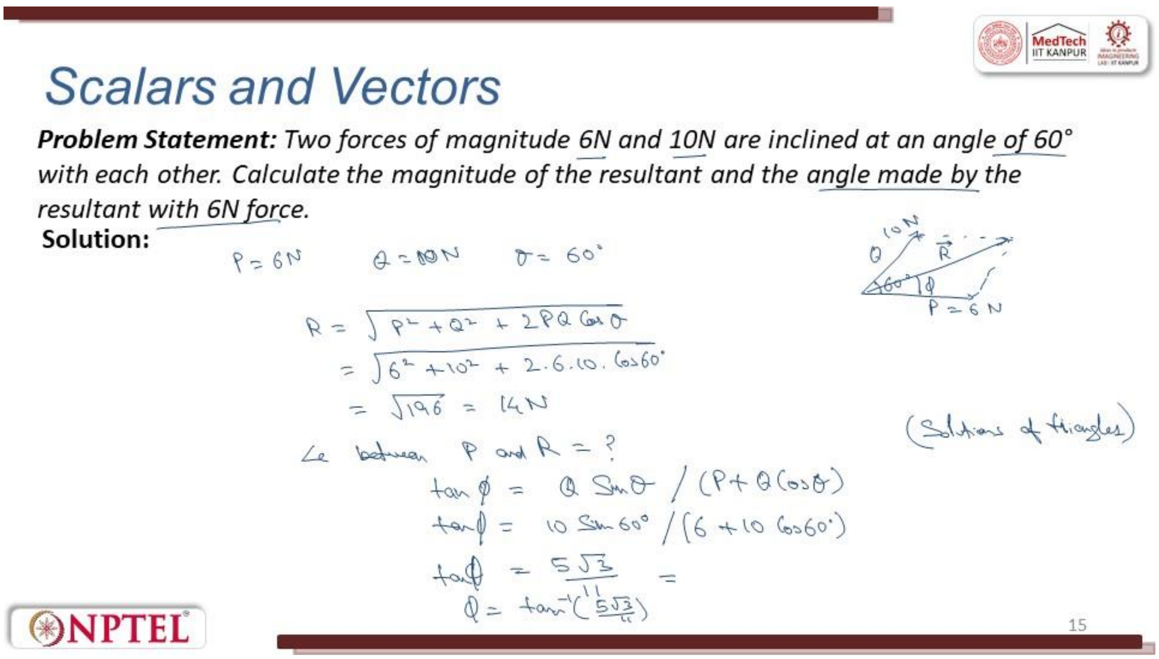
$$\frac{F}{\sin 60°} = \frac{200\ lb}{\sin 45°}$$

F = 245 lb

$$\frac{F_R}{\sin 75°} = \frac{200\ lb}{\sin 45°}$$

F_R = 273 lb

So, this is application of the parallelogram and the triangle law to find the resultant magnitude of a vector.



Now there is another problem statement where it is given the magnitude 6 Newton and 10 Newton are inclined at an angle of 60 degrees with each other, they are two forces. Calculate the magnitude of resultant and the angle made by resultant with 6 and 4, 6 and 4 that is there. So let me try to draw this triangle, two forces are inclined at an angle of around 60 degrees, let me say Q and P, they are inclined at an angle 60 degrees, right. Using the parallelogram law, I will try to find this resultant R and this angle phi they are asking, their resultant and the angle made by the resultant with 6N force.

P = 6N, Q = 10N and θ = 60°

We have,

$$R = \sqrt{P^2 + Q^2 + 2P\,Q\cos\theta}$$

$$R = \sqrt{6^2+10^2+2.6.10cos60}$$

$$\therefore R = \sqrt{196} = 14N$$

Which is the required magnitude. Let ø be the angle between P and R. Then,

$$\text{Tan}\phi = Q\sin\phi\,/\,(P + Q\cos\phi)$$

$$\Rightarrow \tan\phi = 10\sin60°/(6+10\cos60°)$$

$$\Rightarrow \tan\phi = \frac{5\sqrt{3}}{11}$$

$$\therefore \phi = \tan{-1}\left(\frac{5\sqrt{3}}{11}\right)$$

You can put it in this way itself or you can solve and try to get the angle in degrees. You can see I am just using the solutions of triangles theory here to solve these problems.

Scalars and Vectors

Problem Statement: *Consider two vectors such that |a|=6 and |b|=3 and α = 60°. Find their dot product.*

Solution:

$$a.b = |a|\,|b|\cos\alpha$$
$$= 6.3.\cos 60°$$
$$= 6.3.\tfrac{1}{2}$$
$$= 9$$
$$a.b = 9$$

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This is a simple problem statement here where magnitude of a is 6, magnitude of b is 3 and angle between them is 60 degrees.

$$a.b = |a|\ |b| \cos \alpha$$

$$\text{So, } a.b = 6.3.\cos (60°)$$

$$= 18 \times (1/2)$$

$$a.b = 9$$

This is a very simple relationship. Next, I am keeping two three examples for you to solve. In these slides, you will get the solutions, but it is suggested yet you try to understand the problem, you try to solve and try to get the solution and then only refer the lecture notes where the solution of these are given.

Scalars and Vectors

Problem Statement: *Prove that the vectors a = 3i+j-4k and vector b = 8i-8j+4k are perpendicular.*

Solution:

We know that the vectors are perpendicular if their dot product is zero

$$a.b = (3i+j-4k).(8i-8j+4k)$$

$$= (3)(8) + (1)(-8)+(-4)(4)$$

$$=24-8-16 =0$$

Since the scalar product is zero, we can conclude that the vectors are perpendicular to each other.

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Prove that the vectors a = 3i+j-4k and vector b = 8i-8j+4k are perpendicular. So, we have to prove that these vectors are perpendicular, just the solution would be given. Try to solve it and get the solution.

Scalars and Vectors

Problem Statement: *Find the cross product of two vectors a and b if their magnitudes are 5 and 10 respectively. Given that angle between then is 30°.*

Solution:

a x b = a.b. Sin α

a* × *b
= a.b.sin (30)
= (5) (10) (1/2)
= 25
perpendicular to ***a*** *and* ***b***

Another problem statement is find the cross product of two vectors a and b if their magnitudes are 5 and 10 respectively. Given that angle between then is 30°. I am now talking about the cross product $a \times b = |a|\ |b| \sin \alpha$.

This you can actually solve, this is very simple solution sine of 30 degrees into 5×10. Try to solve it and you would get solutions there.

Scalars and Vectors

Problem Statement: Find the area of a parallelogram whose adjacent sides are
a = 4i+2j -3k
b= 2 i +j-4k

a x b

Solution:

The area is calculated by finding the cross product of adjacent sides
$a \times b = x(a_2b_3 - b_2a_3) + y(a_3b_1 - a_1b_3) + z(a_1b_2 - a_2b_1)$
= i(-8+3) + j(-6+16) + k(4-4)
= -5i +10j
Therefore, the magnitude of area is

$\sqrt{(5^2 + 10^2)}$

$\sqrt{(25+100)} = \sqrt{(125)} = = 5\sqrt{5}$

Another problem statement that I have here is Find the area of a parallelogram whose adjacent sides are

a = 4i+2j -3k

b= 2 i +j-4k

The area is always a cross product if you remember area is a into b.

So, with this I am closing this lecture, I will try to talk about the Statistics, Kinetics and Kinematics and Moment of Inertia in the second part of the tutorial 1.

Thank you.