Basics of Mechanical Engineering-1

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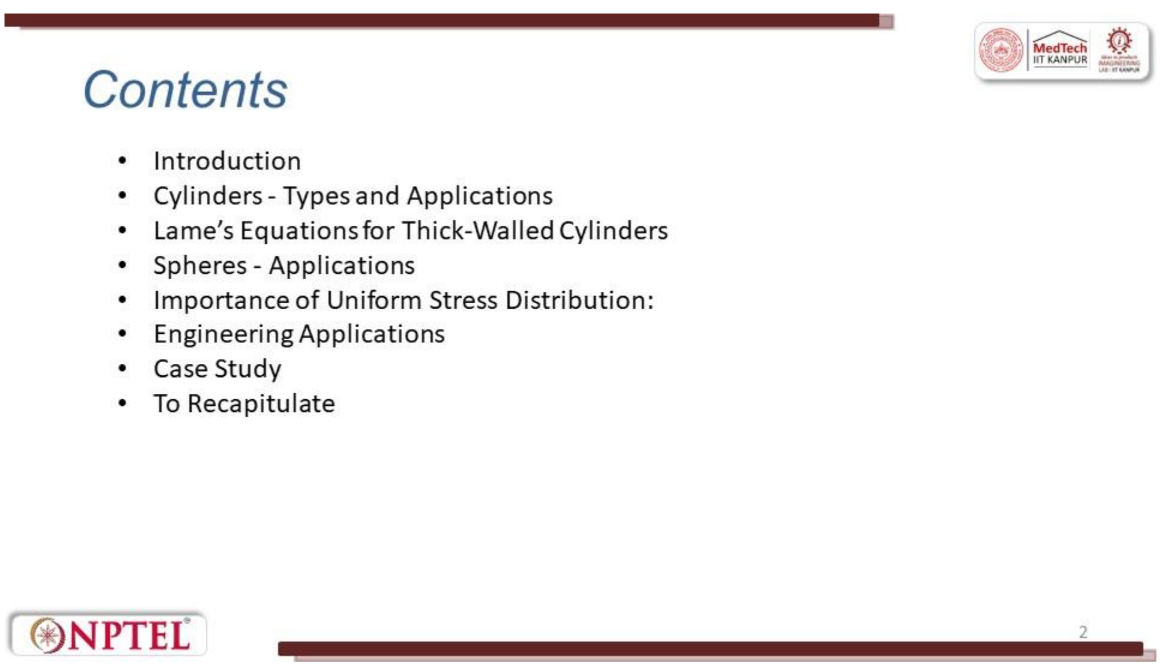
Week 05

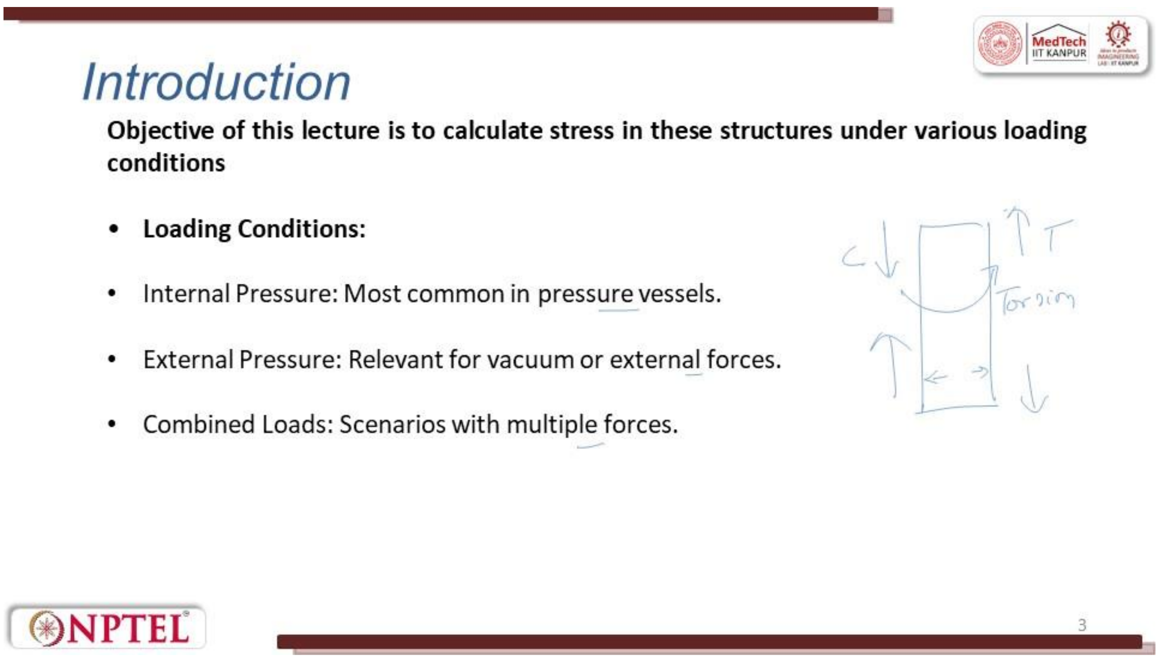
Lecture 19

Stress in Cylinders and Spheres (Part 1 of 2)

Welcome to the next lecture on Stress in Cylinders and Spheres. When we are talking about Cylinder, it is like your bottle, plastic bottle where you fill it up with water or you fill it up with some aerated drinks and then have it. So that is a typical Cylinder and then a Sphere is a ball. So, many of the structures if you see, they are always cylindrical in shape or they are spherical in shape. When we do construction industry, we always look forward for cylindrical one. Cylinder is easy to manufacture. As we discussed in the previous lectures, cylinders does not have any sharp edges. So, moment there are not many sharp edges, the stress concentration points are going to be low.

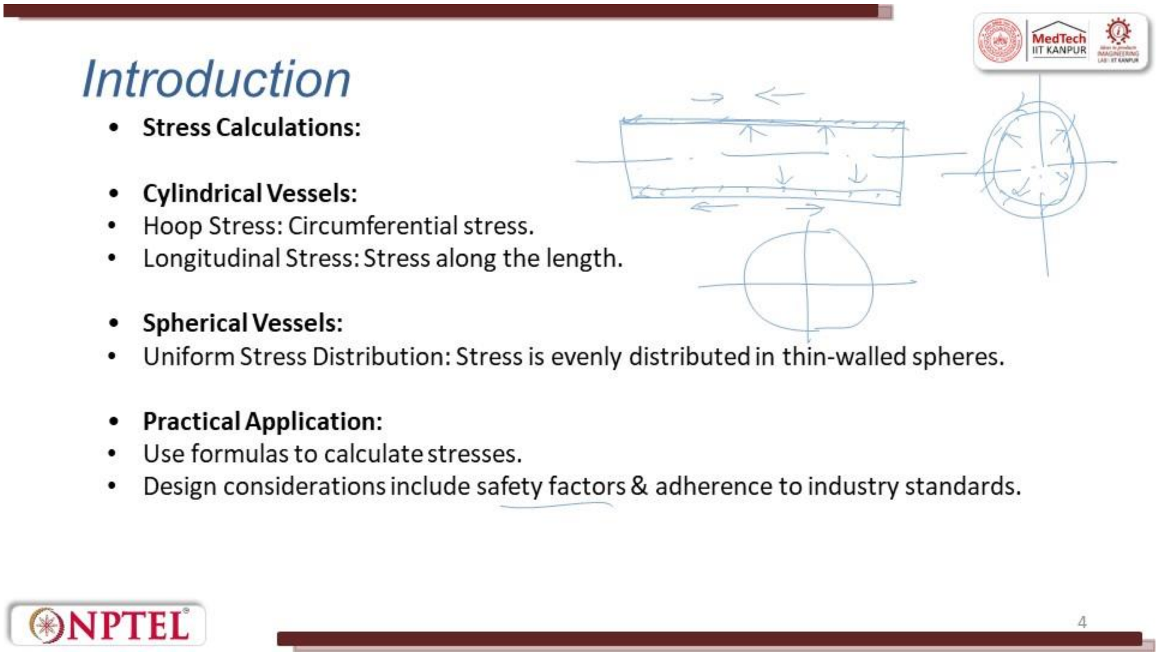


This lecture will follow the content like introduction, then Cylinders, types and its application, Lame's equation for Thick Walled Cylinder, Spheres and its Application, Importance of Uniform Stress Distribution, Engineering Application, Case Studies, one or two problems to solve and finally we will have a recap.



The objective of this lecture is to calculate stress in these structures under various loading conditions. When we take a Cylinder, you can have tensile load, you can have Compressive load, you can have Torsion load. This is Tensile, this is compression, this is Torsion or Shear. You can also have internal stress. So you can see here so many different forces or loading conditions can happen when you fill the cylinder with water or when you are trying to fill it up with pressurized air.

So the loading conditions can be internal pressure, most common in pressure vessels. Then you can have external pressure relevant for vacuum or external forces or you can try to have a combination scenarios with multiple forces. You can have varying loading conditions.

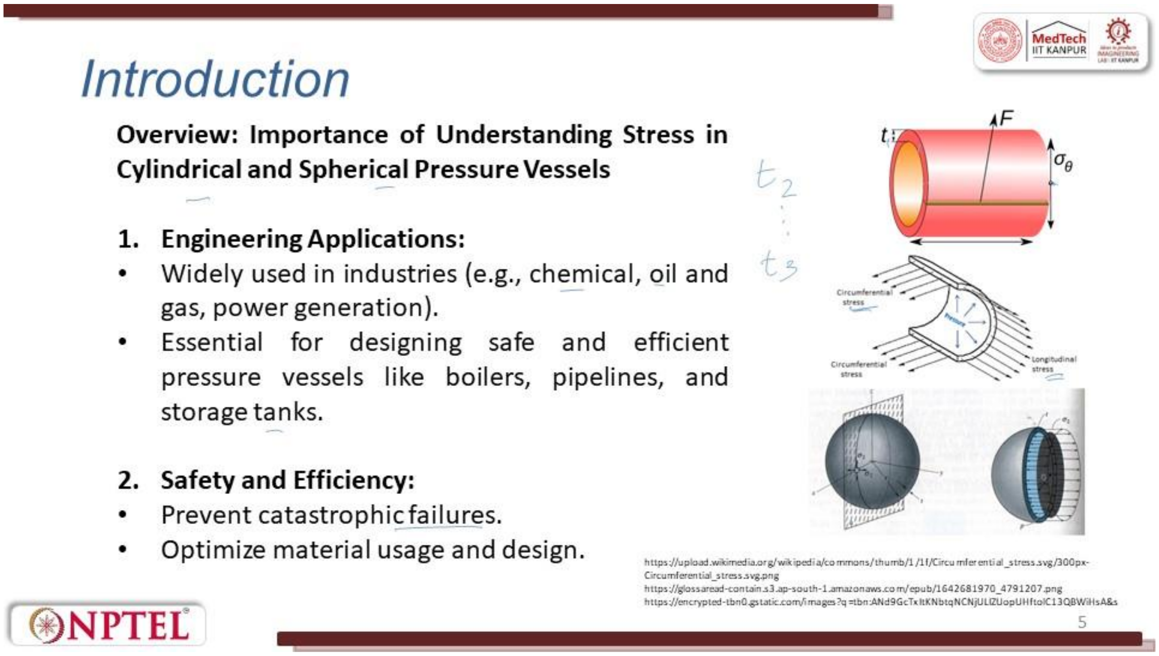


Stress Calculations. We will try to see in a cylindrical vessel and in a spherical vessel, what are all the stresses which are going to happen?

When we try to talk about a cylinder, you can have two type of Stresses. I said a cylinder, the cylinder will have some amount of thickness and we discussed that Thin Wall Cylinder, Thick Wall Cylinder. So, Thin Wall Cylinder you can try to have. So in a Cylinder you can have hoop stress which is otherwise called as circumferential stress and then you can have longitudinal stress. It can be Tensile, it can be Compression.

So Hoop stress it is otherwise called as Circumferential stress. Then you have longitudinal stress which is the stress acting along the length of a cylinder. Spherical vessel. So Spherical vessel are, when you have spherical vessels you can see where they store petroleum products or a tank, it will always be spherical in nature. It will be spherical in nature.

So it will have uniform stress distribution. The stress is evenly distributed in the thin walls. Practical applications wherein which you can use formulas to calculate the stresses and design consideration includes factor of safety and adhere to the industrial standards.

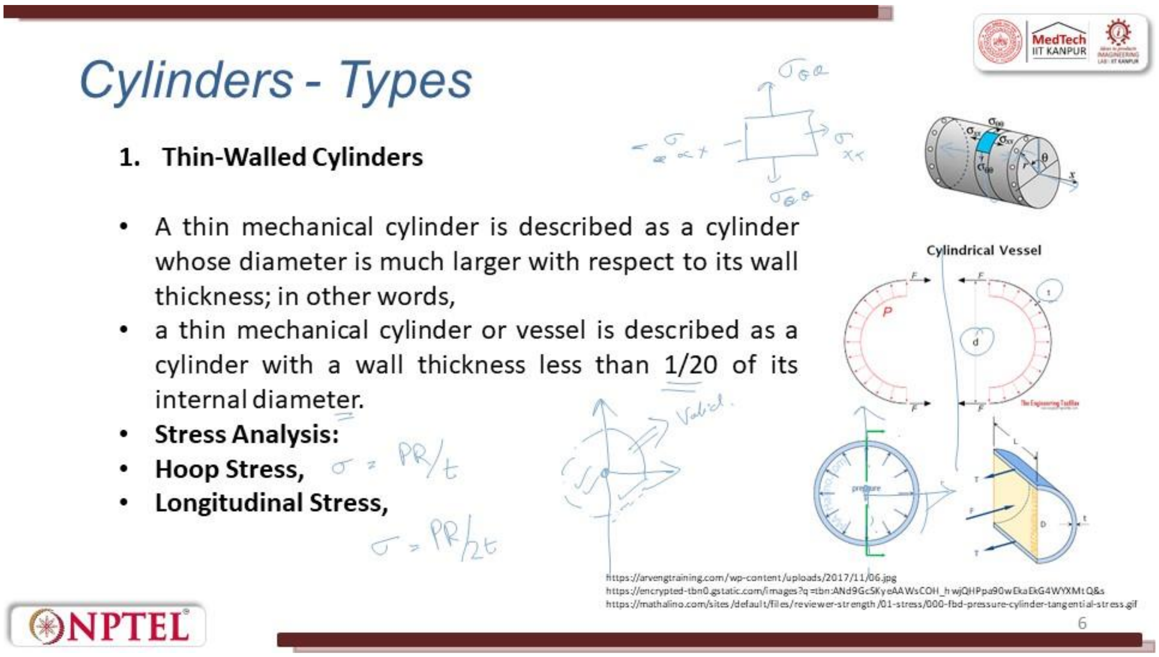


The importance and understanding stress in cylinders and spherical pressure vessels. The engineering applications are going to be, it is used in chemical industry, oil and gas industry, power generation industry, everywhere we will be using the cylinders.

Essentially for designing safe and efficient pressure vessels like boilers, pipelines and storage tanks. So these are general engineering applications wherein which we try to use cylinders and hemispheric pressure vessels. So if you try to take a cylinder as I already discussed, T is the thickness, the stress which is acting along the direction and which is perpendicular to the axis and then you will also have the θ which is there, right. So the stresses are going to be Circumferential stress. And this is along the direction which is called as Longitudinal stress.

When you try to take a cylinder and when you try to fill it up with liquid or with air, you are trying to pressurize. So this pressure will lead to circumferential stress and longitudinal stress. The safety and efficiency it should make sure prevent such that there is no catastrophical failure and then we also have to optimize the amount of material which is used in the cylinder. For example you can make T instead of T1 you can go to T2. So T can be T2 it can be T3.

So the T3 can be larger thickness. So you have to find out what should be the minimum thickness I should apply such that it tries to avoid catastrophical failure. So cylinder types, there are thin wall, then we have thick wall cylinders.



So thin wall cylinders will be taken for discussion first. Thin Wall Mechanical cylinders is described as a cylinder whose diameter is much larger with respect to its wall thickness.

In other words, the thin mechanical cylinder or vessel is described as a cylinder with a wall thickness less than 1/20th of the internal diameter. So this 1/20th is very important because when you try to go smaller and smaller, for example, when you try to go in millimeter diameter, so then the thickness you should know what should be the thickness. So it should be 1/20th of the internal diameter, if you have thickness then it is called as

Thin Wall Cylinders and if you remember back we did a Principal Strain, Principal Stress calculation the same thing will also happen here. Since it is an introductory course, we just superficially scrape through it. Otherwise, we try to take a single element and try to solve with this single element what are all the stresses which come and how do we solve it.

So, if you try to look into this small element, I will just draw it here. It talks about, because it is in this running direction, see θ, σ_θ, θ and then it is σ_{xx} is in this direction and maybe σ xx minus will be in this direction, right. So a single unit cell is taken back to the principal stress and then we try to understand what is coming in the unit cell or in the in the single element and then we try to come back and then take it because we assume that several of these unit cells integrate together and form a thin wall and then first we try to integrate along this direction and then we try to integrate along this direction or then we try to get what is the value which comes, right. So here you can see here this is θ and then this is θ direction and this is x direction here is the radius which is given.

So when we try to section a cylinder, we will try to see how are the forces coming when the pressure is acting, D is the diameter and T is the thickness. So generally in a pressure vessel, it always happens like this when we try to take a cross section. Because whatever you try to solve in this, in this direction, this is half. When we try to solve this half, so if you multiply it by 2, this half also comes or you try to do it only with this half. In fact, when there is a cylinder, we always try to solve the problem for a quarter.

The quarter we multiply it into 2. So it is first we try to solve like this. Then we try to multiply and try to solve like this. Then what we do is we try to solve it for 180 degrees. So when something is valid here, it is also valid here and then here.

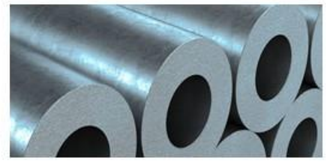
So it is valid here and then it is also valid here. Why? Because it is a small segment which is spinned around an axis and there is uniform pressure all around. So we will try to generalize and do it only for a quarter. So Stress Analysis, then we have hoop stress.

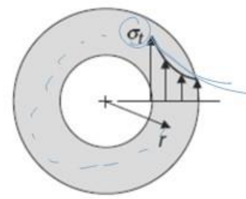
Hoop stress is defined as $\sigma = \frac{PR}{t}$. The Longitudinal stress is defined as $\sigma = \frac{PR}{2t}$. This is for Longitudinal stress.

Cylinders - Types

2. Thick-Walled Cylinders

- A thick-walled cylinder is defined as having a wall thickness greater than one-twentieth of its inner diameter. This design is used in high-pressure applications to ensure structural integrity and safety under significant internal pressures.
- **Stress Analysis:**
- Uses Lame's equations for more complex stress distributions.
- **Radial Stress (σ_r)** and **Hoop Stress (σ_h)** are determined based on internal and external pressures.





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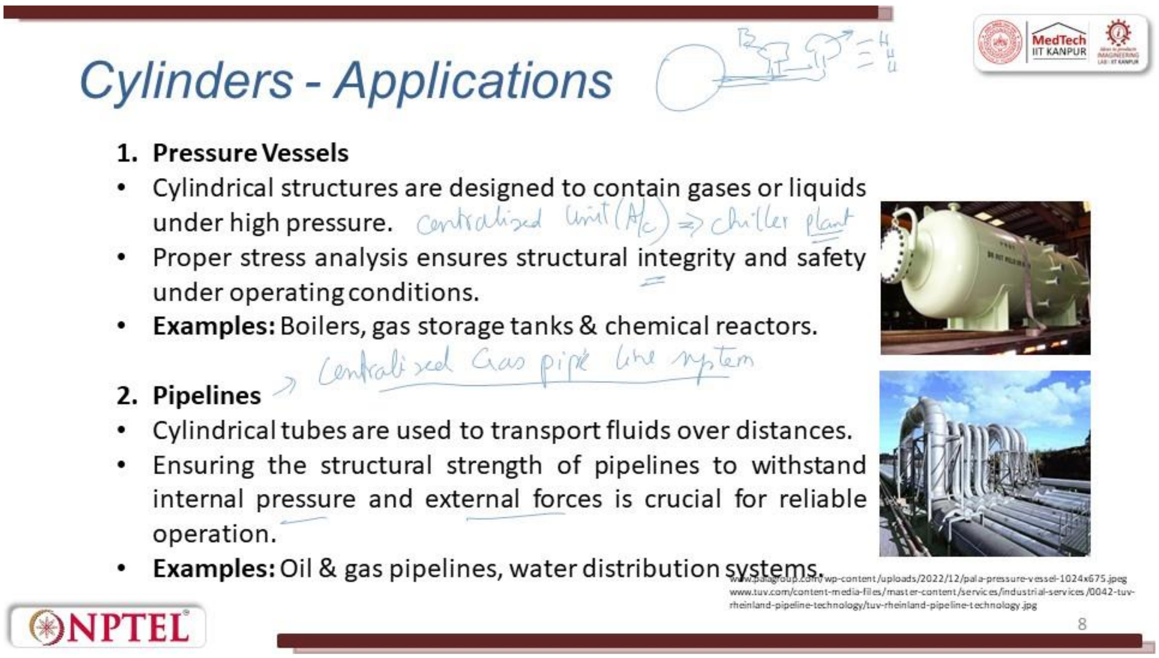
When we talk about Thick Wall Cylinders, Thick Wall Cylinders is defined as having a wall thickness greater than 1/20th of its inner diameter. This design is used in high pressure applications to ensure structural integrity and safety under a significant higher internal pressures. So, the thickness goes like this and if you see stress it follows a distribution, it exponentially falls down.

So, in our equation you will have an exponential factor, this is the radius and when we talk about the radius, we try to take the center of this thickness. The Stress Analysis, we use Lame's equation for more complex stress distribution. The Radial stress (σ_r) and the Hoop stress (σ_h) are determined based on internal and external pressure.



So, as I told you the cylinder applications, this is a boiler. So, you can see where there is an application. The cylindrical structure are designed to contain gases or liquids under very high pressure. For example, when we are transporting liquid nitrogen, we transport it at a very high pressure.

When we are transporting petrol, we also try to pressurize it and then send it. When we are trying to use in thermal power stations, where in which the heat which is getting generated by burning a coal, this in turn heats the water, converts it into steam, there also we use these boilers, right.

So, these boilers are used for a heat exchange, right. The heat exchange can be from a higher temperature to a lower temperature or from a lower temperature to a higher temperature. So, these boilers are used. When we are talking about centralized AC units, which is AC used in malls, used in classrooms or used in marriage halls, used in auditorium, we always have a centralized unit. The centralized unit will many a times have a chiller plant.

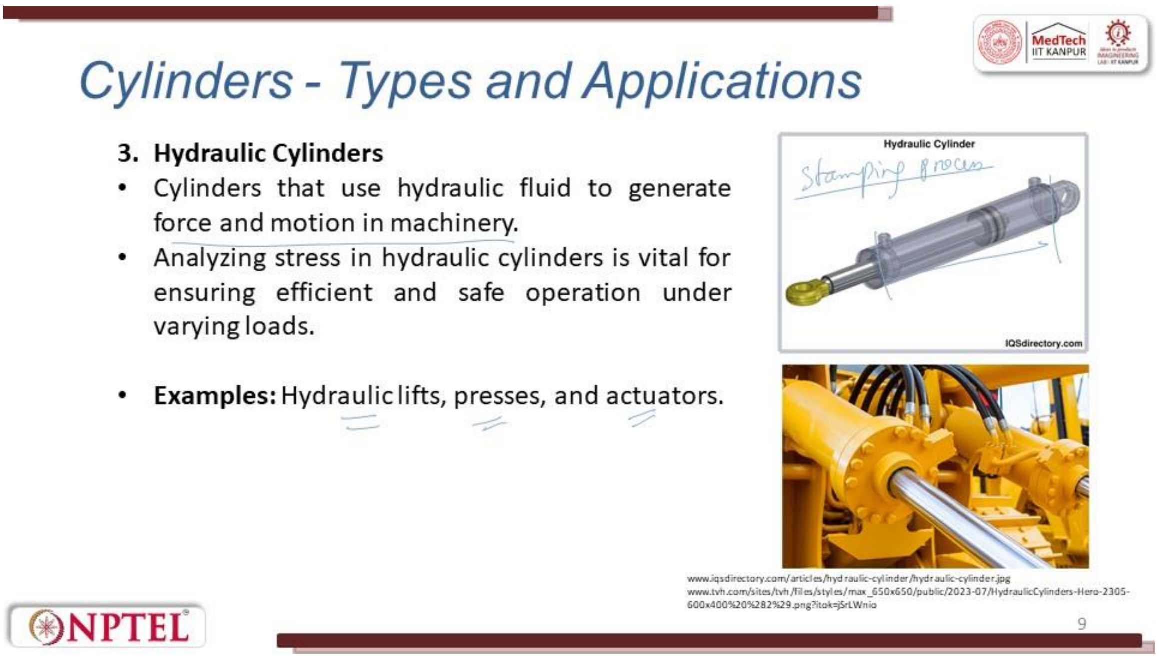
So the chiller plant also will have a cylindrical structure through which there is again a heat transfer. The proper stress analysis ensures Structural Integrity and safety. What is Structural Integrity? It should not crack, it should not burst. So the stresses should be distributed.

So structural integrity and safety under operating conditions. Examples like boiler gas storage tanks, chemical reactors are some of the examples of pressure vessels pipelines are also very important. Through these pipelines again, you can move liquid, you can move gases or you can make move steam, right. So the cylindrical tubes are used to transport fluid over long distance so for example we have centralized gas pipeline system where in which gas pipeline system which is used to connect from the station to all the individual houses, it is used.

So you will have a centralized gas pressure extraction system. From there we try to send it to every house, every block and then this every block you will have also. So this is a block and in the block, you will have houses. So now this gas whatever is generated here is pressurized and pushed to every house. So that is why today we are trying to move towards centralized pipelining system wherein the cylinders are getting removed. These centralized tubes are used to transport fluid over distance.

Ensuring the structural strength of the pipeline to withstand internal pressure and external force is crucial for reliable operation. For example, when the gas is pushed inside the pipeline, it will be always pressurized. For example, the water drain system what is there in your house uses a pipeline wherein which the pressures are not very high. These days the intensity of rain is very high. So, within a short span a huge discharge of water has to go that also goes through the cylindrical pipelines.

Here it will be made out of concrete or cast iron or PVC whatever it is. So, again here the internal pressure and the external force has to be balanced. So, used in oil and gas pipelines, water distribution systems also.



There are Hydraulic systems. For example, this Hydraulic cylinder is used in stamping process. In Stamping what we do is we keep a plate and then we have a die. So it tries to hit at the die and then we have a die. So we have a work piece which is resting on a die. So then we have a punch which comes from a certain distance. It pierces the sheet and then enters into the die thus making a hole on the surface.

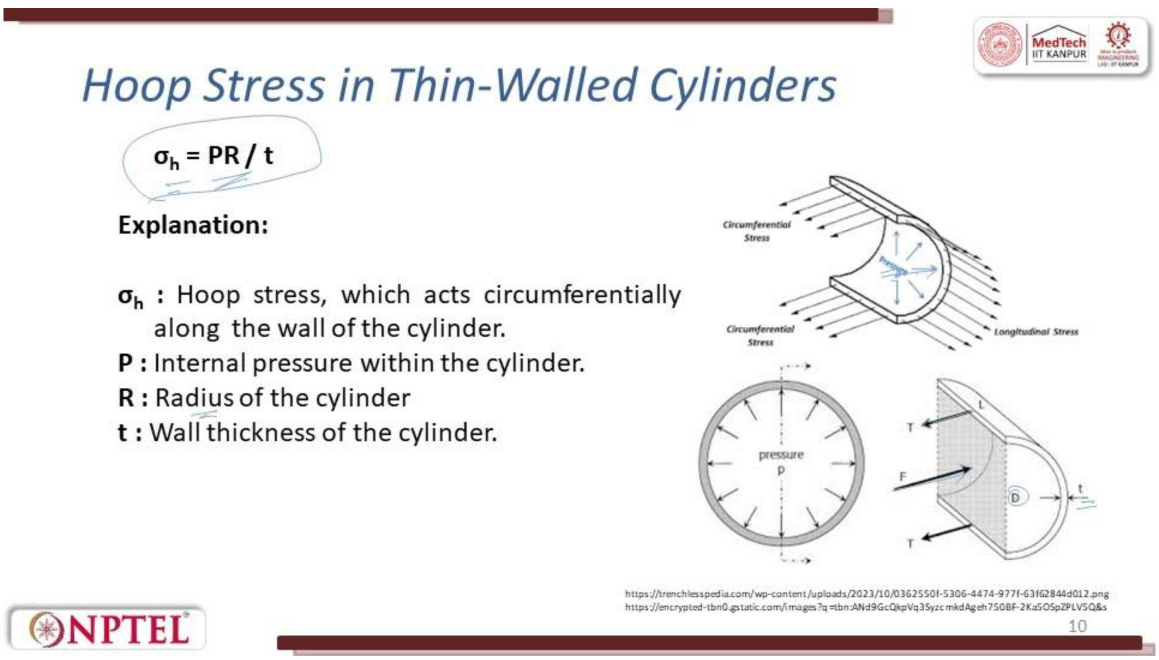
So now it is reciprocation which is done and here we use Hydraulic systems. The force required to shear a plate you know what is the Compressive stress required from the universal testing system or machine you would have found out what is the compression strength required to compress a material such that it deforms. So shearing must have much heavier loads.

So, Hydraulics are used, the hydraulics move vertically up and down. So, these cylinders that uses Hydraulic fluid to generate force and motion in machinery also tries to have a cylinder.

So, inside the cylinder, you will have liquid which is pressurized. As and when you compress, the liquid is pressurized. So, once it is compressed, the pressure is exerted on the wall. So, you try to, you also convert this into a cylinder, thin wall cylinder. Analyzing stresses in Hydraulic cylinder is vital for ensuring efficient and safe operation under varying load.

So, Hydraulic lifts which are used in your forklifter. For example, when you are moving a material from one place to the other inside a room. So, we always use forklifts. So, the forklifts are used to lift the object so that you can walk through and then get to the other place. So, press which I have already discussed and actuators.

These actuators are very much used in cars. These actuators are used in the luggage space in aeroplane or when you are trying to use your hood space in your car. When you try to open your hood, you see a Hydraulic cylinder which tries to automatically move the hood lid.

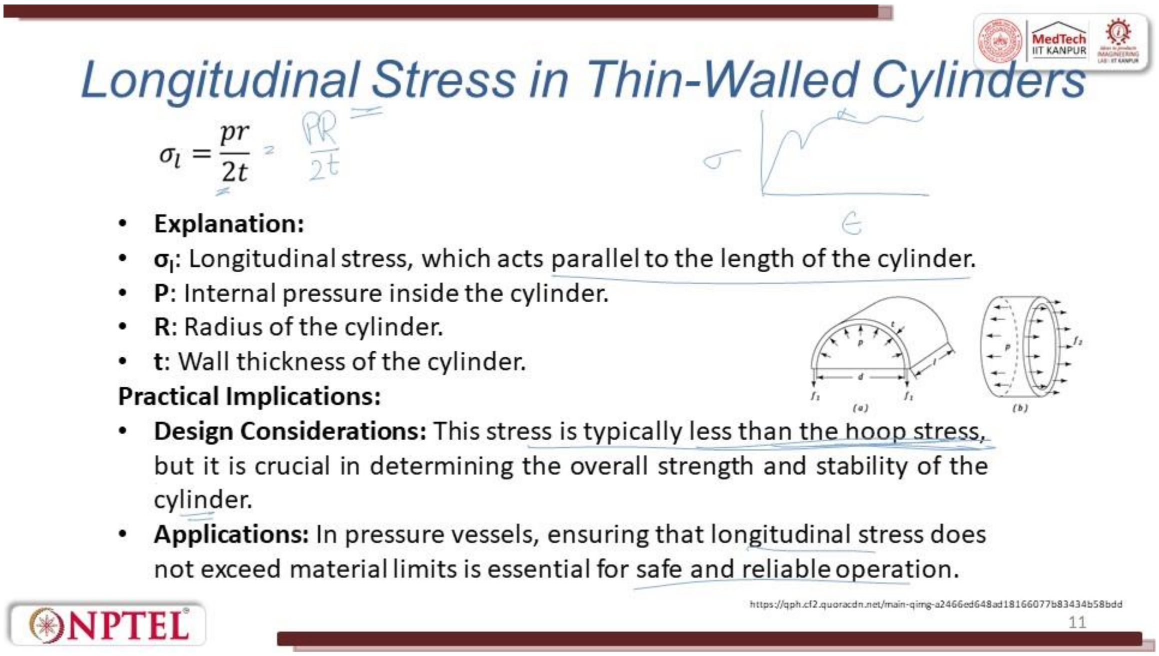


So the Hoop stress in a thin wall cylinder is written like this where in which you have

$$\sigma_h = \frac{PR}{t}$$

So what is P? P is the Internal pressure which is exerting on the thin shell. Then R is the Radius of the cylinder and t is the Wall thickness. So the stress whatever is getting measured or whatever is getting calculated will be equal to pressure multiplied by radius of the circle or the cylinder and divided by the wall thickness t. So this is t. This is force which is acting and this is the torsion t or circumferential. You can try to take this is t, this is force and then you try to take it along a length L So, this is D where in which we are trying to look at the radius.

So, it is D by 2, right. So, this is how we calculate the Hoop stress. Again here, there are lot of approximations, but this is a first thumb rule. So, in the first thumb rule, we try to calculate the generic hoop stress which is getting exerted in a thin wall.



So let us find out what is the Longitudinal stress which comes in a thin wall cylinder which is $\sigma_l = \frac{pr}{2t}$.

So I have represented it by a small p and small r but it has to be $\frac{PR}{2t}$. So σ_l is the Longitudinal stress which acts parallel to the length of a cylinder. Internal pressure is P which is acting on the cylinder and then R is the radius of the cylinder, t is the wall thickness. So it is nothing but because it is acting along the length it is 2 times t. So Practical Implication, Design Consideration is this stress is typically less than the Hoop stress, but it is crucial in determining the overall strength and the stability of the cylinder. So the thumb rule is the longitudinal stress will be typically lesser than the Hoop stress.

When you are trying to do in the examination, if I ask you to find out, the Hoop stress and the Longitudinal stress for a single cylinder. If the answer for the Longitudinal stress is higher, then you have to take a pause, relook into your calculations and go. This is a general thumb rule, okay. Application: in pressure vessels, it ensures that the longitudinal stresses does not exceed the material limit is essential for safe and reliable operation. So, from here we are trying to link back to the Stress-Strain diagram which we studied.

For a typical material, whatever it is, we studied like this. So, we should always make sure the longitudinal stress does not exceed the material limits. So, it should not exceed the ultimate tensile strength for a safe handling. So let us now try to solve a numerical problem.

Numerical Problem

Calculate the hoop and longitudinal stress for a thin-walled cylinder with the following parameters:

Internal Pressure (p): 2000 kPa; Radius (r): 0.5 m;

Wall Thickness (t): 0.01 m

$$\sigma_h = \frac{P \times R}{t} = \frac{2\times10^3\,\text{Pa} \times 0.5\,\text{m}}{0.01\,\text{m}} = 1\times10^5\,\text{kPa}$$

$$= \frac{2\times10^3 \times 0.5\times100}{0.01\times100} = \frac{2\times10^3\times50}{1}$$

$$= 100\times10^3$$

$$= 1\times10^5\,\text{kPa}$$

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Numerical Problem

Longitudinal Stress

$$\sigma_t = \frac{P \times R}{2t} = \frac{2\times10^3 \times 0.5}{2\times0.01}$$

$$= \frac{2\times10^3 \times 5\times100}{2\times0.01\times100}$$

$$= \frac{2\times10^3\times50}{2\times1} = 50\times10^3\,\text{kPa}$$

$$= 50\,\text{MPa}$$

- Hoop Stress = 100 MPa
- Longitudinal Stress = 50 MPa

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Calculate the hoop and the longitudinal stress for a thin wall cylinder with the following parameters where the pressure P is 200 kilopascals, radius is 0.5 meters and the wall thickness is 0.01 meters.

$$\sigma_h = \frac{PR}{t}$$

$$= \frac{2 \times 10^3 \; Pa \times 0.5m}{0.01m} = 1 \times 10^5 KPa$$

$$\frac{2 \times 10^3 \times 0.5 \times 100}{0.01 \times 100} = 2 \times 10^3 \times 50 = 100 \times 10^3 = 1 \times 10^5 KPa$$

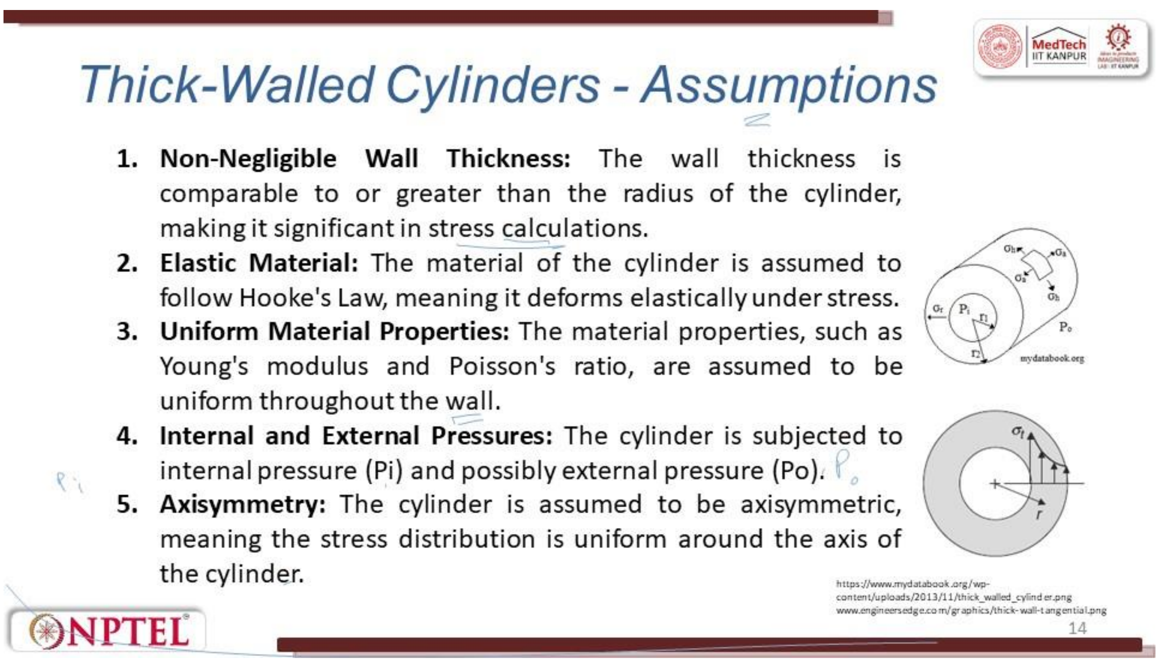
Longitudinal Stress,

$$\sigma_t = \frac{PR}{2t} = \frac{2 \times 10^3 \times 0.5}{2 \times 0.01} = \frac{2 \times 10^3 \times 5 \times 100}{2 \times 0.01 \times 100} = \frac{2 \times 10^3 \times 50}{2 \times 1} = 50 \times 10^3 KPa$$

$$= 50 MPa$$

Hoop Stress = 100 MPa

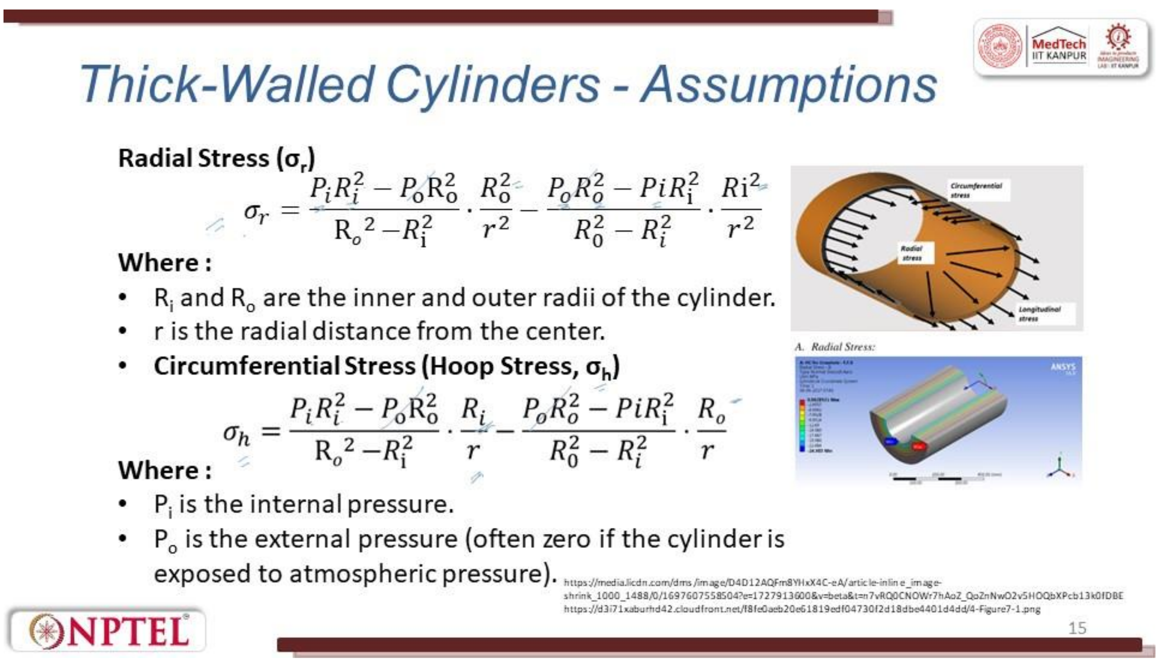
Longitudinal Stress = 50 MPa



Next we will move to Thick Walled cylinders. In Thick Walled cylinders we have assumptions made. So what are the assumptions? Non-negligible wall thickness. The wall thickness is comparable to or greater than the radius of the cylinder making it significant in stress calculations. The elastic material, the material of the cylinder is assumed to follow Hooke's law, meaning it deforms elastically under stress.

Then we have an assumption Uniform Material Property. The material properties such as Young's modulus and Poisson's ratio are assumed to be uniform throughout the wall thickness. Then Internal and External Pressure. The cylinder is subjected to internal pressure P i and external pressure P naught. So the P i is P suffix i and it is P suffix naught.

Then it is Axisymmetry. The cylinder is assumed to be axisymmetric, meaning the stress distribution is uniform around the axis of the cylinder. So, as I told you in the previous slides itself, you try to take a single element. So, this single element tries to have σ θ and then this is hoop, this is longitude.



So, the Radial stresses which are getting formed in the thick wall cylinders are

$$\sigma_r = \frac{P_i R_i^2 - P_o R_o^2}{Ro^2 - R_i^2} \cdot \frac{R_o^2}{r^2} - \frac{P_o R_o^2 - P i R_i^2}{R_0^2 - R_i^2} \cdot \frac{Ri^2}{r^2}$$

where Ri and R0 are the inner and outer radius of a cylinder, ok. R is the radial distance from the center. So, this is how we calculate the radial stress. When we try to calculate the Circumferential stress or the Hoop stress, the Hoop stress is nothing but in the same way.

$$\sigma_h = \frac{P_i R_i^2 - P_o \mathrm{R}_\mathrm{o}^2}{\mathrm{R}o^2 - R_\mathrm{i}^2} \cdot \frac{R_i}{r} - \frac{P_o R_o^2 - PiR_\mathrm{i}^2}{R_0^2 - R_i^2} \cdot \frac{R_o}{r}$$

Thank you very much.