Basics of Mechanical Engineering-1

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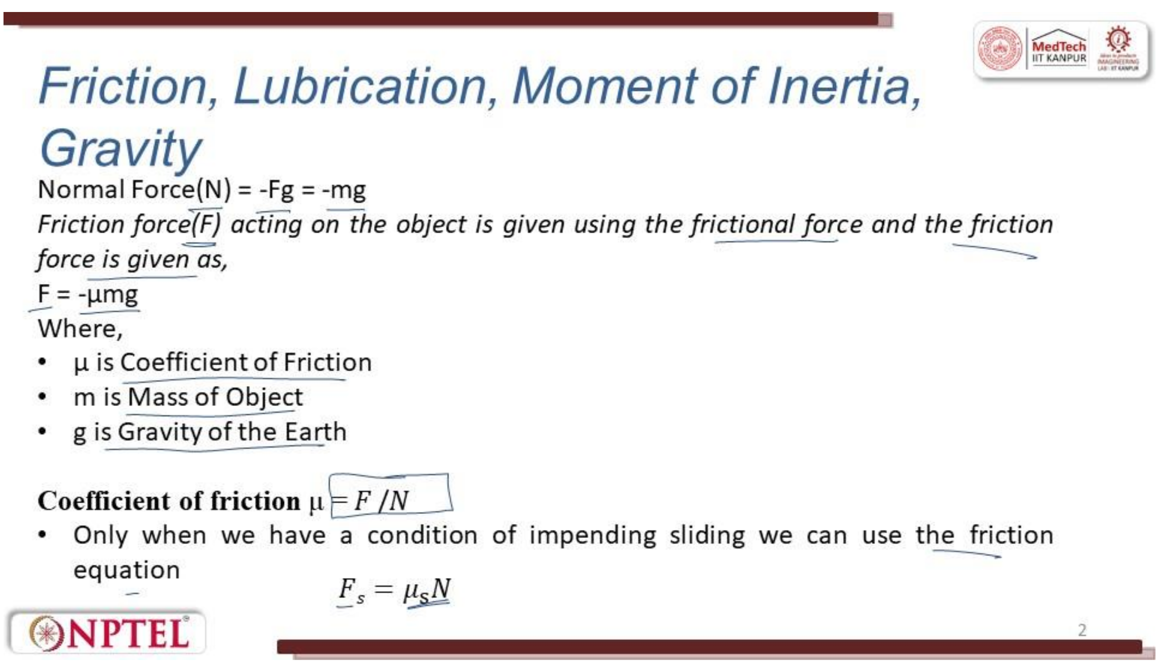
Week 04

Lecture 17

Tutorial 2 (Part 1 of 2)

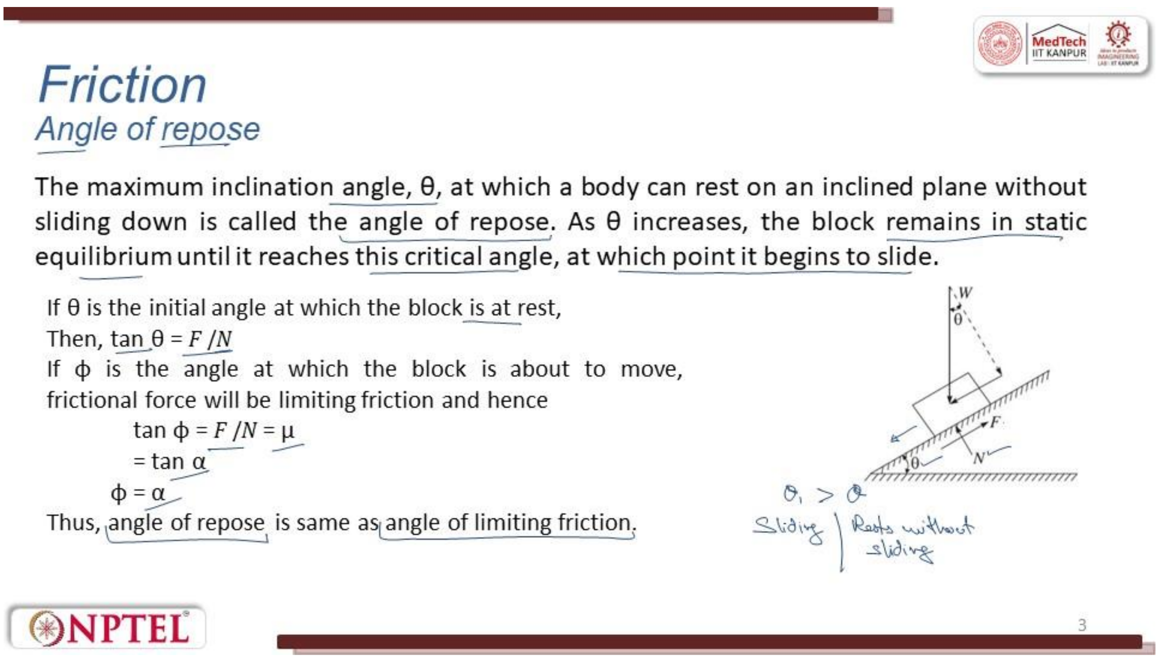
Welcome to the second tutorial session in the course Basics of Mechanical Engineering-1. We have discussed about the Units and Dimensions, Scalar and Vector quantity in the first tutorial. In this session, I will try to cover the tutorials on the topics discussed in the week 1, 2 and 3, which is Friction, Lubrication. I will talk about some of the environmental problems on Moment of Inertia and Gravity. I am Dr. Amandeep Singh Oberoi from Indian Institute of Technology, Kanpur.



Let us try to recall the concepts that we went through in the week 2, that is Friction, Lubrication, Moment of Inertia and Gravity. We understood that the Normal Force (N) =

-Fg = -mg which is minus of mass into the acceleration due to gravity. Friction force F acting on the object is given using the frictional force and the friction force is given as F = -μmg where μ is the coefficient of friction, M is the mass of the object. And g is gravity of the earth. That is acceleration due to gravity.

Coefficient of friction therefore turns to be $\mu = F/N$. That is frictional force over the normal force acting on an object. We will see certain examples and we will try to understand it further using the solving of the examples. Now, only when we have a condition of impending sliding, we can also use the friction equation that is $Fs = \mu_s N$.



Along with the concept of friction, we need to understand something known as Angle of Repose. If there is a body that is sliding over a slant surface with a slant angle of θ and the normal force acting on the body from the bottom is N. Frictional force that is acting on the body F is determined here.

So, maximum inclination angle here is θ at which body can rest on an inclined plane without sliding down is called Angle of Repose. So, if it is at angle θ, it will stay here. Anything more than θ, that is suppose if it is θ one more than the given angle θ, it will start sliding but at angle θ or till the angle θ it does not slide. So, it rests without sliding.

This is known as Angle of Repose as θ increases the block remains in static equilibrium until it reaches its critical angle at which point it begins to slide.

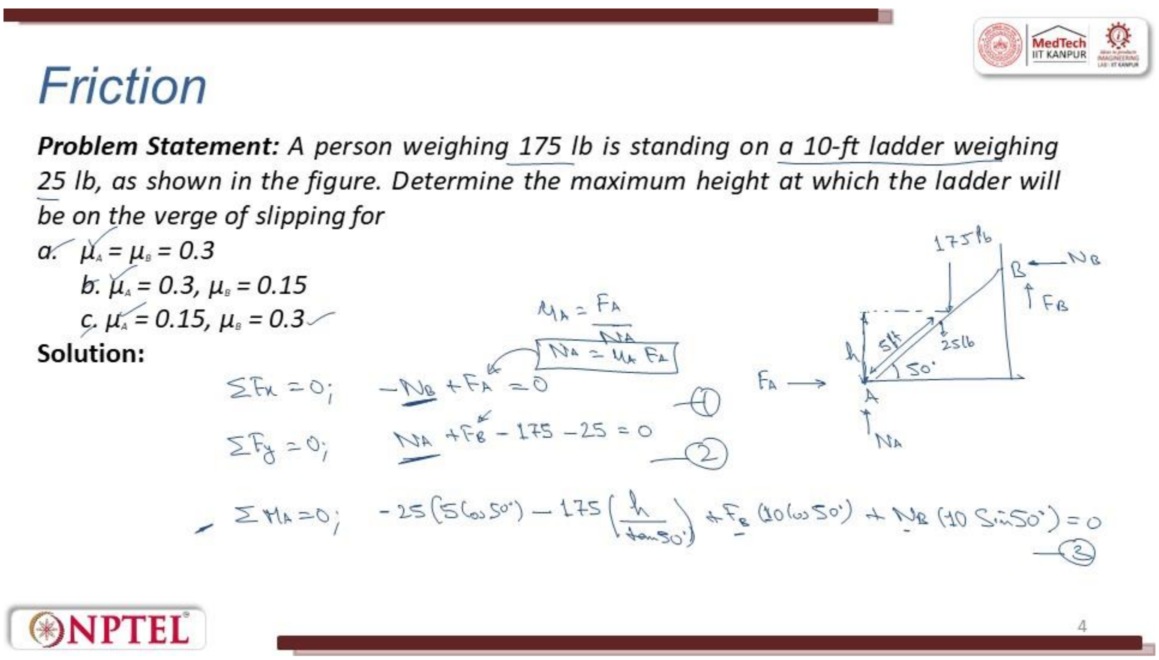
θ is the initial angle at which the block is at rest. So, tan θ = F/N where F is the frictional force, N is the normal force acting at the bottom in the direction normal to the base surface of the object. If φ is the angle at which the block is about to move, frictional force will be limiting friction and hence

$$\tan\varphi = F/N = \mu$$

$$= \tan\alpha$$

$$\varphi = \alpha$$

Thus, angle of repose is same as angle of limiting friction, right. So, I talked about Angle of Friction, I talked about Angle of Repose.



Let us now try to see some examples on Friction and Moment of Inertia. So, here is one problem statement where a person weighing 175 pound is standing on a 10 feet ladder weighing 25 pound as shown in figure. I will draw a figure here. This is a ladder that is 10 feet long and a person is standing here who is weighing 175 pound. Determine the maximum height at which ladder would be on verge or slipping for.

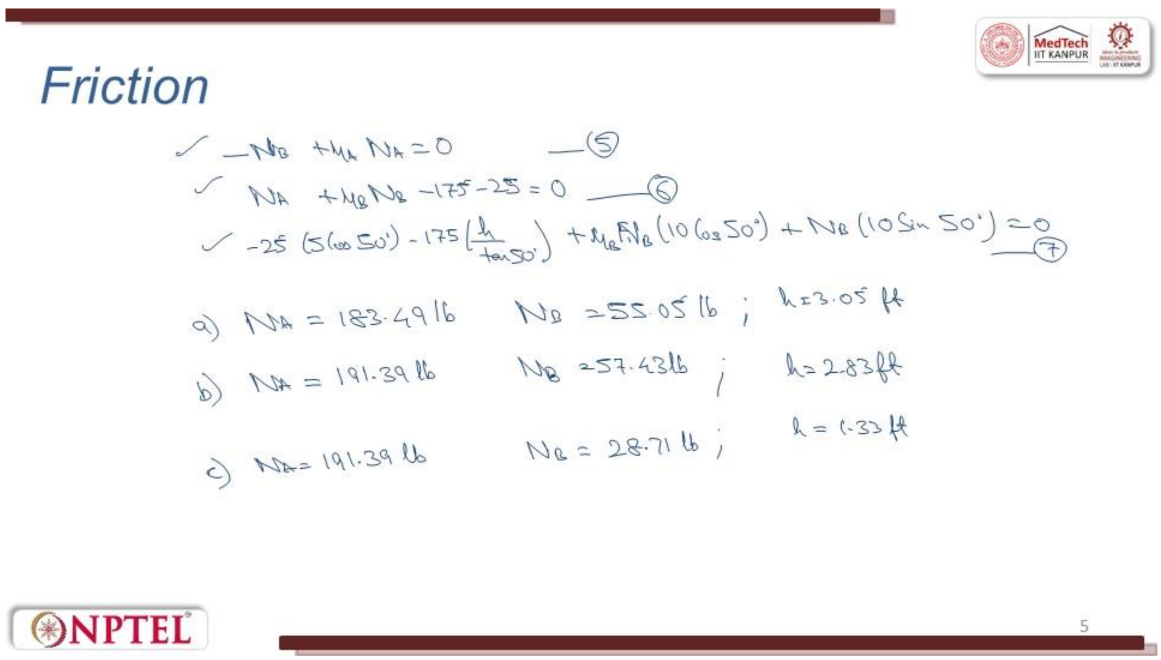
We begin by drawing the free-body diagram of the ladder and writing the equilibrium equations. Note that the friction force at each surface of contact is shown in opposite direction to that of impending motion. This is crucial as we will be using the friction equations along with those of equilibrium to solve the problem.

Equations of equilibrium:

$+\rightarrow \Sigma F_X = 0 \quad = \quad - N_B + F_A = 0$

$+\uparrow \Sigma F_y = 0 \quad = \quad N_B + F_A - 175 - 25 = 0$

$+\downarrow \Sigma M_A = 0 \; = \; - 25(5\cos50°) - 175\left(\frac{h}{tan50°}\right) + F_B(10\cos50°) + N_B\,(10\sin50°) = 0$



$- N_B + \mu_A N_A = 0$

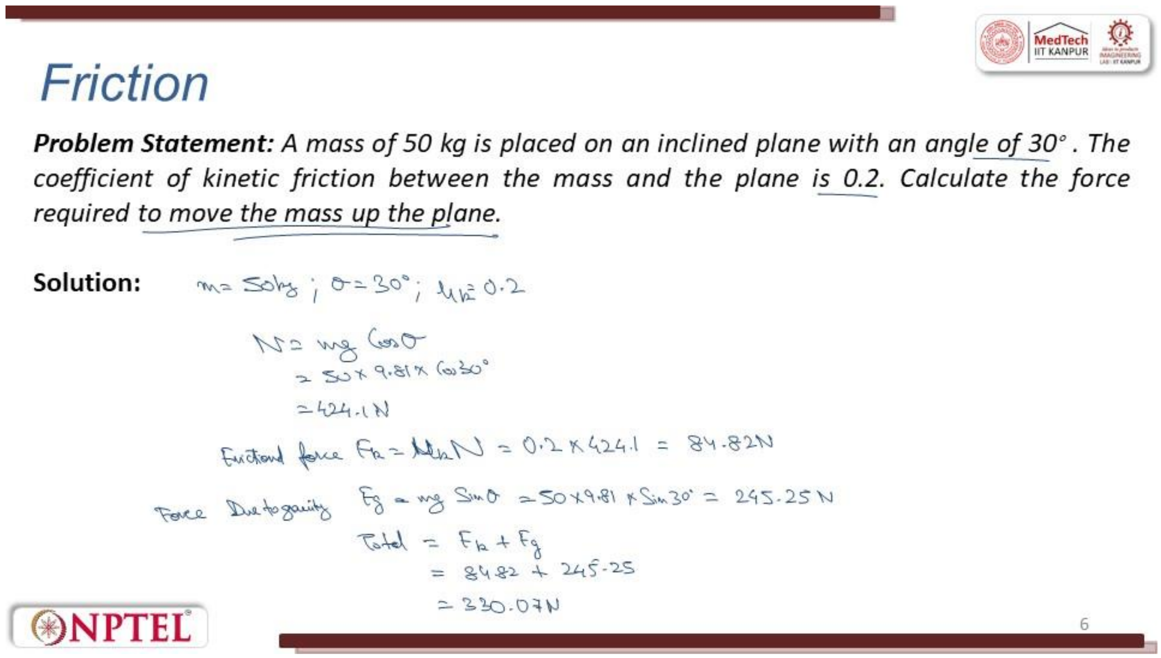
$N_A + \mu_B N_B - 175 - 25 = 0$

$-25\,(5\,\text{Cos}\,50°) - 175\,\left(\frac{h}{tan50}°\right) + \mu_B N_B\,\,(10\,\text{Cos}\,50°)\,N_B\,(10\,\sin 50°) = 0$

a) $N_A = 183.49$ lb $\qquad N_B = 55.05$ lb; $\qquad h = 3.05$ ft

b) $N_A = 191.39$ lb $\qquad N_B = 57.43$ lb; $\qquad h = 2.83$ ft

c) $N_A = 191.39$ lb $\qquad N_B = 28.71$ lb; $\qquad h = 1.33$ ft



So, there is another problem that I could see a mass of 50 kg is placed on an inclined plane with an angle of 30 degrees. The coefficient of kinetic friction between the mass and the plane is 0.2. Calculate the force required to move the mass up the plane.

So, this is again angle of repose is given either you increase the angle or you try to find the force at which this could be pulled up. So, it is trying to pull up or it is trying to move mass up the plane. So, that means g will come into the existence here.

Given: $m=50kg$, $\theta=30^\circ$, $\mu_k=0.2$,

$N=mg\cos\theta = 50 \times 9.81 \times \cos30^\circ = 424.1$ N

Frictional force $F_k = \mu_k N = 0.2 \times 424.1 = 84.82$ N

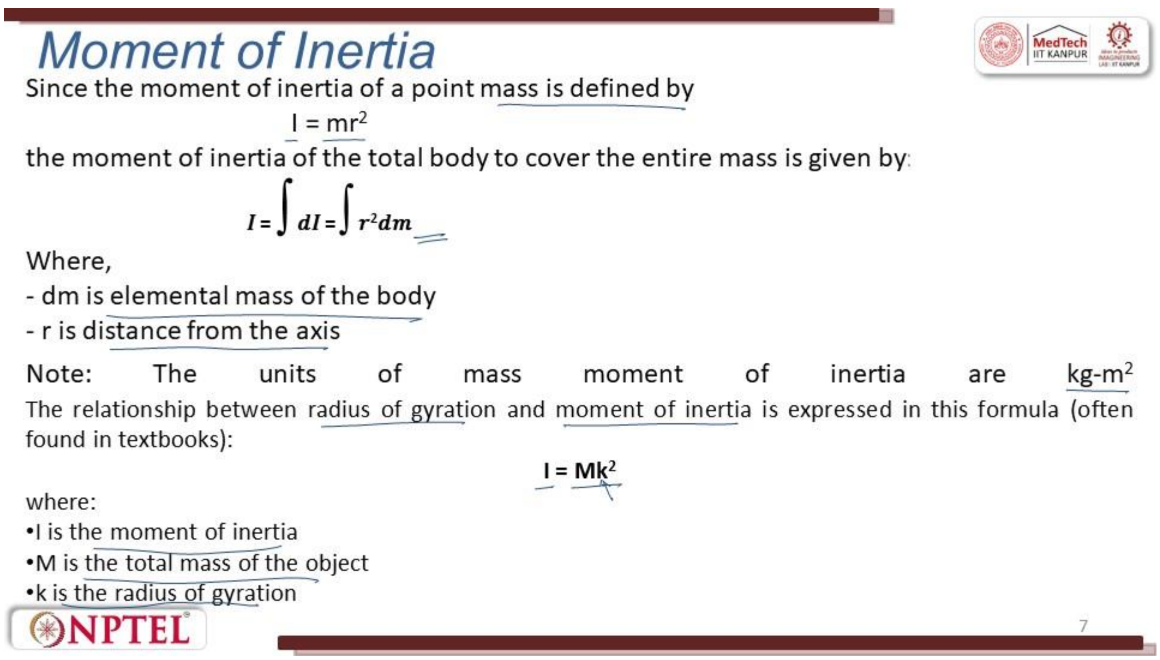
Force due to gravity along the plane $F_g = mg\sin\theta = 50 \times 9.81 \times \sin30^\circ = 245.25$ N

Total force, $F = F_k + F_g$

$= 84.82 + 245.25$

$= 330.07$ N

So, these were some problems on the friction and also I try to include the gravity into it. Now, let us now try to see the moment of inertia though I have calculated the moments in the previous numerical as well.



Let us try to recall the relations which were given in the Moment of Inertia in the theory. So, since the Moment of Inertia of a point mass is defined by I = mr^2. m is mass of the body, r is distance from the axis.

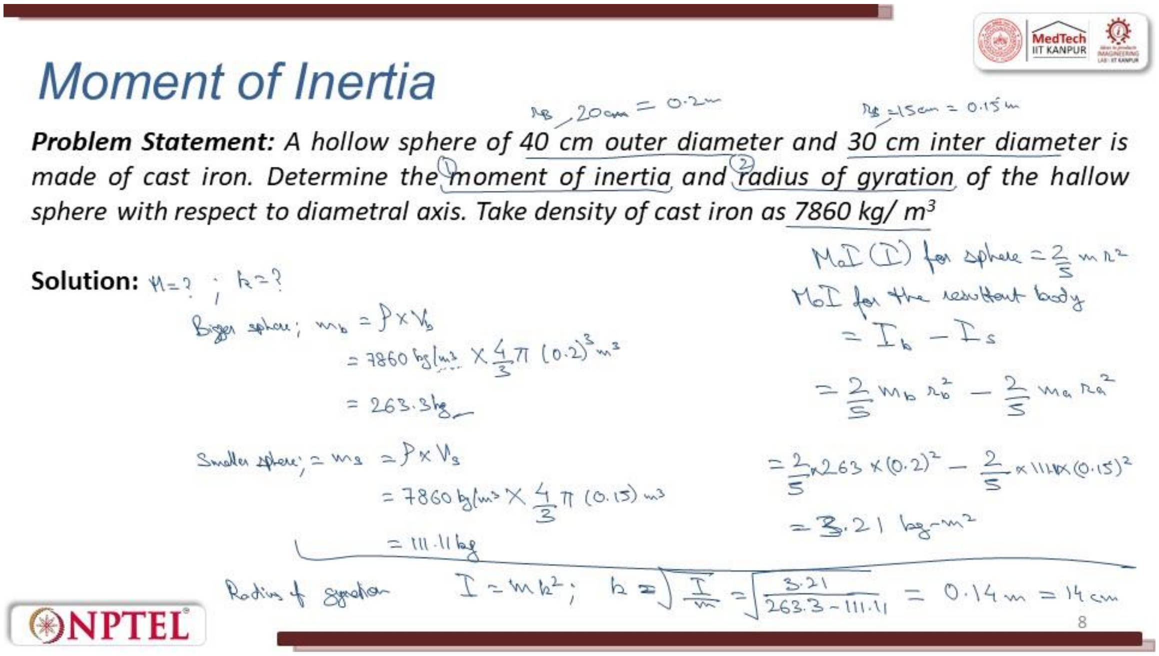
The movement of inertia of the total body to cover the entire mass is given by

$$I = \int dI = \int r2dm$$

where dm is elemental mass of the body and r is distance from the axis. If suppose moment is to be taken around one axis, this distance if the moment of some small mass is there, suppose this is here, this is mass of the body around this axis and this is the distance here. So, there we determine the moment here. The units of mass of moment of inertia are kg meter square. The relationship between the Radius of Gyration and Moment of Inertia is expressed in the formula often found in textbook studies.

I = Mk^2

where k is my radius of gyration. It is given here; k is radius of gyration. I is moment of inertia; m is the total mass of the object.



Let us try to look at certain problem statements on this and try to have a solution on them. A hollow sphere of 40 centimeter outer dia. and 30 centimeter internal dia. is made of cast iron. Determine the Moment of Inertia and Radius of Gyration. That is I have to find value of m? I have to find the value of k? The hollow sphere with respect to the diametral axis take density of cast iron as 7860 kg/m^3. So, there is a larger sphere, there is a smaller sphere, larger sphere is the outer dia., smaller sphere is the internal dia. So, there is a hollow sphere, we have internal dia. and the external dia.

i. Large sphere

Mass of the sphere (M) = $\rho \times v$

$$= 7860 \times \frac{4}{3}\pi r^3 = 7860 \times \frac{4}{3}\pi(20 \times 10^{-2})^3$$

$$= 263.3 \text{ kg}$$

ii. Smaller sphere

Mass of the sphere (M) = $\rho \times v$

$$= 7860 \times \frac{4}{3}\pi r^3 = 7860 \times \frac{4}{3}\pi(15 \times 10^{-2})^3$$

$$= 111.11 \text{ kg}$$

For sphere $I = \frac{2}{5} MR^2$

Moment of inertia of the resulting solid

$$= I_{(\text{larger sphere})} - I_{(\text{Smaller sphere})}$$

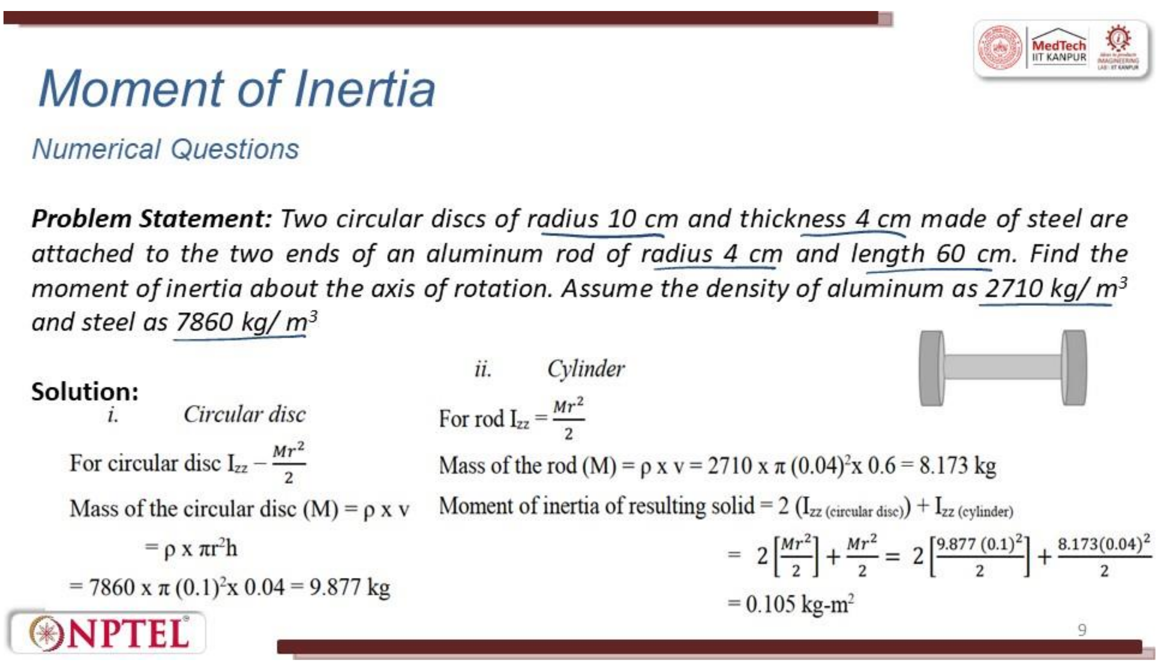
$$I = \frac{2}{5} MR^2 - \frac{2}{5} MR^2$$

$$= \frac{2}{5} \times 263.3 \times 0.2^2 - \frac{2}{5} \times 111.11 \times 0.15^2$$

$$= 3.21 \text{ kg-m}^2$$

Radius of gyration $k = \sqrt{\frac{3.21}{263.30 - 111.11}} = 0.14$ m

This was one problem to calculate the Moment of Inertia, the Radius of Gyration and we tried to calculate it.



So, there is another problem statement where two circular discs of radius 10 centimeter and thickness 4 centimeter. These are made of steel. These are attached to two ends of an aluminum rod of radius 4 centimeter. The length of the aluminum rod is 60 centimeter. Find the moment of inertia about the axis of rotation.

Assume the density of aluminum as 2710 kg per meter cube and for steel it is 7860 kg per meter cube. This is a problem for your home practice. You can also make a diagram that how are these being connected here and how do we have the circular disc of the given radius and thickness and the density also given for aluminum steel. Try to solve it and you will get the solution.

With this I am also ending the first part of the tutorial tool where I have discussed the Friction, the Gravity, the Moment of Inertia related practice questions. I will now try to come to the second part of the second tutorial session where I will try to talk about the Stress, Strain, Residual stresses, Poisson's ratio. All these parts would be discussed in the second part.

Thank you.