Basics of Mechanical Engineering-1

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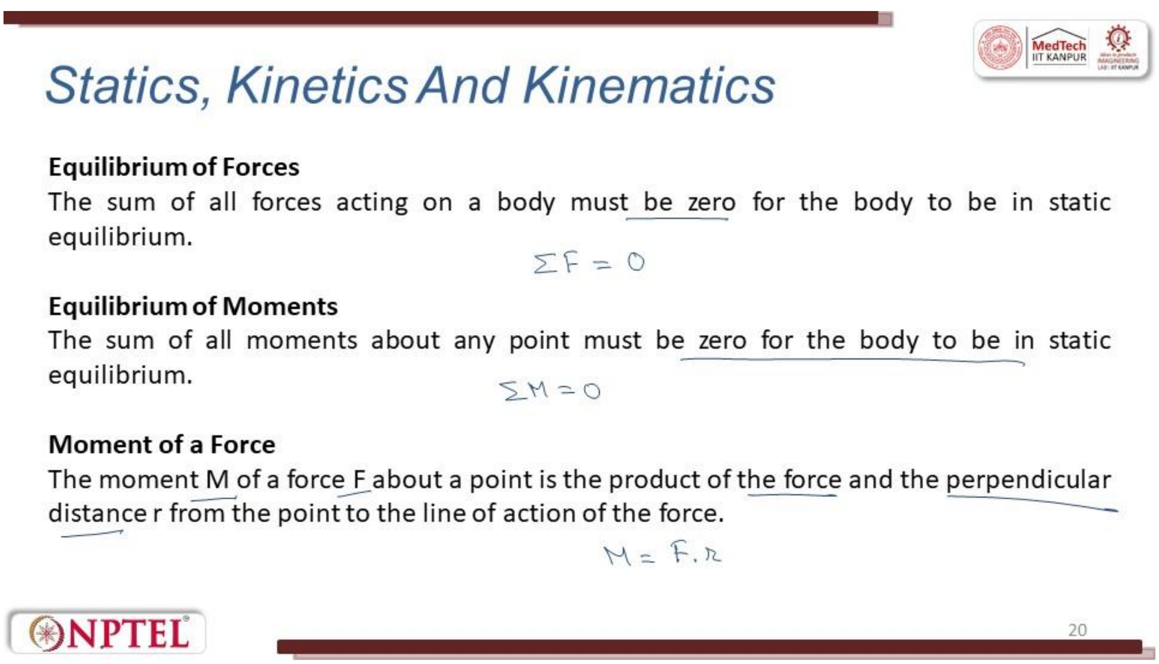
Week 04

Lecture16

Tutorial - 1 (Part 2 of 2)

Welcome to the second part of the Tutorial session of the content that was covered in week 1. We had discussed about Units and Dimensions and Scales and Vectors in the previous part of the tutorial. Here I will discuss about the Statics, Kinetics and Kinematics and Laws of Motion. Some problem statements would be solved.

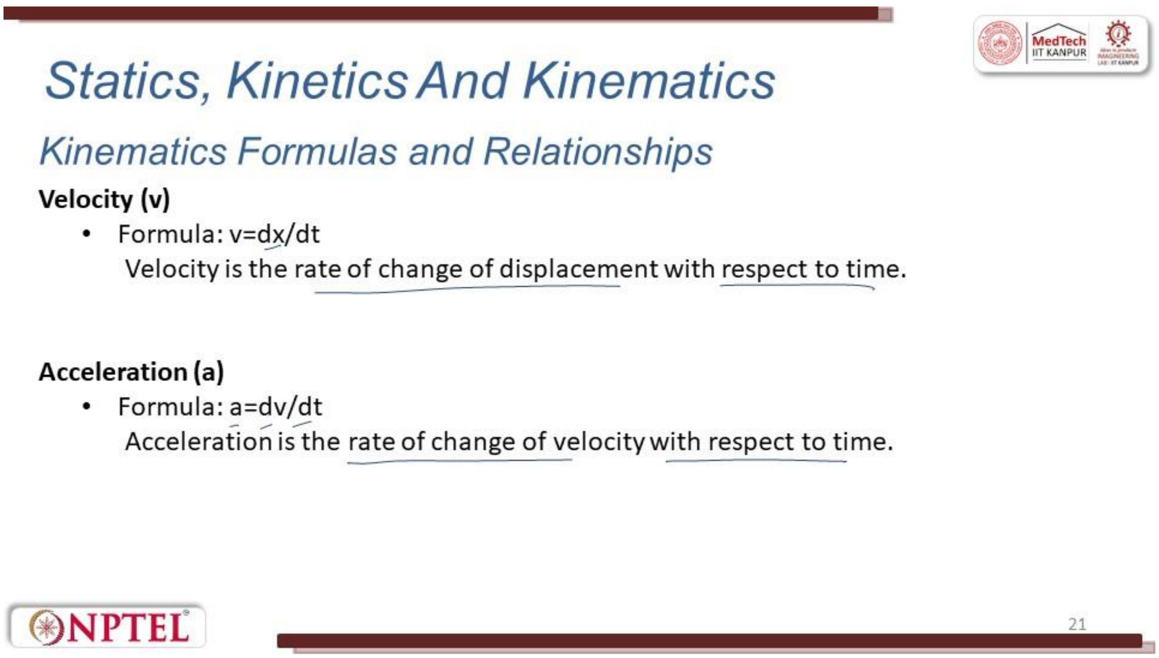


Just to recall the basic theory or in statics, Equilibrium of forces, sum of all the forces acting on a body must be 0 for the body to be in a static equilibrium.

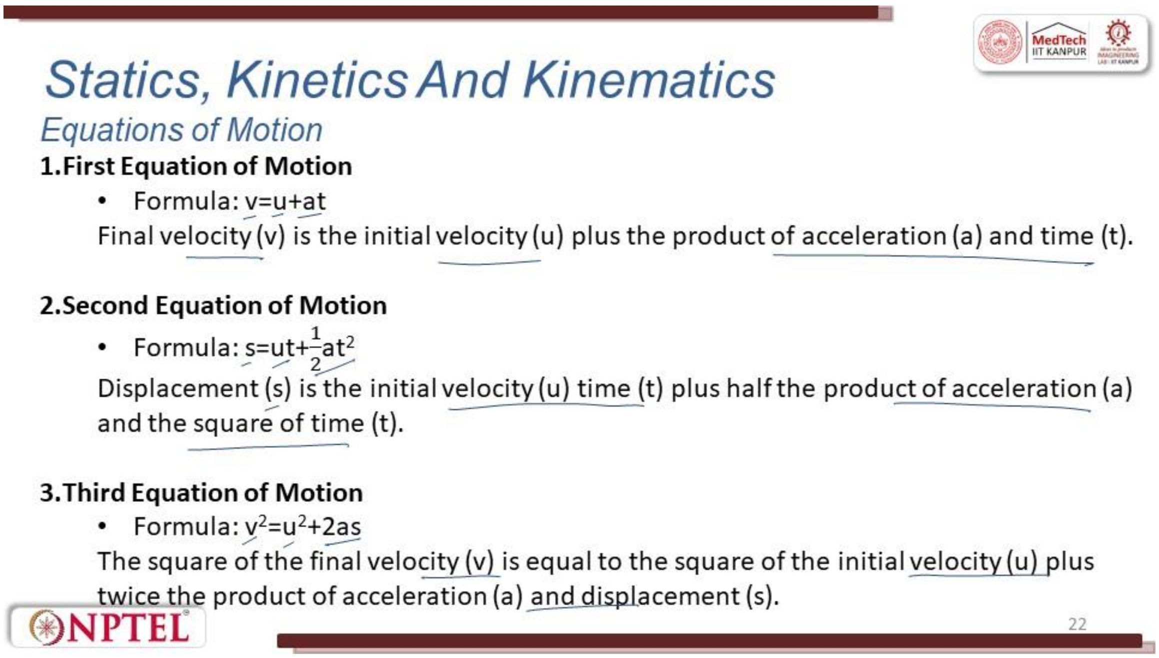
That means ∑F=0. Equilibrium of movements, sum of all movements about any point must be 0 for the body to be in static equilibrium. So that means ∑M=0, moment of a force.

M=F.r

These are certain principles that we discussed in the week 1.



Simple relationships for the velocity which is change in the position of the object that is displacement per unit change in time. This is velocity of rate of change of displacement with respect to time. Similarly acceleration a = dv/dt that is change in velocity per unit change in time, it is rate of change of velocity with respect to time then the equations of motion that we discussed.



First equation of motion that is $v = u+at$. Final velocity v is the sum of initial velocity and the product of the acceleration and time.

Second equation of motion is $s = ut+\frac{1}{2}at^2$.

Third equation of motion that is $v^2 = u^2+2as$

These are general formula that you had been reading in your senior secondary education as well and these would be applied in engineering mechanics further.

Statics, Kinetics And Kinematics
Kinetics Formulas and Relationships

Newton's Second Law

- Formula: F=ma
  The force (F) acting on an object is equal to the mass (m) of the object multiplied by its acceleration (a).

Work-Energy Principle

- Formula: W=ΔK
  The work (W) done on an object is equal to the change in its kinetic energy (ΔK).

Power

- Formula: P=dW/dt
  Power (P) is the rate at which work (W) is done over time (t).

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Then we had Newton's second law that is F=ma

Work-energy principle that is W=ΔK that is work done in an object is equal to change in the kinetic energy. Then P = dW/dt. It is rate at which work is done over time.

Statics, Kinetics And Kinematics
Numerical Questions

Problem Statement: *A 10 kg block is in equilibrium on a frictionless surface with forces F_1=20 N and F_2. Find the magnitude and direction of F_2.*

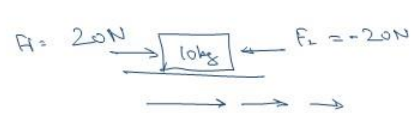
Solution:

$\Sigma F = 0$

$F_1 + F_2 = 0$

$20N + F_2 = 0$

$F_2 = -20N$ (in the opposite direction)



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Now, let us try to use these to solve some numerical questions. A 10 kg block is in equilibrium on a frictionless surface with forces F_1=20 N and F_2. Find the magnitude and direction of F_2. The body stays in equilibrium.

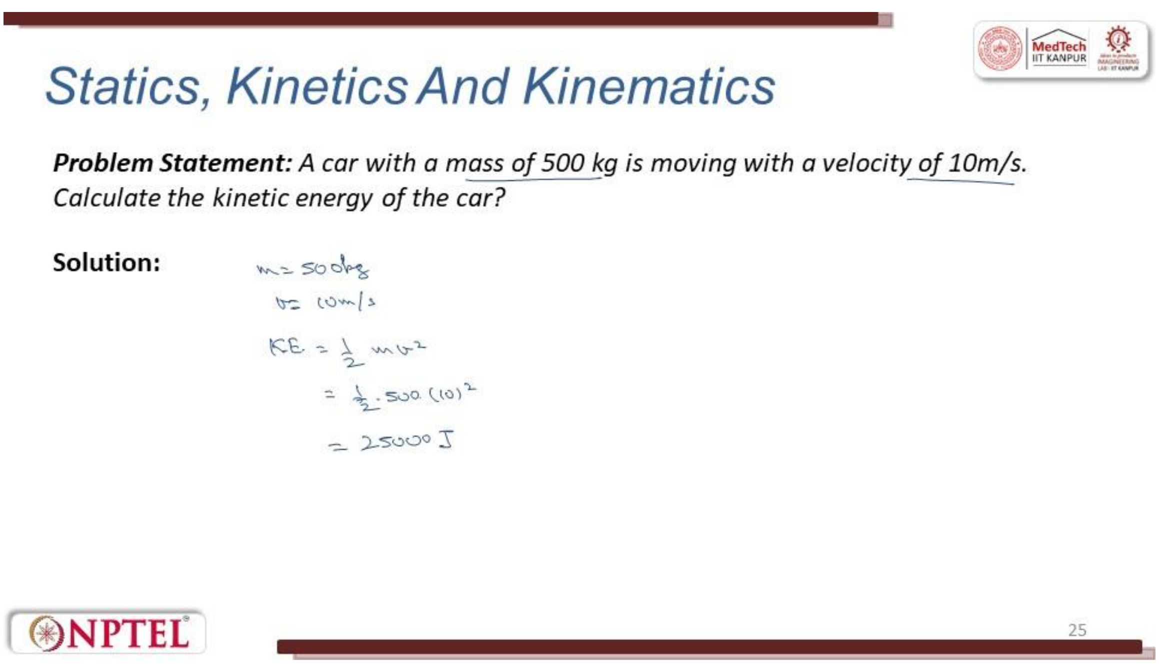
That is the first principle of equilibrium that we saw. That means suppose this is the body. Frictional surface is there. 10 kg block, F_1 force is 20 Newton which will try to push this in this direction. To keep it in equilibrium, we have to find what will be the value of F_2.

$$\sum F=0$$

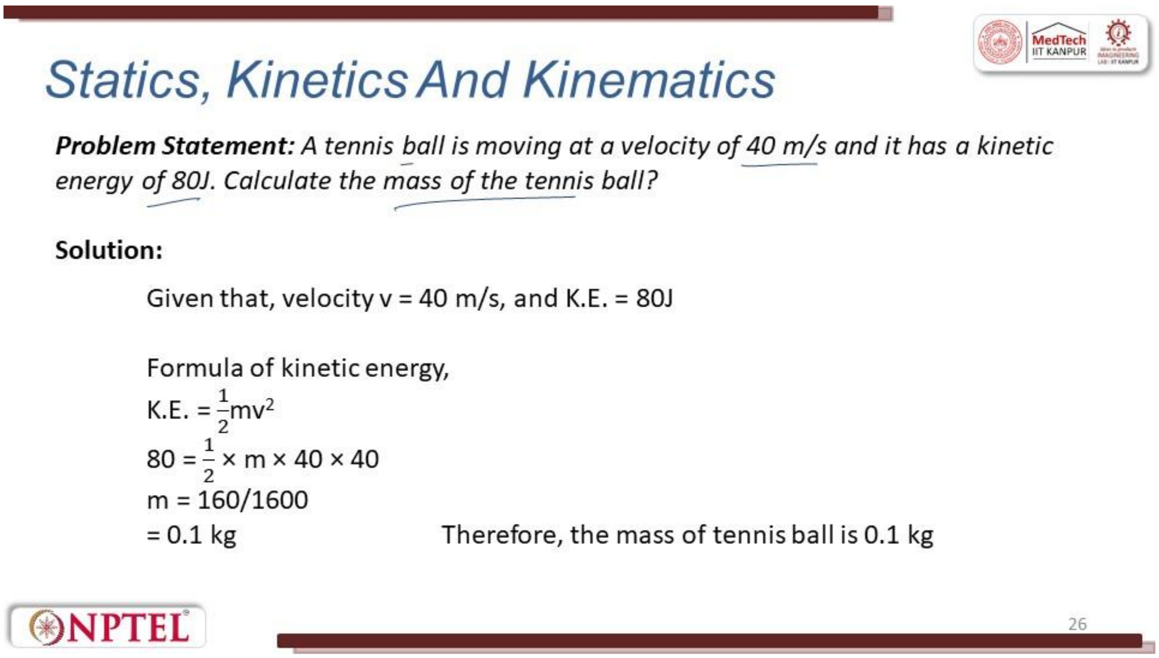
$$20\text{ N}+F_2=0$$

$$F_2 = -20\text{ N (opposite direction)}$$

This is a simple application of the equilibrium of forces principle to find the force acting in the opposite direction.

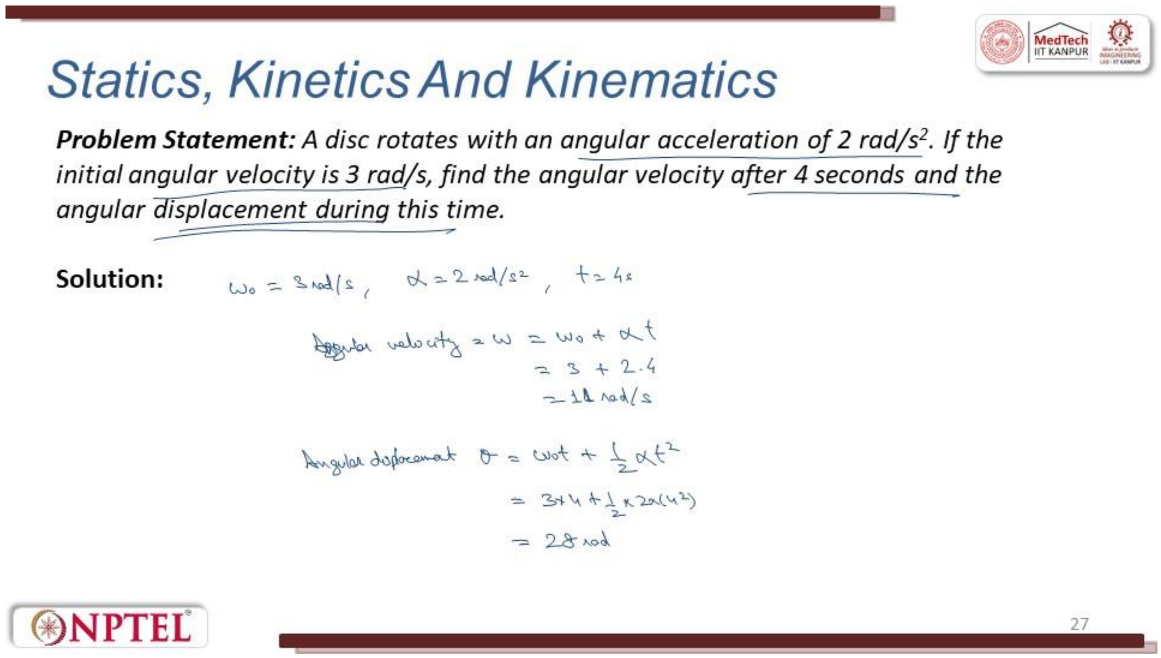


So, there is a problem statement where it is mentioned a car with a mass of 500 kg is moving with a velocity of 10 meters per second. Calculate the kinetic energy. We all know this is a simple relation between kinetic energy and the mass and velocity. Mass given is 500 kg and velocity given is 10 meters per second. Very slow moving car as now that is given kinetic energy relationship we know is equal to half mv square which is equal to half into mass into square of the velocity everything is given in SI units meter per second 10 square this is equal to 25000 joules. This is a very simple problem statement.



I will give you another problem statement where you have a velocity of a tennis ball. It is moving at 40 m/s and kinetic energy is given 80 joules. Calculate the mass.

Using the same relationship, you need to find the mass of the tennis ball. This problem you solve by yourself.



Another problem statement is known the rotation now, the angular velocity. This rotates with an angular acceleration of 2 radians/sec. If the initial angular velocity is 3 radians/second, find the angular velocity after 4 seconds and angular displacement during this time.

ω_0=3 rad/s, α=2 rad/s^2, t=4 s,

Angular velocity $\omega = \omega_0 + \alpha t$

$= 3 + 2 \times 4$

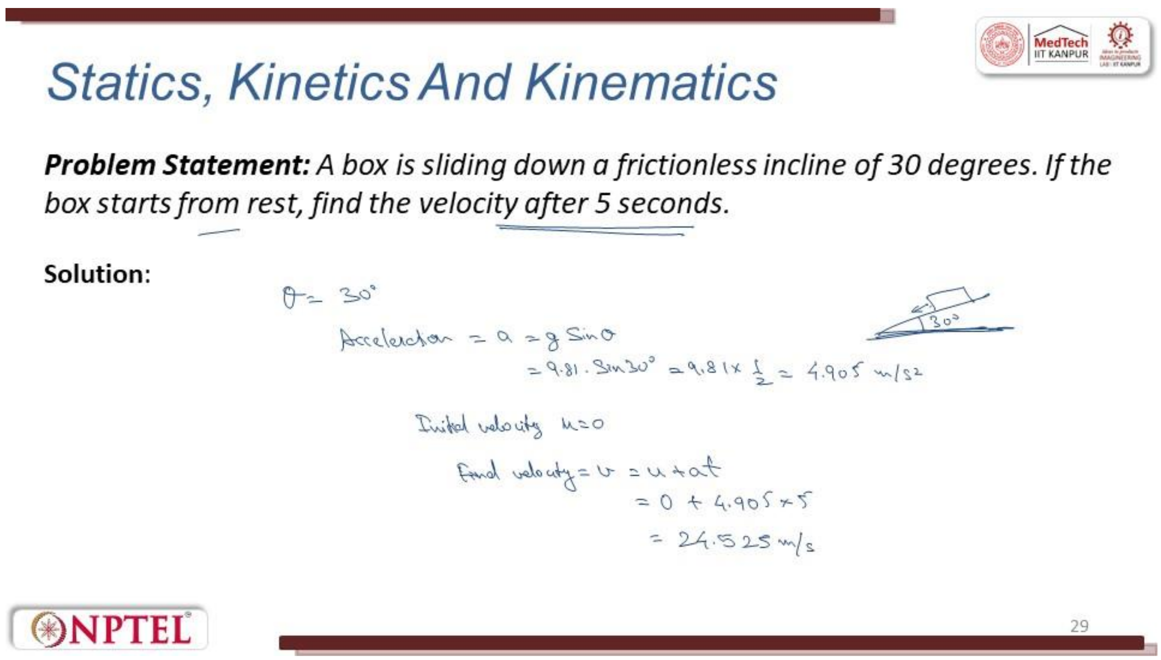
$= 11$ rad/s

Angular displacement $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 3 \times 4 + \frac{1}{2} \times 2 \times 4^2$

$= 12 + 16$

$= 28$ rad



There is another problem statement the box is sliding down a frictionalized incline of 30 degrees. If the box starts from rest find the velocity after 5 seconds. So, here incline angle theta is equal to 30 degrees and the box starts from rest. So, here we know acceleration because the angle given is 30 degrees. This is a plane surface at 30 degrees.

Incline angle $\theta = 30^\circ$

Acceleration $a = g \sin\theta$

$= 9.81 \times \sin 30^\circ$

$= 9.81 \times 0.5$

$= 4.905\ m/s^2$

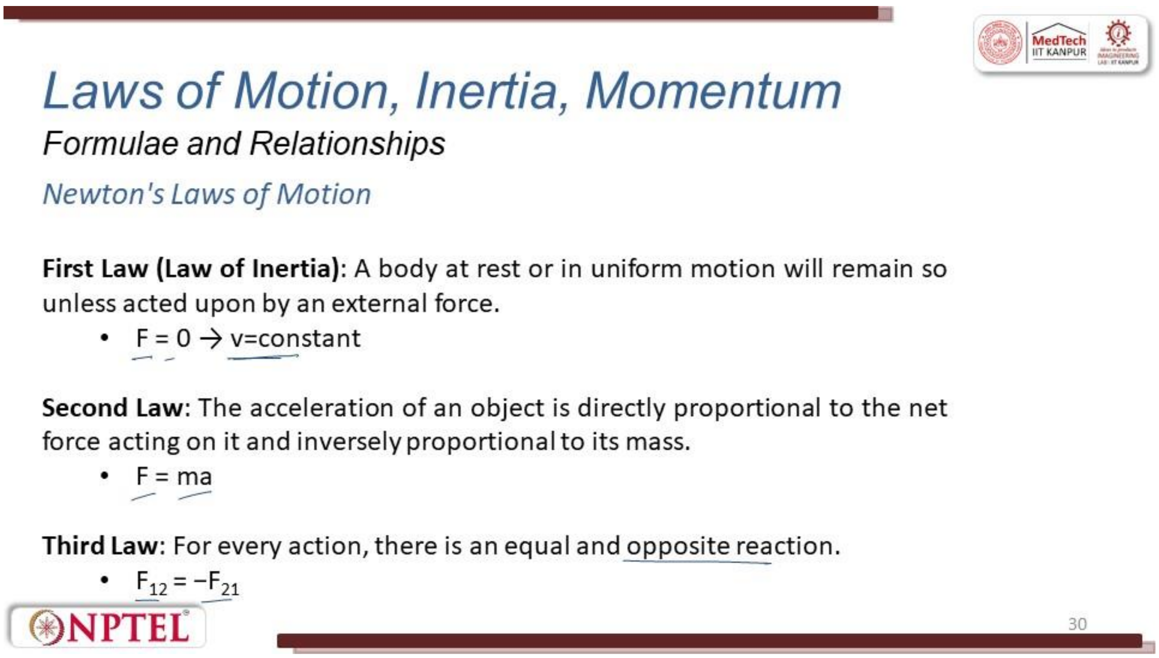
Initial velocity u=0

Final velocity $v = u + at$

$= 0 + 4.905 \times 5$

$= 24.525\ m/s$

So, there are certain other problems that you can solve by yourself.



Next topic is Laws of Motion, Inertia and Momentum. Certain relationships which were discussed the first law of inertia that the body at rest or in uniform motion will remain, so unless acted upon by an external force that is $F = 0 \rightarrow v = constant$.

Second law that we have already taken in the previous slides as well. The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass which is $F=ma$.

Third law of the motion given by Newton is of the equivalent opposite reaction. It is for every action there is an equivalent opposite reaction that is $F_{12} = -F_{21}$.

Laws of Motion, Inertia, Momentum

Momentum and Impulse

Linear Momentum (p):

- p=mv
- Units: kg·m/s

Impulse (J):

- J = Δp = FΔt
- Impulse-momentum theorem: FΔt = mΔv

Conservation of Momentum:

- In a closed system with no external forces: $\sum p_{initial} = \sum p_{final}$

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Then we talked about the linear momentum that is p = mv; the momentum units is kg.m/s.

Mass into velocity impulse is J that is change in momentum which is J = Δp = FΔt. So, there is a theorem known as Impulse Momentum Theorem that is FΔt=mΔv.

Conservation of momentum that is initial momentum and final momentum is same in a closed system with no external forces.

Laws of Motion, Inertia, Momentum

Inertia

Moment of Inertia (I):

- Rotational analog of mass for linear motion.
- For a point mass: $I = mr^2$
- Units: $kg{\cdot}m^2$

Rotational Inertia:

- $\tau = I\alpha$ (Torque is equal to the moment of inertia times angular acceleration)

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Inertia, Moment of inertia, rotational analog of mass of linear motion, we know $I = mr^2$, units are $kg.m^2$, rotational inertia $\tau = I\alpha$, that is torque is equal to the moment of inertia times the angular acceleration. Let us see a few numerical problems here.

Laws of Motion, Inertia, Momentum

Numerical Questions

Problem Statement: *Find out how much net force will be needed to accelerate a 2500 kg truck at 5.50 m/s^2.*

Solution:

$a = 5.5\ m/s^2$

$m = 2500\ kg$

$F = m \times a$

$= 2500 \times 5.5$

$= 13750 N$

2500kg truck ~~at~~ $5.5\ m/s^2 \longrightarrow 13750$ N

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Find out how much net force will be needed to accelerate a 2500 kg truck at 5.5 meters per second square.

Given,

Acceleration (a) = 5.50 m/s^2

Mass of the Truck (m)= 2500 kg

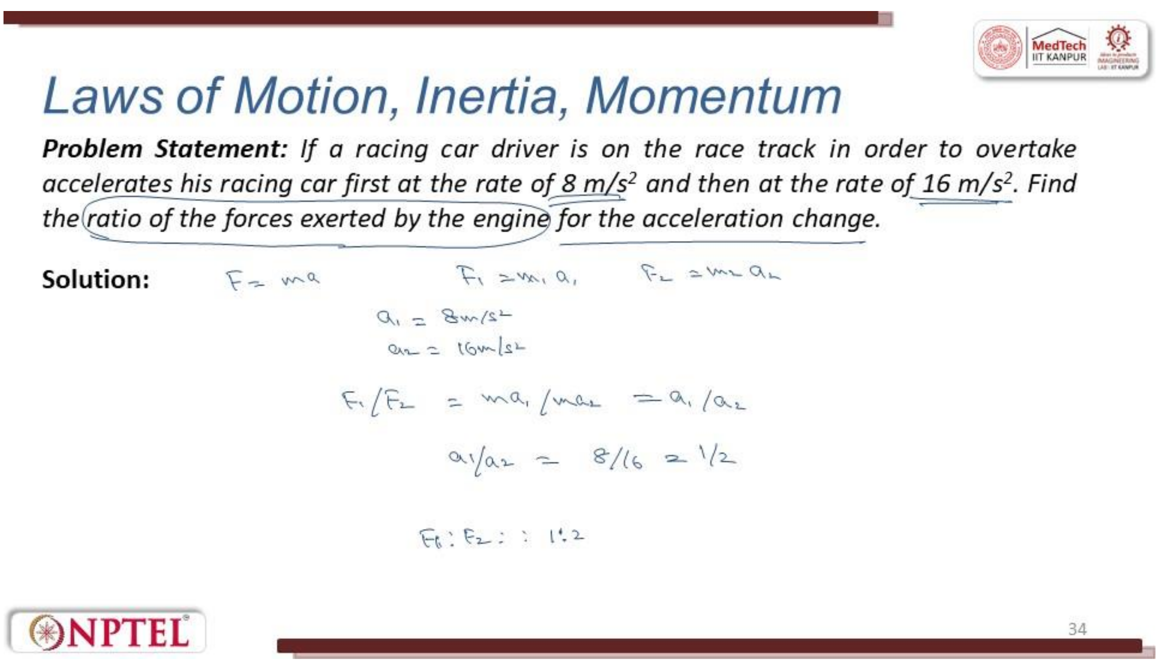
Hence,

Force = Mass × Acceleration

F = 2500 × 5.5

F = 13750 N

Net force will be needed to accelerate a 2500 kg truck at 5.50 m/s^2 is 13750 N.



Let me try to see another example where a racing car driver is on a race track in order to overtake he accelerates his racing car first at the rate of 8 m/s^2 then at the rate of 16 m/s^2. Find the ratio of the forces exerted by the engine for the acceleration change. So, we need to find the ratio of the forces which it has exerted. So, it is simple ratio to be taken.

Given,

$a1 = 8 \text{ m/s}^2$

$a2 = 16 \text{ m/s}^2$

We have to find the ratio of F_1/F_2

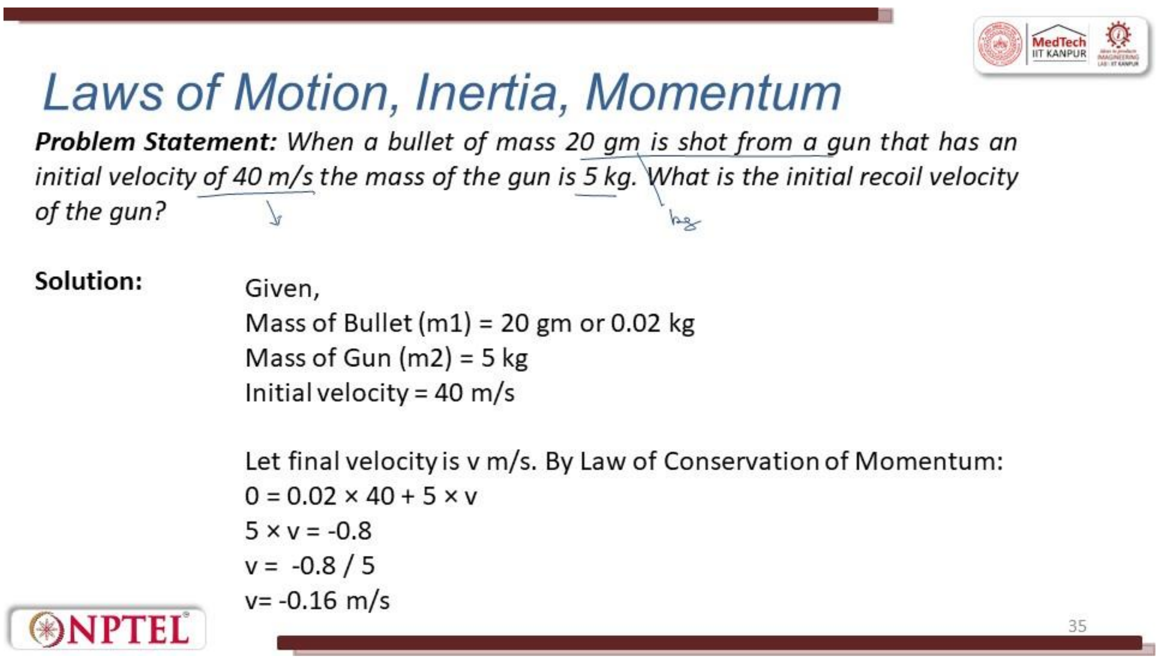
$$F_1/F_2 = ma_1/ma_2$$

Mass of the racing car is same in both the cases,

$$F_1/F_2 = a_1/a_2$$

$$F_1/F_2 = 8/16$$

$$F_1/F_2 = 1/2 = 1:2$$

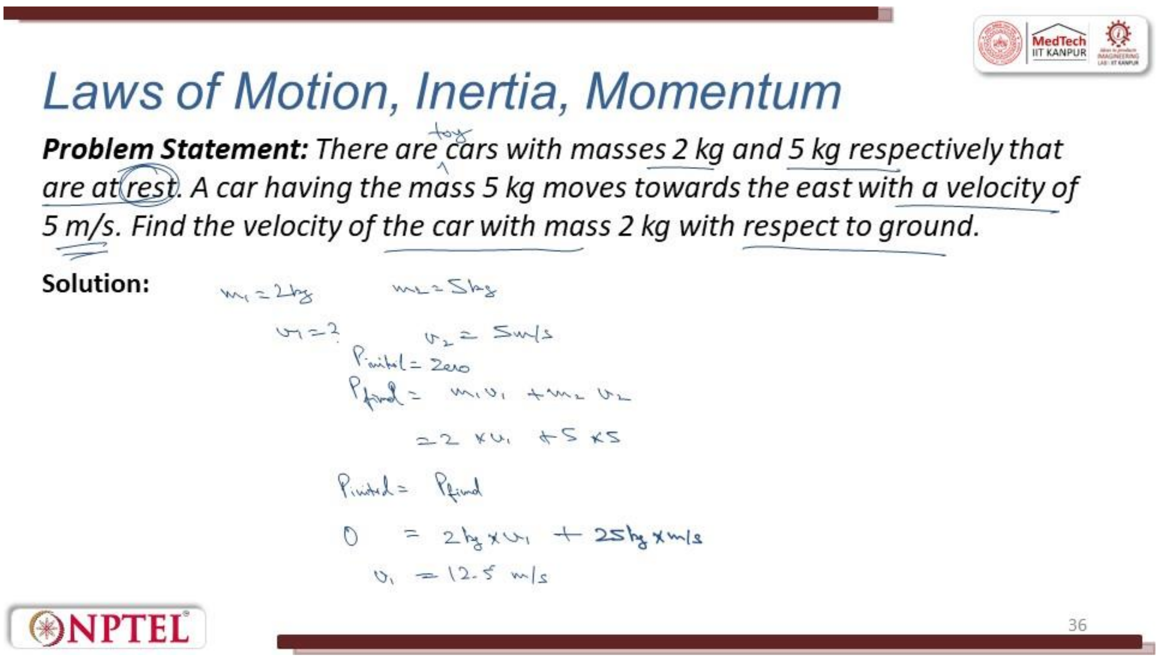


There is another example that is here. When a bullet of mass 20 gram is shot from a gun that has an initial velocity of 40 m/s, the mass of the gun is 5 kg. What is the initial recoil velocity of the gun?

So, it has been given the mass of the bullet as 20 grams. We have to convert this into kg. And we have been given the mass of the gun that is 5 kg right and initial velocity is given for the bullet is 40 m/s.

Recoil velocity of the gun we have to taken. So, we will use the Law of the Conservation of Momentum and try to solve this problem. You can solve it by yourself and try to find what is the initial recoil velocity of the gun. Try to solve it by yourself. Try to use the Law of the Conservation of Momentum and get the solution.

You will get the solution in the lecture notes. There you will find it and this is for you as an home assignment.



Another law of conservation of momentum problem here is there are cars with masses 2 kg and 5 kg respectively the toy cars that are at rest. A car having mass 5 kg moves towards the east with a velocity of 5 meters per second find the velocity of the car with mass 2 kg with respect to ground. So, we have m1 as 2 kg and m2 as 5 kg.

$m_1 = 2$ kg

$m_2 = 5$ kg

$v_1 = ?$

$v_2 = 5$ m/s

We know from the law of conservation of momentum that,

$P_{initial} = 0$, as the cars are at rest

$$P_{final} = p_1 + p_2$$

$P_{final} = m_1\ v_1 + m_2\ v_2$

$= 2\ kg \times v_1 + 5\ kg \times 5\ m/s$

$P_i = P_f$

$0 = 2\ kg\ .\ v_1 + 25\ kg.\ m/s$

$v_1 = 12.5\ m/s$

This tutorial session had very trivial problems. Trivial problems means the problems that you have been studying in the first year engineering or maybe in your pre-engineering senior secondary education.

In the forthcoming lectures, where we will talk about the tutorial sessions on the Stress-Strain Diagram, about the different Mechanical Components, Crutches, Brakes, Belts, etc., that will be having application of all these laws that we have studied here. So, I will give you a sheet of the numerical problems and statements that you will also solve for your home assignment. With this, I am ending this lecture.

Thank you.