

Basics of Mechanical Engineering-1

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Week 04

Lecture 18

Tutorial 2 (Part 2 of 2)

Now, we are in the second part of the second tutorial session. We have been practicing certain numerical problems on the Friction, Moment of Inertia, Gravity in the first part.

Mechanical Properties: Stress, Strain, Residual Stress



Formulae and Relationships

Stress (σ):

- Force per unit area.
- Formula: $\sigma = F/A$

Where, F is the force applied and A is the Cross-sectional area

- Units: Pascal (Pa) or N/m²

Strain (ϵ):

- Deformation per unit length.
- Formula: $\epsilon = \Delta L/L_0$

Where, ΔL is change in length and L_0 is initial length

- Units: Dimensionless (or sometimes expressed as a percentage)



Here, I will focus more on the Stress, E and Residual stresses. Just to recall certain relations and formula, stress is denoted by $\sigma = F/A$ where F is force applied and A is the cross sectional area of the body. Your units of the stress are in Pascal (Pa) or N/m².

Strain is deformation per unit length that is the $\epsilon = \Delta L / L_0$. The initial length where delta is in the change in length and L_0 is the initial length. Units are dimensionless or sometimes it is expressed in percentage ϵ , the percentage elongation or may be in compression the percentage compression.



Mechanical Properties: Stress, Strain, Residual Stress

Formulae and Relationships

Hooke's Law:

- Relationship between stress and strain.
- Formula: $\sigma = E\epsilon$

Where E is the Young's Modulus of the material (Pa)

Residual Stress :

Stresses that remain in a material after the original cause of the stresses has been removed.

- Sources: Manufacturing processes such as welding, machining, and HT.
- Measurement: Often measured using X-ray diffraction or hole-drilling methods.



Hooke's law was also discussed that is the relationship between stress and ϵ we know. $\sigma = E\epsilon$ where E is the young modulus of a material for which the units are Pascal. Residual stresses are the stresses that remain in a material.

It remain in a material after the original cause of the stress has been removed even. So, sources of the Residual stress are manufacturing processes such as welding, machining and heat treatment. Measurement of residual stress is often using the X-ray diffraction or hole drilling and many optical methods are there that helps us to find the residual stresses in the bodies and these residual stresses also determine the properties and the applications where the bodies are to be used.

Mechanical Properties:

Stress, Strain, Residual Stress

Numerical Questions

Problem Statement: A square steel rod 20mm x 20 mm in section is to carry an axial load (compressive) of 100 kN. Calculate the shortening in a length of 50 mm. $E = 2.14 \times 10^8 \text{ kN/m}^2$

Solution: $\delta l = ?$

$$\begin{aligned} \text{Area} \rightarrow A &= 0.02 \times 0.02 = 0.0004 \text{ m}^2 \\ \text{Length} \rightarrow l &= 50 \text{ mm} = 0.05 \text{ m} \\ \text{Load} \rightarrow P &= 100 \text{ kN} \\ E &= 2.14 \times 10^8 \text{ kN/m}^2 \end{aligned}$$



Let us now try to see some of the numerical problems or the relations which were discussed in the previous weeks. This is a problem statement where it is given a square steel rod of 20 mm into 20 mm in section is to carry an axial load which is compressive of 100 kilo Newton. We need to calculate the shortening in a length of 50 mm. I will try to show you the tensile strength test, compression test and some other tests in the forthcoming weeks. We will try to use these relations to find out what is a Young's modulus, what is the Tensile strength, Compressive strength, impact strength of various materials.

Area, $A = 0.02 \times 0.02 = 0.0004 \text{ m}^2$

Length, $l = 50 \text{ mm or } 0.05 \text{ m}$

Load, $P = 100 \text{ kN}$

$$E = 2.14 \times 10^8 \text{ kN / m}^2$$

Mechanical Properties: Stress, Strain, Residual Stress

Shortening of rod δl :

$$\text{Stress } (\sigma) = \frac{P}{A} = \frac{100 \text{ kN}}{0.0004 \text{ m}^2} = 250000 \text{ kN/m}^2$$

$$\text{Strain } (\epsilon) = \frac{\text{Stress}}{E}$$

$$\therefore \text{Strain} = \frac{\text{Stress}}{E} = \frac{250000}{2.14 \times 10^8}$$

$$\frac{\delta l}{l} = \frac{250000}{2.14 \times 10^8}$$

$$\delta l = \frac{250000}{2.14 \times 10^8} \times l = \frac{250000}{2.14 \times 10^8} \times 0.05$$

$$= 0.0000584 \text{ m } (\approx) 0.0584 \text{ mm}$$

$$\text{Shortening } (\delta l) = 0.0584 \text{ mm}$$



Shortening of the rod δl :

$$\text{Stress, } \sigma = \frac{P}{A}$$

$$\sigma = \frac{100}{0.0004} = 250000 \text{ kN/m}^2$$

Also,

$$E = \frac{\text{Stress}}{\text{Strain}}$$

or,

$$\text{Strain} = \frac{\text{Stress}}{E} = \frac{250000}{2.14 \times 10^8}$$

or,

$$\frac{\delta l}{l} = \frac{250000}{2.14 \times 10^8}$$

$$\delta l = \frac{250000}{2.14 \times 10^8} \times l = \frac{250000}{2.14 \times 10^8} \times 0.05$$

$$= 0.0000584 \text{ m or } 0.0584 \text{ mm}$$

Hence, the shortening of the rod = 0.0584 mm

Mechanical Properties: Stress, Strain, Residual Stress

Problem Statement: A hollow cast-iron cylinder 4 m long, 300 mm outer diameter, and thickness of metal 50 mm is subjected to a central load on the top when standing straight. The stress produced is 75000 kN/m². Assume Young's modulus for cast-iron as 1.5 × 10⁸ kN / (m²) and find

- (i) Magnitude of the load,
 (ii) Longitudinal strain produced, and
 (iii) Total decrease in length.

Solution:

$$\begin{aligned}
 D &= 300 \text{ mm} = 0.3 \text{ m} \\
 t &= 50 \text{ mm} = 0.05 \text{ m} \\
 l &= 4 \text{ m} \\
 \text{Stress, } \sigma &= 75000 \text{ kN/m}^2 \\
 E &= 1.5 \times 10^8 \text{ kN/m}^2 \\
 d &= D - 2t = 0.3 - 2 \times 0.05 = 0.2 \text{ m}
 \end{aligned}$$

(i) Magnitude of load.

$$\begin{aligned}
 \sigma &= \frac{P}{A} \\
 P &= \sigma \times A \\
 &= 75000 \times \frac{\pi}{4} (D^2 - d^2) \\
 &= 75000 \times \frac{\pi}{4} ((0.3)^2 - (0.2)^2) \\
 &= 2945.2 \text{ kN}
 \end{aligned}$$



I have a similar problem statement here where A hollow cast-iron cylinder 4 m long, 300 mm outer diameter, and thickness of metal 50 mm is subjected to a central load on the top when standing straight. The stress produced is 75000 kN/m². Assume Young's modulus for cast-iron as 1.5 × 10⁸ kN / (m²) and find

- (i) Magnitude of the load,
 (ii) Longitudinal strain produced, and
 (iii) Total decrease in length

Outer diameter, $D = 300 \text{ mm} = 0.3 \text{ m}$

Thickness, $t = 50 \text{ mm} = 0.05 \text{ m}$

Length, $l = 4 \text{ m}$

Stress produced, $\sigma = 75000 \text{ kN/ m}^2$

$E = 1.5 \times 10^8 \text{ kN/ m}^2$

Inner diameter of the cylinder, $d = D - 2t = 0.3 - 2 \times 0.05 = 0.2 \text{ m}$

i. Magnitude of the load p:

Using the relation, $\sigma = \frac{P}{A}$

or,
$$P = \sigma \times A = 75000 \times \frac{\pi}{4} (D^2 - d^2)$$
$$= 75000 \times \frac{\pi}{4} (0.3^2 - 0.2^2)$$
$$P = 2945.2 \text{ kN (Ans.)}$$

ii. Longitudinal strain produced, e:

Using the relation

Strain,
$$e = \frac{\text{Stress}}{E} = \frac{75000}{1.5 \times 10^8}$$
$$= 0.0005 \text{ (Ans.)}$$

iii. Total decrease in length, δl :

Using the relation,

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta l}{l}$$

$$0.0005 = \frac{\delta l}{l}$$

$$\delta l = 0.0005 \times 4 \text{ m} = 0.002 \text{ m} = 2\text{mm}$$

Hence, decrease in length = 2mm (Ans.)

Mechanical Properties: Stress, Strain, Residual Stress

Problem Statement: The safe stress, for a hollow steel column which carries an axial load of 2.1×10^3 kN is 125 MN/m^2 . If the external diameter of the column is 30 cm, determine the internal diameter.

Solution: Safe stress, $\sigma = 125 \text{ MN/m}^2 = 125 \times 10^6 \text{ N/m}^2$
 Axis load, $P = 2.1 \times 10^3 \text{ kN} = 2.1 \times 10^6 \text{ N}$
 External diameter, $D = 30 \text{ cm} = 0.30 \text{ m}$

Let d = Internal diameter
 Area of cross-section of the column,

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (.30^2 - d^2) \text{ m}^2$$

Mechanical Properties: Stress, Strain, Residual Stress

Using $\sigma = \frac{P}{A}$

$$125 \times 10^6 = \frac{2.1 \times 10^6}{\frac{\pi}{4} (.30^2 - d^2)} \quad \text{or} \quad (.30^2 - d^2) = \frac{4 \times 2.1 \times 10^6}{\pi \times 125 \times 10^6}$$

$$0.09 - d^2 = 213.9 \quad \text{or} \quad 0.09 - 0.02139 = d^2$$

$$\begin{aligned} d &= \sqrt{0.09 - 0.02139} \\ &= 0.2619 \text{ m} \\ &= 26.19 \text{ cm. Ans} \end{aligned}$$

Now, there are numericals in the similar fashion that I have here. The safe stress, for a hollow steel column which carries an axial load of 2.1×10^3 kN is 125 MN/m^2 . If the external diameter of the column is 30 cm, determine the internal diameter.

Safe stress, $\sigma = 125 \text{ MN/m}^2 = 125 \times 10^6 \text{ N/m}^2$

Axis load, $P = 2.1 \times 10^3 \text{ kN} = 2.1 \times 10^6 \text{ N}$

External diameter, $D = 30 \text{ cm} = 0.30 \text{ m}$

Let $d = \text{Internal diameter}$

Area of cross-section of the column,

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (.30^2 - d^2) \text{ m}^2$$

Using $\sigma = \frac{P}{A}$

$$125 \times 10^6 = \frac{2.1 \times 10^6}{\frac{\pi}{4} (.30^2 - d^2)} \quad \text{or} \quad (.30^2 - d^2) = \frac{4 \times 2.1 \times 10^6}{\pi \times 125 \times 10^6}$$

$$0.09 - d^2 = 213.9 \quad \text{or} \quad 0.09 - 0.02139 = d^2$$

$$d = \sqrt{0.09 - 0.02139}$$

$$= 0.2619 \text{ m}$$

$$= 26.19 \text{ cm. Ans}$$

Mechanical Properties: Stress, Strain, Residual Stress

Problem Statement: Find the Young's Modulus of a brass rod of diameter 25 mm and of length 250 mm which is subjected to a tensile load of 50 kN when the extension of the rod is equal to 0.3 mm

Solution: Given: Dia. Of rod, $D = 25 \text{ mm}$

$$\text{Area of rod, } A = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2$$

$$\text{Tensile load, } P = 50 \text{ kN} = 50 \times 1000 = 50,000 \text{ N}$$

$$\text{Extension of rod, } \delta L = 0.3 \text{ mm}$$

$$\text{Length of rod, } L = 250 \text{ mm}$$

Mechanical Properties: Stress, Strain, Residual Stress

Stress (σ) is given by

$$\sigma = \frac{P}{A} = \frac{50,000}{490.87} = 101.86 \text{ N/mm}$$

Strain (e) is given by

$$e = \frac{\delta L}{L} = \frac{0.3}{250} = 0.0012$$

Young's Modulus (E) is obtained, as

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{101.86 \text{ N/mm}^2}{0.0012} = 84883.33 \text{ N/mm}^2$$

$$= 84883.33 \times 10^6 \text{ N/m}^2. \text{ Ans. } (\because 1 \text{ N/mm}^2 = 10^6 \text{ N/m}^2)$$

$$= 84.883 \times 10^9 \text{ N/m}^2 = 84.883 \text{ GN/m}^2. \text{ Ans. } (\because 10^9 = \text{G})$$

Similarly, I have given you another numerical statement here. Find the Young's Modulus of a brass rod of diameter 25 mm and of length 250 mm which is subjected to a tensile load of 50 kN when the extension of the rod is equal to 0.3 mm.

Given: Dia. Of rod, $D = 25 \text{ mm}$

$$\text{Area of rod, } A = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2$$

$$\text{Tensile load, } P = 50\text{kN} = 50 \times 1000 = 50,000 \text{ N}$$

$$\text{Extension of rod, } \delta L = 0.3 \text{ mm}$$

$$\text{Length of rod, } L = 250 \text{ mm}$$

Stress (σ) is given by

$$\sigma = \frac{P}{A} = \frac{50000}{490.87} = 101.86 \text{ N/mm}^2$$

Strain (e) is given by

$$e = \frac{\delta l}{l} = \frac{0.3}{250} = 0.0012$$

Young's Modulus (E) is obtained, as

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{101.86 \text{ N/mm}^2}{0.0012} = 84883.33 \text{ N/mm}^2$$

$$= 84883.33 \times 10^6 \text{ N/m}^2. \text{ Ans.} \quad (\because 1\text{N/mm}^2 = 10^6 \text{ N/m}^2)$$

$$= 84.883 \times 10^9 \text{ N/m}^2 = 84.883 \text{ GN/m}^2. \text{ Ans.} \quad (\because 10^9 = \text{G})$$

Mechanical Properties: Stress, Strain, Residual Stress

Problem Statement: A hydraulic press exerts a total load of 3.5 MN. This load is carried by two steel rods, supporting the upper head of the press. If the safe stress is 85 MPa and $E = 210 \text{ kN/mm}^2$, find :

1. diameter of the rods, and 2. extension in each rod in a length of 2.5 m.

Solution: Given: $P = 3.5 \text{ MN} = 3.5 \times 10^6 \text{ N}$; $\sigma_s = 85 \text{ MPa} = 85 \text{ N/mm}^2$; $E = 210 \text{ kN/mm}^2 = 210 \times 10^3 \text{ N/mm}^2$; $l = 2.5 \text{ m} = 2.5 \times 10^3 \text{ mm}$

1. Diameter of the rods

Let $d =$ Diameter of the rods in mm.

$$\text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854 d^2$$

Since the load P is carried by two rods, therefore load carried by each rod,

$$P_1 = \frac{P}{2} = \frac{3.5 \times 10^6}{2} = 1.75 \times 10^6 \text{ N}$$

Mechanical Properties: Stress, Strain, Residual Stress

We know that load carried by each rod (P_1),

$$1.75 \times 10^6 \text{ N} = \sigma_t \cdot A = 85 \times 0.7854 d^2 = 66.76 d^2$$

$$d^2 = \frac{1.75 \times 10^6}{66.76} = 26213 \text{ or } d = 162 \text{ mm}$$

2. Extension in each rod

Let $\delta l =$ Extension in each rod.

We know that Young's modulus (E),

$$210 \times 10^3 = \frac{P_1 \times l}{A \times \delta l} = \frac{85 \times 2.5 \times 10^3}{\delta l} = \frac{212.5 \times 10^3}{\delta l} \quad \left(\because \frac{P_1}{A} = \sigma_t \right)$$

$$\therefore \delta l = \frac{212.5 \times 10^3}{(210 \times 10^3)} = 1.012 \text{ mm. Ans.}$$

A hydraulic press exerts a total load of 3.5 MN. This load is carried by two steel rods, supporting the upper head of the press. If the safe stress is 85 MPa and $E = 210 \text{ kN/mm}^2$, find :

1. diameter of the rods, and 2. extension in each rod in a length of 2.5 m.

Given: $P = 3.5 \text{ MN} = 3.5 \times 10^6 \text{ N}$; $\sigma = 85 \text{ MPa} = 85 \text{ N/mm}^2$; $E = 210 \text{ kN/mm}^2 = 210 \times 10^3 \text{ N/mm}$; $l = 2.5 \text{ m} = 2.5 \times 10^3 \text{ mm}$

1. Diameter of the rods

Let $d = \text{Diameter of the rods in mm.}$

Area, $A = \frac{\pi}{4} \times d^2 = 0.7854 d^2$

Since the load P is carried by two rods, therefore load carried by each rod,

$$P_1 = \frac{P}{2} = \frac{3.5 \times 10^6}{2} = 1.75 \times 10^6 \text{ N}$$

We know that load carried by each rod (P_1),

$$1.75 \times 10^6 \text{ N} = \sigma \cdot A = 85 \times 0.7854 d^2 = 66.76 d^2$$

$$d^2 = 1.75 \times 10^6 / 66.76 = 26\,213 \text{ or } d = 162 \text{ mm}$$

2. Extension in each rod

Let $\delta l = \text{Extension in each rod.}$

We know that Young's modulus (E),

$$210 \times 10^3 = \frac{P_1 \times l}{A \times \delta l} = \frac{85 \times 2.5 \times 10^3}{\delta l} = \frac{212.5 \times 10^3}{\delta l} \quad \left(\because \frac{P_1}{A} = \sigma \right)$$

$$\therefore \delta l = 212.5 \times 10^3 / (210 \times 10^3) = 1.012 \text{ mm. Ans.}$$

Stress-Strain Curve, Elasticity, Poisson's Ratio

Formulae and Relationships

Elasticity:

- **Hooke's Law:** $\sigma = E\epsilon$
- **Young's Modulus (E):** Measure of material's stiffness.
- Units: Pascal (Pa) or N/m^2

Poisson's Ratio (ν):

- Definition: Ratio of transverse strain to axial strain.
- Formula: $\nu = -\epsilon_{\text{transverse}} / \epsilon_{\text{axial}}$
- Typical values: 0.25 to 0.35 for most metals

Now, another thing that we discussed in the previous weeks are Hooke's law. Hooke's law, Young's modulus, these were discussed.

The Elasticity of the material is given here. Young's modulus is the measure of material stiffness. The unit sparsity is already discussed. Next is Poisson's ratio. The definition is ratio of transverse ϵ to axial ϵ . $\nu = -\epsilon_{\text{transverse}} / \epsilon_{\text{axial}}$ is Poisson's ratio. Typical values are 0.25 to 0.35 for most metal.

Stress-Strain Curve, Elasticity, Poisson's Ratio

Numerical Questions

Problem Statement: A steel rod of cross-sectional area 2 cm^2 is subjected to a tensile force of $10,000 \text{ N}$. Calculate the stress in the rod.

Solution:

$$A = 2 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$$

$$F = 10000 \text{ N}$$

$$\sigma = F/A = 10000 / 2 \times 10^{-4} \\ = 50 \text{ MPa}$$

Let us try to see some problem statements on this. A steel rod of cross sectional area 2 centimeter is subjected to a tensile force of this. Calculate the stress in the rod.

Given: $A = 2 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$

$$F = 10,000 \text{ N}$$

$$\text{Stress } \sigma = F/A$$

$$= 10,000 / 2 \times 10^{-4}$$

$$= 50 \text{ MPa}$$

Stress-Strain Curve, Elasticity, Poisson's Ratio

Problem Statement: A 1-meter-long aluminum bar is stretched by 0.2 mm under a tensile load. Calculate the strain.

Solution:

$$0.02\%$$

Given: $L_0 = 1 \text{ m}$, $\Delta L = 0.2 \text{ mm} = 0.0002 \text{ m}$

Strain $\epsilon = \Delta L / L_0 = 0.0002 / 1 = 0.0002$ (or 0.02%)

Similarly, there is a small very short form articles. 1 meter long aluminum bar is stretched by 0.2 millimeter under intense siloed

- Given: $L_0 = 1 \text{ m}$, $\Delta L = 0.2 \text{ mm} = 0.0002 \text{ m}$
- Strain $\epsilon = \Delta L / L_0 = 0.0002 / 1 = 0.0002$ (or 0.02%)

I have given very short numericals here because some problems might also come in your exams. It will be very short to get marks in your exam. These problems would be easy for you to score.

Stress-Strain Curve, Elasticity, Poisson's Ratio

Problem Statement: A mild steel rod of 12 mm diameter was tested for tensile strength with the gauge length of 60 mm. Following observations were recorded : Final length = 80 mm; Final diameter = 7 mm; Yield load = 3.4 kN and Ultimate load = 6.1 kN.

Calculate : 1. yield stress, 2. ultimate tensile stress, 3. percentage reduction in area, and 4. percentage elongation

Solution:

Given:

$$\begin{aligned} D &= 12 \text{ mm}; \\ l &= 60 \text{ mm}; \\ L &= 80 \text{ mm}; \\ d &= 7 \text{ mm}; \\ W_y &= 3.4 \text{ kN} = 3400 \text{ N} \\ W_u &= 6.1 \text{ kN} = 6100 \text{ N} \end{aligned}$$

$$\text{Original Area; } A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} (12)^2 = 113 \text{ mm}^2$$

$$\text{Final Area; } a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} (7)^2 = 38.5 \text{ mm}^2$$

$$\begin{aligned} \text{1. Yield stress} &= \frac{W_y}{A} = \frac{3400}{113} = 30.1 \text{ N/mm}^2 \\ &= 30.1 \text{ MPa} \end{aligned}$$

Stress-Strain Curve, Elasticity, Poisson's Ratio

2. Ultimate tensile stress; $\frac{W_u}{A} = \frac{6100}{113} \approx 54 \text{ N/mm}^2 = 54 \text{ MPa}$

3. % reduction in Area $\frac{A-a}{A} = \frac{113-38.5}{113} \approx 0.66$
 $= 66\%$

4. % elongation $\frac{L-l}{L} = \frac{80-60}{80} = 0.25$
 $= 25\%$



Now comes another problem statement. A mild steel rod of 12 mm diameter was tested for tensile strength with the gauge length of 60 mm. Following observations were recorded : Final length = 80 mm; Final diameter = 7 mm; Yield load = 3.4 kN and Ultimate load = 6.1 kN. Calculate : 1. yield stress, 2. ultimate tensile stress, 3. percentage reduction in area, and 4. percentage elongation

Given: $D = 12\text{mm}$; $l = 60\text{mm}$; $L = 80\text{mm}$; $d = 7\text{mm}$; $W_y = 3.4 \text{ kN} = 3400 \text{ N}$;

$W_u = 6.1 \text{ kN} = 6100 \text{ N}$

We know that original area of the rod,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (12)^2 = 113 \text{ mm}^2$$

and final area of the rod,

$$a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (7)^2 = 38.5 \text{ mm}^2$$

1. Yield stress:

We know that yield stress = $\frac{W_y}{A} = \frac{3400}{113} = 30.1 \text{ N/mm}^2 = 30.1 \text{ MPa}$ Ans.

2. Ultimate tensile stress:

$$\text{We know the ultimate tensile stress} = \frac{W_u}{A} = \frac{6100}{113} = 54 \text{ N/mm}^2 = 54 \text{ MPa} \quad \text{Ans.}$$

3. Percentage reduction in area:

$$\text{We know that the percentage reduction in area} = \frac{A - a}{A} = \frac{113 - 38.5}{113} = 0.66 \text{ or } 66\%$$

Ans.

4. Percentage elongation:

$$\text{We know that percentage elongation} = \frac{L - l}{L} = \frac{80 - 60}{80} = 0.25 \text{ or } 25\% \quad \text{Ans.}$$



Stress-Strain Curve, Elasticity, Poisson's Ratio

✓ **Problem Statement:** A 2.0 m long metal wire is loaded, resulting in a 4 mm elongation. Find the change in diameter of wire when elongated if the diameter of wire is 1.5 mm and the Poisson's ratio of wire is 0.24.

Solution:

$$l = 2.0 \text{ m}$$
$$\Delta l = \Delta L_{\text{wire}} = 0.004 \text{ m}$$
$$d = 1.5 \text{ mm}$$
$$\nu = 0.24$$
$$\text{longitudinal strain} = \frac{\Delta l}{l}$$
$$= \frac{0.004}{2.0}$$
$$= 0.002$$

$$\nu = \frac{\epsilon_t}{\epsilon_a}$$
$$0.24 = \epsilon_t / 0.002$$
$$\epsilon_t = 0.00048$$
$$= \frac{\Delta d}{d}$$
$$0.00048 = \frac{\Delta d}{1.5}$$
$$\Delta d = 1.5 \times 0.00048$$
$$\Delta d = 0.00072 \quad \text{Ans.}$$

$$\nu = \frac{\epsilon_{\text{transverse}}}{\epsilon_{\text{axial}}}$$



These one kind of problem statements keep coming when we try to talk about Stress-E curve or Elasticity, we will also try to solve something in Poisson ratio now. So, in Poisson ratio, you see this numerical statement that is there a two point 0 mm long metal wire is loaded resulting in a 4 millimeter elongation. Find the change in diameter of wire when elongated if diameter of wire is 1.5 millimeter and Poisson ratio of wire is 0.24.

Given: Length of wire, L is 2.0 m.

Change in length, ΔL is 4 mm = 0.004 m

Diameter of wire, D is 1.5 mm.

Poisson's ratio, ν is 0.24.

The longitudinal strain in the wire is given as:

$$\begin{aligned}\text{Longitudinal strain} &= \Delta L / L \\ &= 0.004 / 2.0 \\ &= 0.002\end{aligned}$$

The Poisson's Ratio formula is as follows:

$$\nu = \text{Lateral strain} / \text{longitudinal strain}$$

Substitute the given values to find the lateral strain.

$$0.24 = \text{lateral strain} / 0.002$$

$$\text{Lateral strain} = 0.00048$$

The lateral strain in a wire is given as:

$$\text{Lateral strain} = \Delta D / D$$

$$0.00048 = \Delta D / 1.5 \text{ mm}$$

$$\Delta D = 0.00072 \text{ mm}$$

The change in diameter of the wire is 0.00072 mm.

Stress-Strain Curve, Elasticity, Poisson's Ratio

Problem Statement: When a brass rod of diameter 6 mm is subjected to a tension of 5×10^3 N, the diameter changes by 3.6×10^{-4} cm. Calculate the longitudinal strain and Poisson's ratio for brass given that Y for the brass is 9×10^{10} N/m². (elastic modulus)

Solution:

Given: Diameter of rod = $D = 6$ mm, Radius of wire = $6/2 = 3$ mm = 3×10^{-3} m, Load $F = 5 \times 10^3$ N, Change in diameter = $d = 3.6 \times 10^{-4}$ cm = 3.6×10^{-6} m, Y for the brass is 9×10^{10} N/m².

To Find: Longitudinal strain = ?; Poisson's ratio = ?

Stress-Strain Curve, Elasticity, Poisson's Ratio

$Y = \text{Longitudinal Stress} / \text{Longitudinal Strain}$

$$\therefore Y = F / (A \times \text{Longitudinal Strain})$$

$$\therefore \text{Longitudinal Strain} = F / (A \times Y)$$

$$\therefore \text{Longitudinal Strain} = F / (\pi r^2 \times Y)$$

$$\therefore \text{Longitudinal Strain} = 5 \times 10^3 / (3.142 \times (3 \times 10^{-3})^2 \times 9 \times 10^{10})$$

$$\therefore \text{Longitudinal Strain} = 5 \times 10^3 / (3.142 \times 9 \times 10^{-6} \times 9 \times 10^{10})$$

$$\therefore \text{Longitudinal Strain} = 1.96 \times 10^{-3}$$

$$\text{Now, Lateral strain} = d/D = (3.6 \times 10^{-6}) / (6 \times 10^{-3}) = 6 \times 10^{-4}$$

$$\text{Poisson's ratio} = \text{Lateral strain} / \text{Longitudinal strain}$$

$$\therefore \text{Poisson's ratio} = (6 \times 10^{-4}) / (1.96 \times 10^{-3}) = 0.31$$

Ans: Longitudinal strain is 1.96×10^{-3} and Poisson's ratio is 0.31.

Similar problems could be taken like another one When a brass rod of diameter 6 mm is subjected to a tension of 5×10^3 N, the diameter changes by 3.6×10^{-4} cm. Calculate the longitudinal strain and Poisson's ratio for brass given that Y for the brass is 9×10^{10} N/m². (elastic modulus)

Given: Diameter of rod = $D = 6 \text{ mm}$, Radius of wire = $6/2 = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$,

Load $F = 5 \times 10^3 \text{ N}$, Change in diameter = $d = 3.6 \times 10^{-4} \text{ cm} = 3.6 \times 10^{-6} \text{ m}$,

Y for the brass is $9 \times 10^{10} \text{ N/m}^2$.

To Find: Longitudinal strain =? Poisson's ratio = ?,

$Y = \text{Longitudinal Stress} / \text{Longitudinal Strain}$

$\therefore Y = F / (A \times \text{Longitudinal Strain})$

$\therefore \text{Longitudinal Strain} = F / (A \times Y)$

$\therefore \text{Longitudinal Strain} = F / (\pi r^2 \times Y)$

$\therefore \text{Longitudinal Strain} = 5 \times 10^3 / (3.142 \times (3 \times 10^{-3})^2 \times 9 \times 10^{10})$

$\therefore \text{Longitudinal Strain} = 5 \times 10^3 / (3.142 \times 9 \times 10^{-6} \times 9 \times 10^{10})$

$\therefore \text{Longitudinal Strain} = 1.96 \times 10^{-3}$

Now, Lateral strain = $d / D = (3.6 \times 10^{-6}) / (6 \times 10^{-3}) = 6 \times 10^{-4}$

Poisson's ratio = Lateral strain / Longitudinal strain

$\therefore \text{Poisson's ratio} = (6 \times 10^{-4}) / (1.96 \times 10^{-3}) = 0.31$

Ans: Longitudinal strain is 1.96×10^{-3} and Poisson's ratio is 0.31.



Stress-Strain Curve, Elasticity, Poisson's Ratio

Problem Statement: A metal wire of length 1.5 m is loaded and an elongation of 2 mm is produced. If the diameter of the wire is 1 mm, find the change in the diameter of the wire when elongated. $\nu = 0.24$.

Solution: Given: Original length of wire = $L = 1.5 \text{ m}$, Elongation in wire = $2 \text{ mm} = 2 \times 10^{-3} \text{ m}$, Diameter of wire = $D = 1 \text{ mm}$, Poisson's ratio = $\sigma = 0.24$.

To Find: Change in diameter = $d = ?$

Longitudinal strain = $l / L = (2 \times 10^{-3}) / 1.5 = 1.33 \times 10^{-3}$

Poisson's ratio = Lateral strain / Longitudinal strain

$\therefore \text{Lateral strain} = \text{Poisson's ratio} \times \text{Longitudinal strain} = 0.24 \times 1.33 \times 10^{-3} = 3.2 \times 10^{-4}$

Lateral strain = d / D

$\therefore d = \text{Lateral strain} \times D = 3.2 \times 10^{-4} \times 1 \times 10^{-3} = 3.2 \times 10^{-7} \text{ m}$

Ans: The change in diameter is $3.2 \times 10^{-7} \text{ m}$

There is another problem statement where metal wire of length 1.5 meter is loaded and elongation is 2 millimeters. If the diameter of wire is 1 millimeter, find the change in a diameter of the wire when it is elongated.

Given: Original length of wire = $L = 1.5$ m, Elongation in wire = $2 \text{ mm} = 2 \times 10^{-3}$ m,
Diameter of wire = $D = 1$ mm, Poisson's ratio = $\sigma = 0.24$.

To Find: Change in diameter = $d = ?$

$$\text{Longitudinal strain} = l/L = (2 \times 10^{-3})/1.5 = 1.33 \times 10^{-3}$$

$$\text{Poisson's ratio} = \text{Lateral strain} / \text{Longitudinal strain}$$

$$\therefore \text{Lateral strain} = \text{Poisson's ratio} \times \text{Longitudinal strain} = 0.24 \times 1.33 \times 10^{-3} = 3.2 \times 10^{-4}$$

$$\text{Lateral strain} = d / D$$

$$\therefore d = \text{Lateral strain} \times D = 3.2 \times 10^{-4} \times 1 \times 10^{-3} = 3.2 \times 10^{-7} \text{ m}$$

Ans: The change in diameter is 3.2×10^{-7} m

Stress-Strain Curve, Elasticity, Poisson's Ratio

Problem Statement: For a given material. Young's modulus is 110 GN/m^2 and shears modulus is 42 GN/m^2 . Find the Bulk modulus and lateral contraction of a round bar of 37.5 mm diameter and 2.4 m length when stretched 2.5 mm .

Solution:

$$\begin{aligned}
 E &= 110 \text{ GN/m}^2 \\
 C &= 42 \text{ GN/m}^2 \\
 d &= 37.5 \text{ mm} = 0.0375 \text{ m} \\
 l &= 2.4 \text{ m} \\
 \delta l &= 2.5 \text{ mm} = 0.0025 \text{ m}
 \end{aligned}$$

I K is Bulk modulus:

$$K = \frac{mE}{3(m-2)}$$

II Shear modulus:

$$E = 2C \left(1 + \frac{1}{m}\right)$$

Using II: $110 \times 10^9 = 2 \times 42 \times 10^9 \left(1 + \frac{1}{m}\right)$

$$m = \frac{1}{0.31} = 3.22$$

Substituting in I:

$$\begin{aligned}
 K &= \frac{3.22 \times 110 \times 10^9}{3(3.22-2)} \\
 &= 96.77 \text{ GN/m}^2
 \end{aligned}$$

Lateral contraction: δd

$$\text{Long. strain} = \frac{\delta l}{l} = \frac{0.0025}{2.4} = 0.00104$$

$$\text{Lateral strain} = 0.00104 \times \frac{1}{3.22} = \frac{0.00104}{3.22} = 0.000323$$

$$\delta d = 0.000323 \times 37.5 = 0.121 \text{ mm}$$



So, the another problem statement in which Young's modulus is given as $110 \text{ giga Newton's per meter square}$. This shear modulus is $42 \text{ giga Newton's per meter square}$. Find the Bulk modulus and Lateral contraction of a round bar of $37.5 \text{ millimeter diameter}$ and 2.4 meter length when stretched 2.5 millimeter .

Solution: $E = 110 \text{ GN/ m}^2$

Bulk modulus, K :

Shear modulus,

We know that,

$$C = 42 \text{ GN/ m}^2$$

$$E = 2C \left[1 + \frac{1}{m}\right]$$

Diameter of round bar,

$$d = 37.5 \text{ mm} = 0.0375 \text{ m}$$

$$110 \times 10^9 = 2 \times 42 \times 10^9 \left[1 + \frac{1}{m}\right]$$

Length of round bar,

$$l = 2.4 \text{ m}$$

Extension of bar,

$$\delta l = 2.5 \text{ mm} = 0.0025 \text{ m}$$

$$\frac{1}{m} = 1.31 - 1 = 0.31 \text{ or } m = \frac{1}{0.31} = 3.22$$

Substituting this value of m in the equation

$$K = \frac{mE}{3(m-2)} \quad K = \frac{3.22 \times 110 \times 10^9}{3(3.22-2)} = \mathbf{96.77 \text{ GN/m}^2}$$

Lateral contraction, δd :

Longitudinal strain, $\frac{\delta l}{l} = \frac{0.0025}{2.4} = 0.00104$

and, lateral strain $= 0.001045 \times \frac{1}{m} = 0.00104 \times \frac{1}{3.22} = 0.000323$

\therefore Lateral contraction, $\delta d = 0.000323 d$
 $= 0.000323 \times 37.5 = 0.0121 \text{ mm (Ans.)}$

So, this is one where we have calculated the bulk modulus and also we have used the relationship for the shear modulus as well in this problem.

Stress-Strain Curve, Elasticity, Poisson's Ratio



Problem Statement: Calculate the work done in stretching a steel wire 100 cm in length and of cross sectional area 0.03 cm^2 when a load of 100 N is slowly applied before the elastic limit is reached. [$Y_{\text{Steel}} = 2 \times 10^{11} \text{ N/m}^2$]

Solution: Work done (W) = ?

Length of wire (l) = 100 cm = 1 m

Cross-sectional area A = 0.03 cm^2

$$A = 0.03 \times (1 \times 10^{-2})^2 \text{ m}^2$$

$$A = 0.03 \times 10^{-4} \text{ m}^2$$

Force F = 100 N

$$Y (\text{steel}) = 2 \times 10^{11} \text{ Nm}^{-2}$$

We know, Young's modulus of elasticity

$$Y = \frac{Fl}{eR}$$

$$e = \frac{Fl}{YR}$$

$$e = (100 \times 1) / (2 \times 10^{11} \times 0.03 \times 10^{-4})$$

$$\text{Elongation, } e = \frac{1}{6000} \text{ m}$$

Now,

Work done (W) = Energy stored (E)

$$W = \frac{1}{2} F.e$$

$$W = \frac{1}{2} \times 100 \times \frac{1}{6000}$$

$$W = 8.33 \times 10^{-3} \text{ J}$$



So, the last problem statement in this lecture is where we have been given the steel wire of 100 centimeter length and cross sectional area of 0.03 centimeter square and load that is applied on this is 100 Newton and this is slower load that is being applied. The

Young's model of the steel is also given that is $2 \times 10^{11} \text{ N / m}^2$. We need to calculate the work done. Calculate the work done in stretching right that means, we need to calculate the energy stored.

Work done (W) = ?

$$\text{Length of wire (l)} = 100 \text{ cm} = 1 \text{ m}$$

$$\text{Cross-sectional area A} = 0.03 \text{ cm}^2$$

$$A = 0.03 \times (1 \times 10^{-2})^2 \text{ m}^2$$

$$A = 0.03 \times 10^{-4} \text{ m}^2$$

$$\text{Force F} = 100 \text{ N}$$

$$Y (\text{steel}) = 2 \times 10^{11} \text{ Nm}^{-2}$$

We know, Young's modulus of elasticity

$$Y = \frac{Fl}{eR}$$

$$e = \frac{Fl}{YR}$$

$$e = (100 \times 1) / (2 \times 10^{11} \times 0.03 \times 10^{-4})$$

$$\text{Elongation, } e = 16000 \text{ m}$$

Now,

Work done (W) = Energy stored (E)

$$W = \frac{1}{2} F.e$$

$$W = \frac{1}{2} \times 100 \times \frac{1}{6000}$$

$$W = 8.33 \times 10^{-3} \text{ J}$$

With this I am ending this tutorial and I will meet you further where I will take you to further practice sessions and also I will discuss about the laboratory sessions. Laboratory sessions means I will show you the videos on how different tests are being conducted in

laboratory and also we will try to see some simulations on the results on different tests which are there in the forthcoming lectures.

Further we will also try to talk about certain mechanisms in the forthcoming lectures in the forthcoming weeks.

Thank you.