Basics of Mechanical Engineering-1

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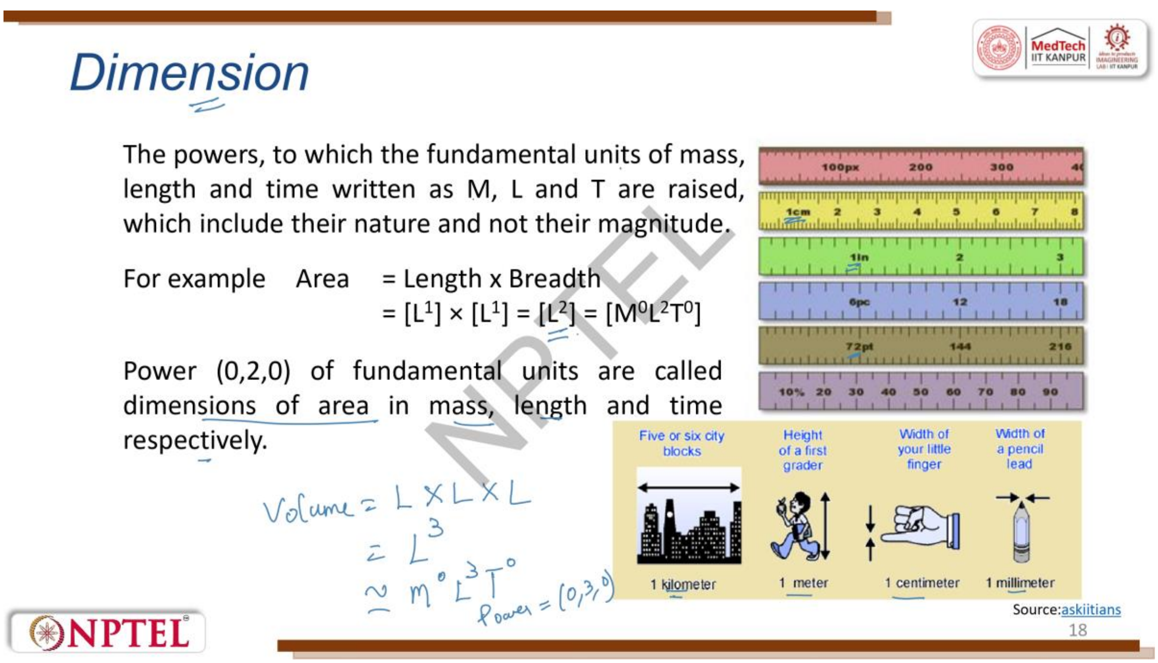
Week 01

Lecture 02

Units, Dimensions and Dimensional Analysis - II

Welcome to the lecture series in the course Basics of Mechanical Engineering I.



The next one is Dimensions. So till now we were looking at various units. First we looked at the basics of units. Then what are the types of units.

Then we tried to look into various unit systems. Then we landed with SI. In SI we saw 7 basic and 2 supplementary. We saw that and then now we are trying to move for dimensions. So, the powers to which the fundamental units of mass, length, time, which is written as M, L, T are raised, which include their nature and not their magnitude.

For example, area is length and breadth. So, Area = Length x Breadth

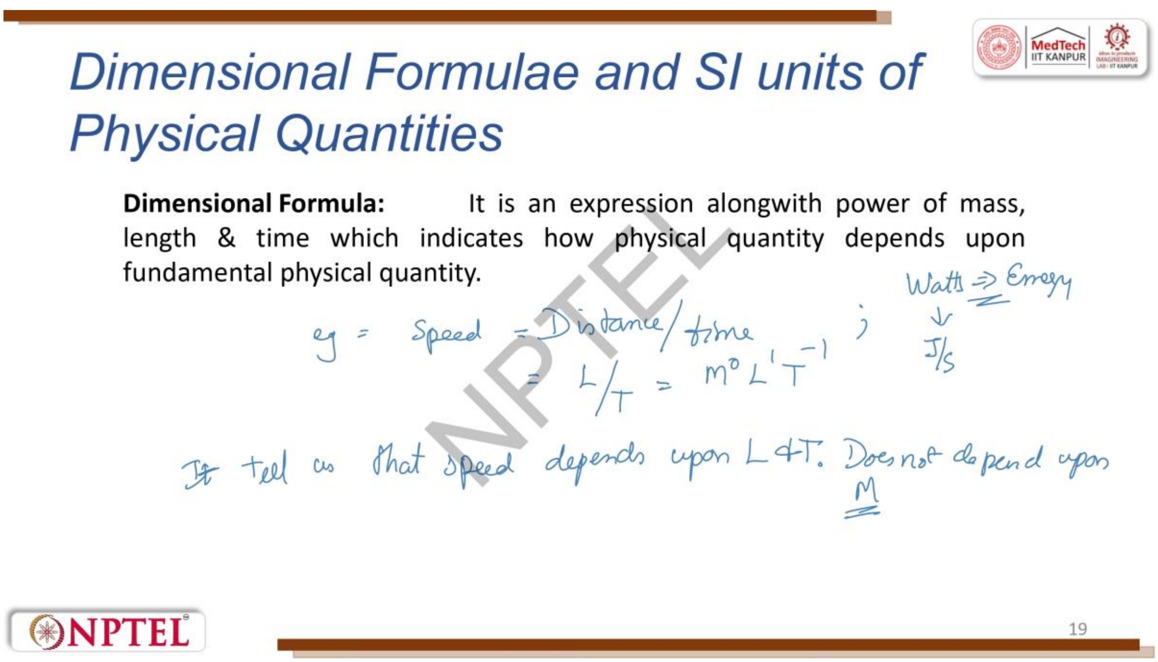
$$= [L^1] \times [L^1] = [L^2] = [M^0L^2T^0]$$

So you can also try to see for volume, which will be length into length into length. So it can be expressed as $[M^0L^3T^0]$.

So, here what we are trying to do is we are trying to talk about the dimensions. So, you can see here these are where this is in centimeter, this is in liter, this is in points. This is in percentage. Okay.

So you have all these things of different scales which are compared. So you can see four or five or six city blocks is one kilometer. The height of a first grader is one meter. The width of your little finger is one centimeter. Width of your pencil tip is 1 millimeter.

So 1 kilometer, 1 meter, 1 centimeter, 1 millimeter. So you can multiply it into 10^3 to get whatever 10^1, 10^2, 10^3. So you have this multiplication which is done. And here we saw about the dimensions.



So in dimensional formulae and SI units of physical quantities, we will try to see dimensional formulae.

It is an expression along with power of mass, length, time, which indicates how physical quantities depend upon fundamental physical quantities. For example, let us take speed. Speed is nothing but distance by time.

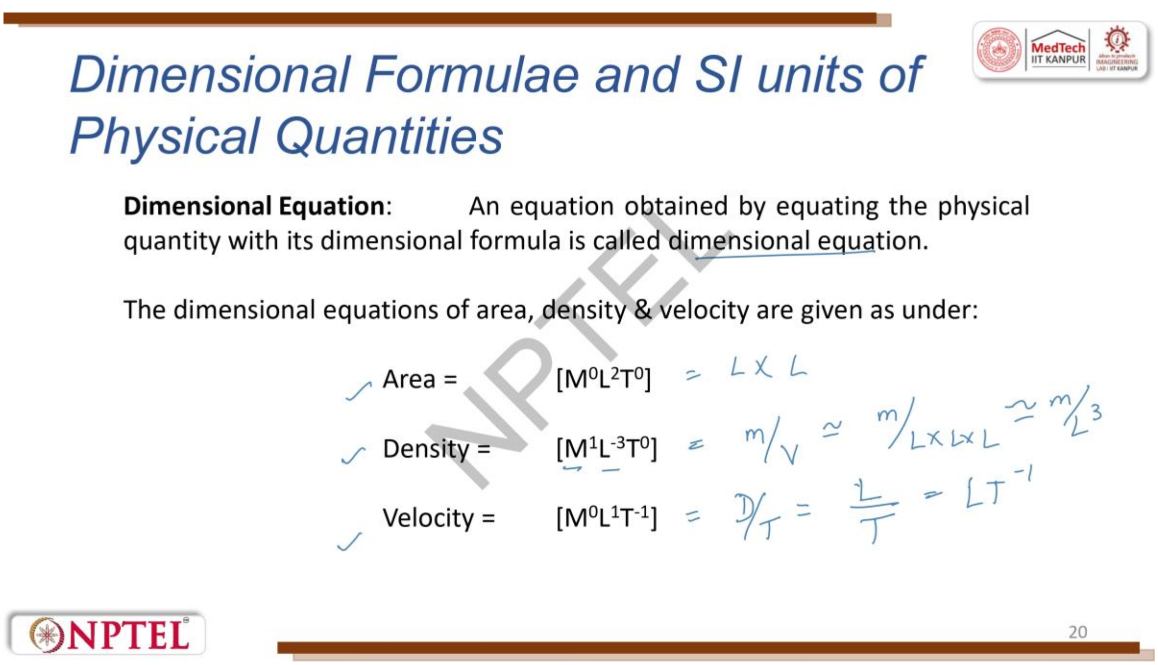
Speed = Distance/Time

$$= [L1] / [T1] = [M^0L^1T^{-1}]$$

It tells us that speed depends upon L & T. It does not depend upon M.

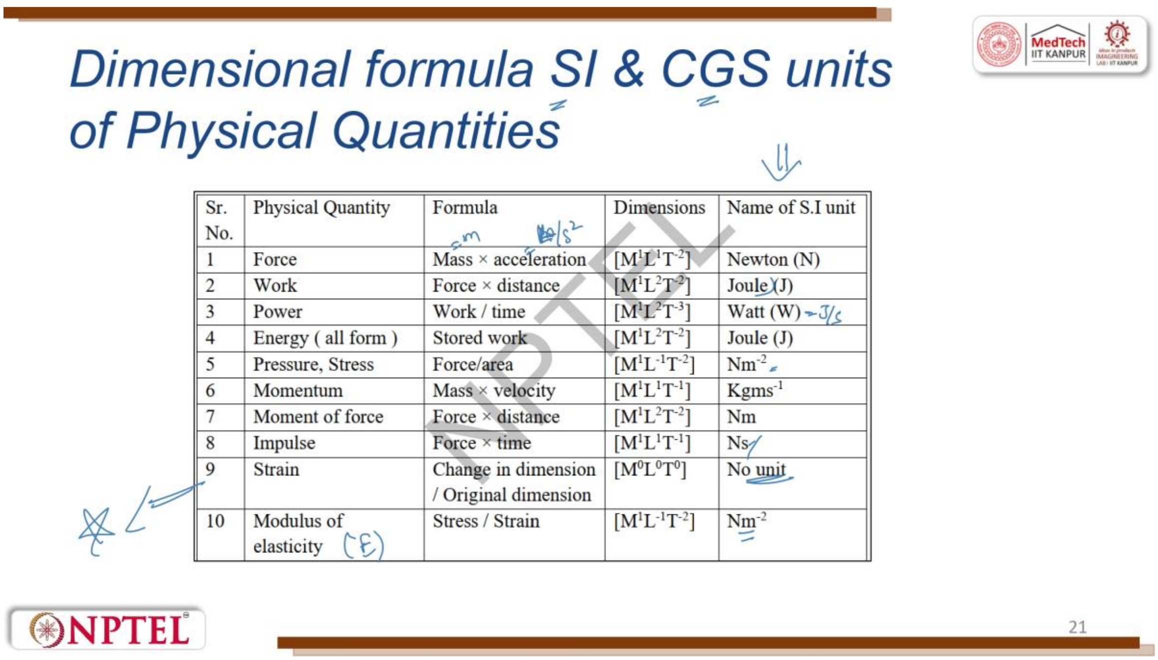
So, this is a dimensional formula which we got. You can try to do for watts. So, which is nothing but energy. You please try for yourself.

And then you will try to see what is the use of it. So, the energy watt can be expressed as joule per second. Now joule you can try to figure out what is it. So from there you will try to see what is the dimensional formula you get.



So, the dimensional formulae and SI units are physical quantity. So, we are trying to express it into dimensional equations. So, here an equation obtained by equating the physical quantities with its dimensional formulae is called as Dimensional Equations. The dimensional equations for area, density, velocity are given here with area is length into length. Density is mass by volume, which is in turn expressed as mass by volume is L into L into L, which is nothing but M/L^3. So, $[M^1\ L^3\ T^0]$ is there.

Velocity is nothing but L (Distance by Time). So it is a meter, it is measured one, so here if we see that you are trying to express it as length by time so which is $[LT^{-1}]$, so it does not depend upon the mass, right. So that is what we are trying to say, so these are dimensional equations of area density and velocity which are given.



Sr. No.	Physical Quantity	Formula	Dimensions	Name of S.I unit
1	Force	Mass × acceleration	$[M^1L^1T^{-2}]$	Newton (N)
2	Work	Force × distance	$[M^1L^2T^{-2}]$	Joule (J)
3	Power	Work / time	$[M^1L^2T^{-3}]$	Watt (W)
4	Energy (all form)	Stored work	$[M^1L^2T^{-2}]$	Joule (J)
5	Pressure, Stress	Force/area	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
6	Momentum	Mass × velocity	$[M^1L^1T^{-1}]$	$Kgms^{-1}$
7	Moment of force	Force × distance	$[M^1L^2T^{-2}]$	Nm
8	Impulse	Force × time	$[M^1L^1T^{-1}]$	Ns
9	Strain	Change in dimension / Original dimension	$[M^0L^0T^0]$	No unit
10	Modulus of elasticity	Stress / Strain	$[M^1L^{-1}T^{-2}]$	Nm^{-2}

So look at the SI units of physical quantities in the below table:

Sr. No.	Physical Quantity	Formula	Dimensions	Name of S.I unit
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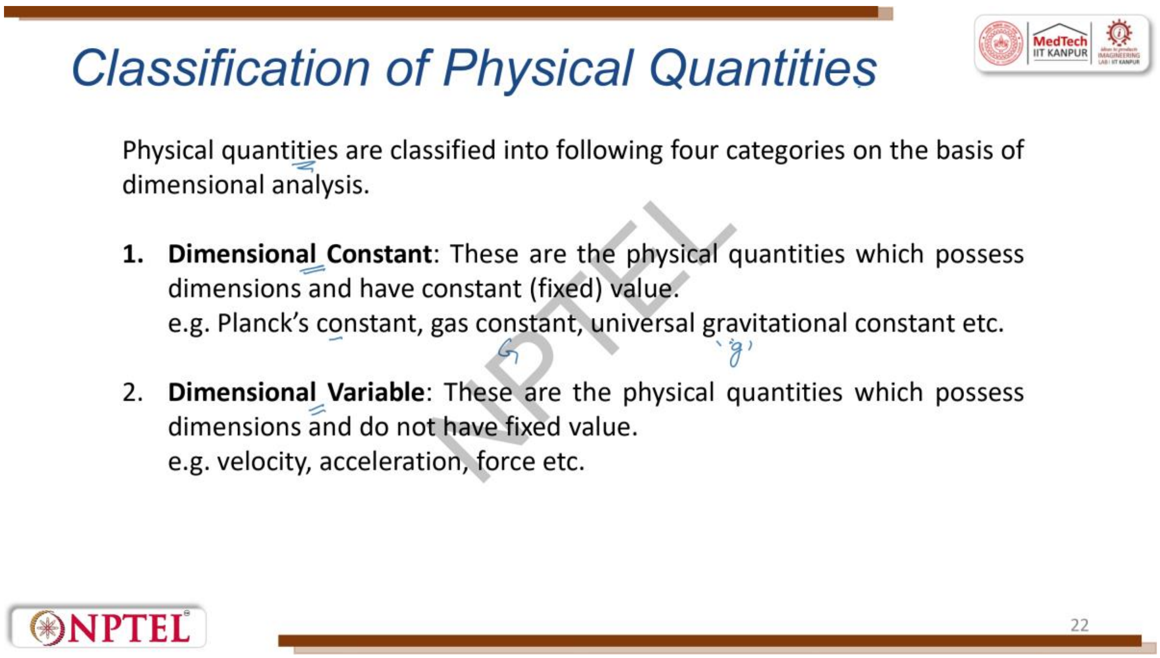
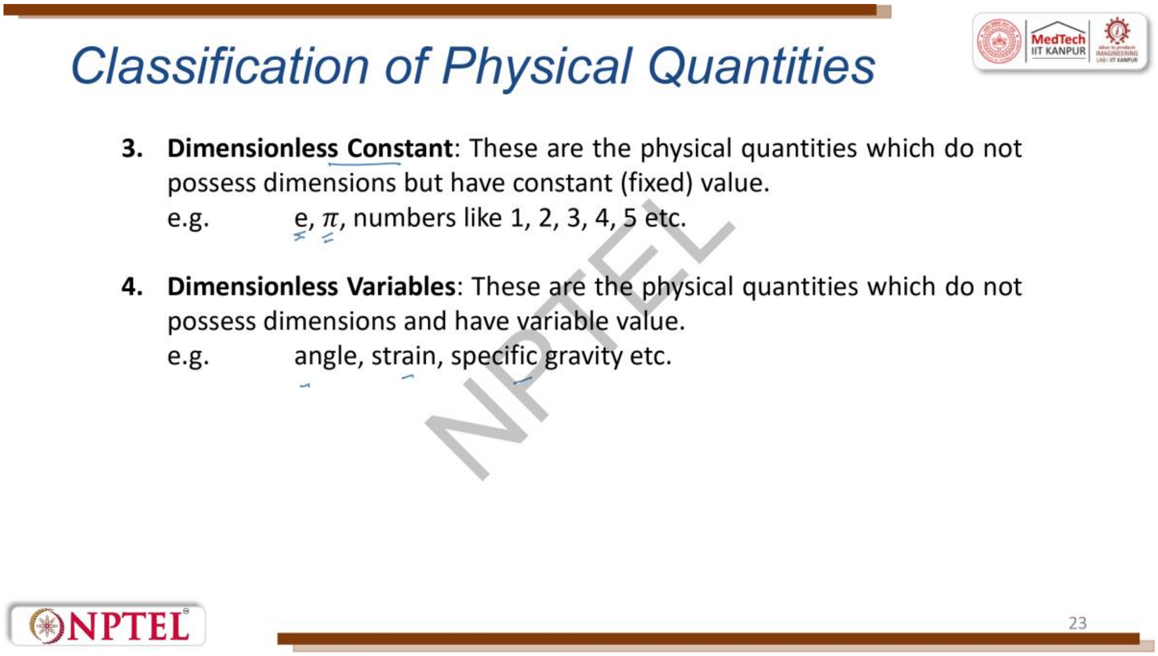


Classification of physical quantities. The physical quantities can be classified into dimensional constant and dimensional variable. There are certain things which is constant. Wherever you go, it is constant. These are physical quantities which possesses dimension and have properties. Constant value for example Planck's constant, gas constant, universal gravity constant (g) are all constants wherever you go it is constant, it does not change but there are something called as Dimensional Variable.

These are physical quantities which possesses dimension and do not have fixed value. For example, a car can have a velocity of this, acceleration of this and force. In today's road condition, it can have this. Tomorrow's road condition, it can have this. Today, at the start of the day, I had a lot of energy.

I punched a second of half of the energy. Evening, I had exponentially low energy. So you see here the same person in a same day having different energy or force to punch whatever it is, right. So these are called as Dimensional Variables, Dimensional Constants. Dimensional constants will not change whatever may be the reason.

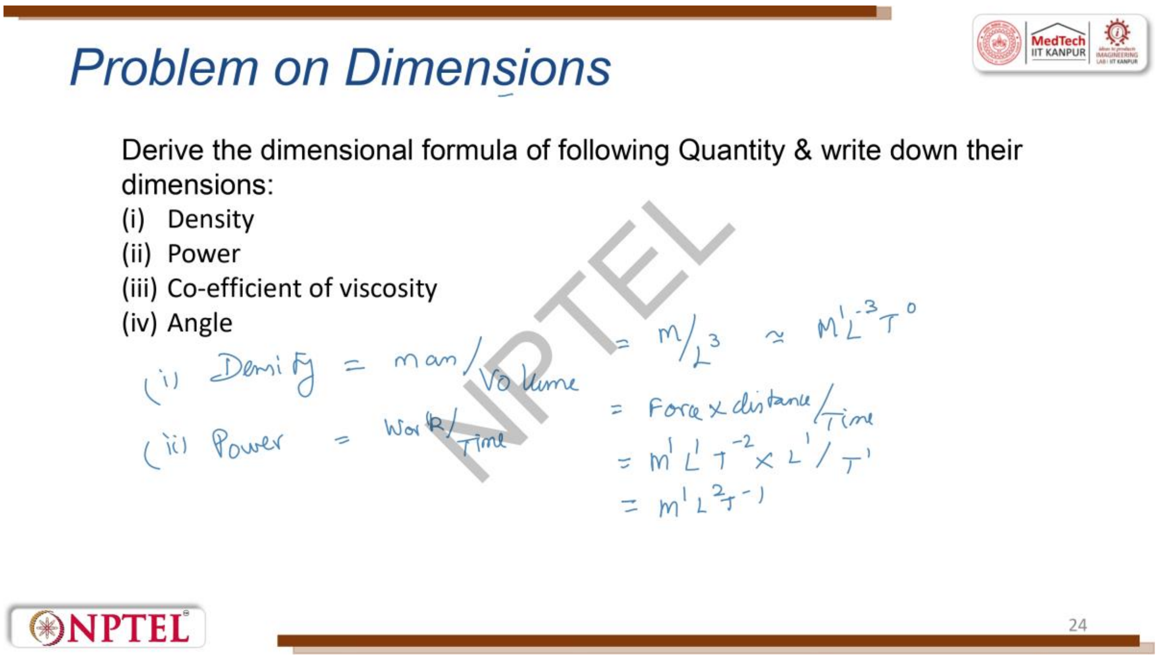
In many of the thermodynamic problems, we always try to talk about gas constant. Then in applied mechanics, we try to use gravity g as 9.8 or approximation we use 10. Planck's constant is also used. But when we try to look into dimensional variables, it keeps changing.



So there are other classification which is called as dimensionless constant. The other one is dimensionless variables. Dimensionless constants are constants. They are physical quantities which do not possess dimensions but have constant values. For example, e, π, numbers like 1, 2, 3, 4, 5, these are called as Dimensionless Constants.

The other one is called as Dimensionless Variables. So, these are the physical quantities which do not possess dimensions and have variable values. For example, angle, strain, specific gravity, etc., right. Variables. Okay, so there are four classifications.

This is very important. Dimensional Constant, Dimensional Variable. In this, we will try to have e, π, etc. and 1, 2, 3, 4, 5 will be the numbers which we are using. So here, there are no dimensions will not be there, specific gravity, strain. Strain is a change in dimension by original dimension, etc.,



So, now let us look at some of the problems which are being discussed on dimensions. Derive the dimensional formulae of following quantities right down their dimensions.

Solution:

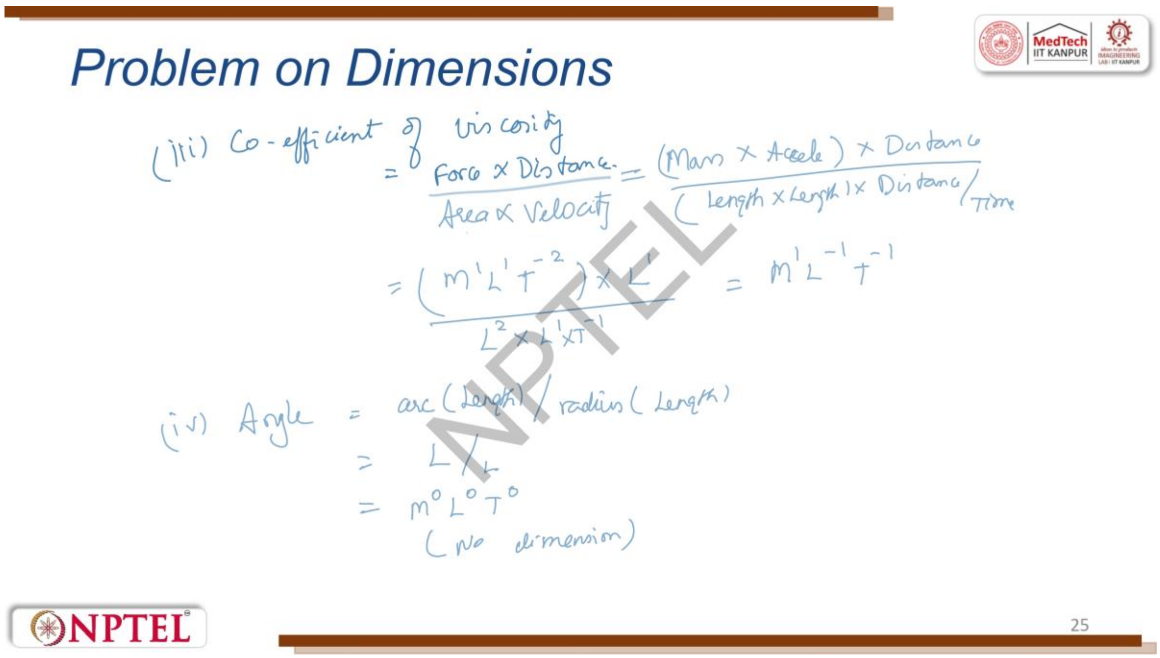
(i) Density = Mass / Volume = $[M] / [L^3]$

$= [M^1L^{-3}T^0]$

(ii) Power = Work / Time = Force x Distance / Time

$= [M^1L^1T^{-2}] \times [L^1] / [T^1]$

$= [M^1L^2T^{-3}]$



(iii) Co-efficient of viscosity

$$= \frac{\text{Force} \times \text{Distance}}{\text{Area} \times \text{Velocity}} = \frac{(\text{Mass} \times \text{Acceleration}) \times \text{Distance}}{(\text{Length} \times \text{Length})\ (\text{Distance} / \text{Time})}$$

$$= \frac{[M^1L^1T^{-2}] \text{ x } [L^1]}{[L^2] \times [L^1T^{-1}]}$$

$$= [M^1L^{-1}T^{-1}]$$

(iv) Angle = arc (length) / radius (length)

$$= [L] / [L]$$

$$= [M^0L^0T^0]$$

(No dimension)

Dimensional Analysis

- A careful examination of the dimensions of various quantities involved in a physical relation is called dimensional analysis.
- The analysis of the dimensions of a physical quantity is of great help to us in a number of ways as discussed under the uses of dimensional equations.

Now let us move on to Dimensional Analysis. A careful examination of dimensions of various quantities involved in the physical relation is called as dimensional analysis.

So this is very important. The analysis of the dimensions of a physical quantity is of a great help to us in a number of ways as discussed under the use of dimensional equations.

Dimensional Analysis

Uses of dimensional equation:

The principle of homogeneity & dimensional analysis has put to the following uses:

(i) Checking the correctness of physical equation.

(ii) To convert a physical quantity from one system of units into another.

(iii) To derive relation among various physical quantities.

The use of dimensional equation, we see the principles of homogeneity and dimensional analysis has put to the following use. Checking the correctness of physical equation to convert the physical quantities from one system of units into another to derive relationship among various physical quantities. So, these are very important principles of homogeneity.

Dimensional Analysis

Limitations of dimensional equation:

The method of dimensions has the following limitations:

- It does not help us to find the value of dimensionless constants involved in various physical relations.
- The values, of such constants have to be determined by some experiments or mathematical investigations.

What are the limitations of dimensional analysis? The method of dimensions have the following limitations. It does not help us to find the value of dimensionless constants involved in various physical relationships. The second one is the value of such constants have to be determined by some experiments or mathematical investigations. So these are the limitations. So you find out it has to be done some experiments and then you try to figure it out.

Dimensional Analysis

Limitations of dimensional equation:

- This method fails to derive formula of a physical quantity which depends upon more than three factors because only three equations are obtained by comparing the powers of M, L and T.
- It fails to derive relations of quantities involving exponential and trigonometric functions.

The other limitations are this method fails to derive formulae of a physical quantity which depends upon more than three factors because only three equations are obtained by comparing the powers of M, L and . And the last pointed it fails to derive relations of quantities involving exponential and trigonometrical functions. So these are the limitations of dimensional equations. So once we saw the limitations of dimensional equations.

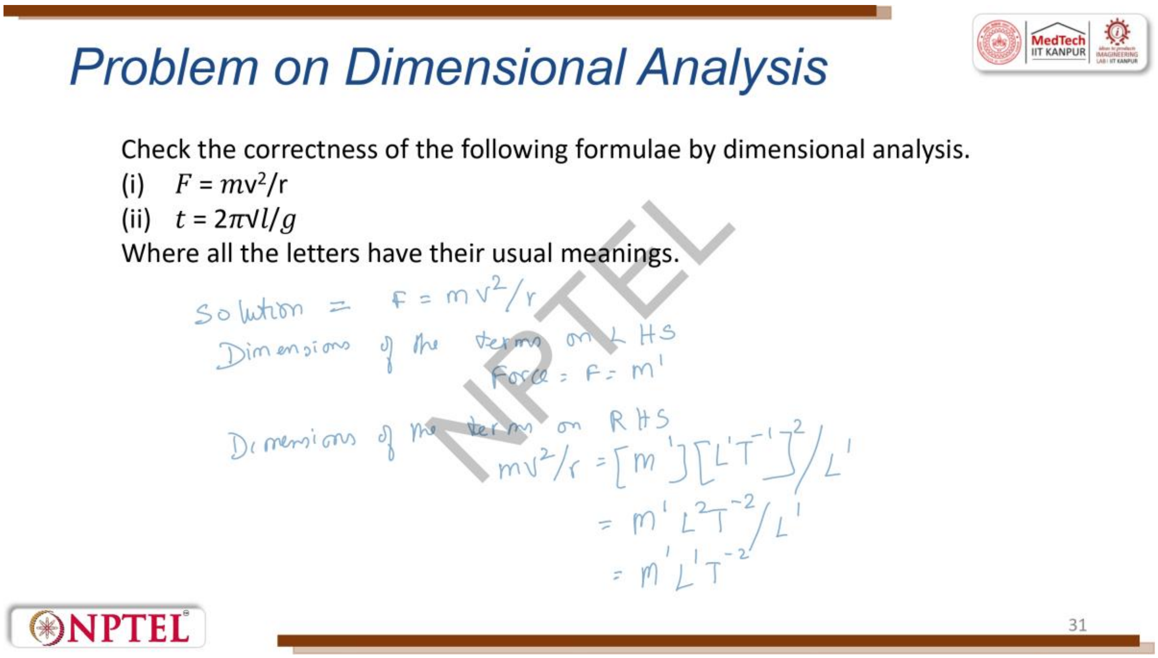
Applications of Dimensional Analysis

Dimensional analysis is a fundamental aspect of measurement and is applied in real-life physics. We make use of dimensional analysis for three prominent reasons:

- To check the consistency of a dimensional equation.
- To derive the relation between physical quantities in physical phenomena.
- To change units from one system to another.

Now let us see the applications of dimensional analysis. Dimensional analysis is a fundamental aspect of measurement and is applied in real life physics. We make use of dimensional analysis for three prominent reasons.

To check the consistency of the dimensional equation, to derive the relationship between physical quantities in physical phenomena and to change units from one system to another.



So, we will try to see some of the problems on dimensional analysis. Check the correctness of the following formulae by dimensional analysis.

Solution: $\boldsymbol{F} = \boldsymbol{m}\mathbf{v}^{2}/\mathbf{r}$

Dimensions of the term on L.H.S

Force, $F = [M^1L^1T^{-2}]$

Dimensions of the term on R.H.S

$\boldsymbol{m}\mathbf{v}^{2}/\mathbf{r} = [M^1][L^1T^{-1}]^2 / [L^1]$

$= [M^1L^2T^{-2}] / [L^1]$

$= [M^1L^1T^{-2}]$

The dimensions of the term on the L.H.S are equal to the dimensions of the term on R.H.S. Therefore, the relation is correct.

(ii) $\boldsymbol{t} = \mathbf{2}\boldsymbol{\pi}\sqrt{\boldsymbol{l}}/\boldsymbol{g}$

Here,

Dimensions of L.H.S,

t (time) = $[T^1] = [M^0L^0T^1]$

Dimensions of the terms on R.H.S

2π being constant, have no dimensions

l (length) = $[L^1]$

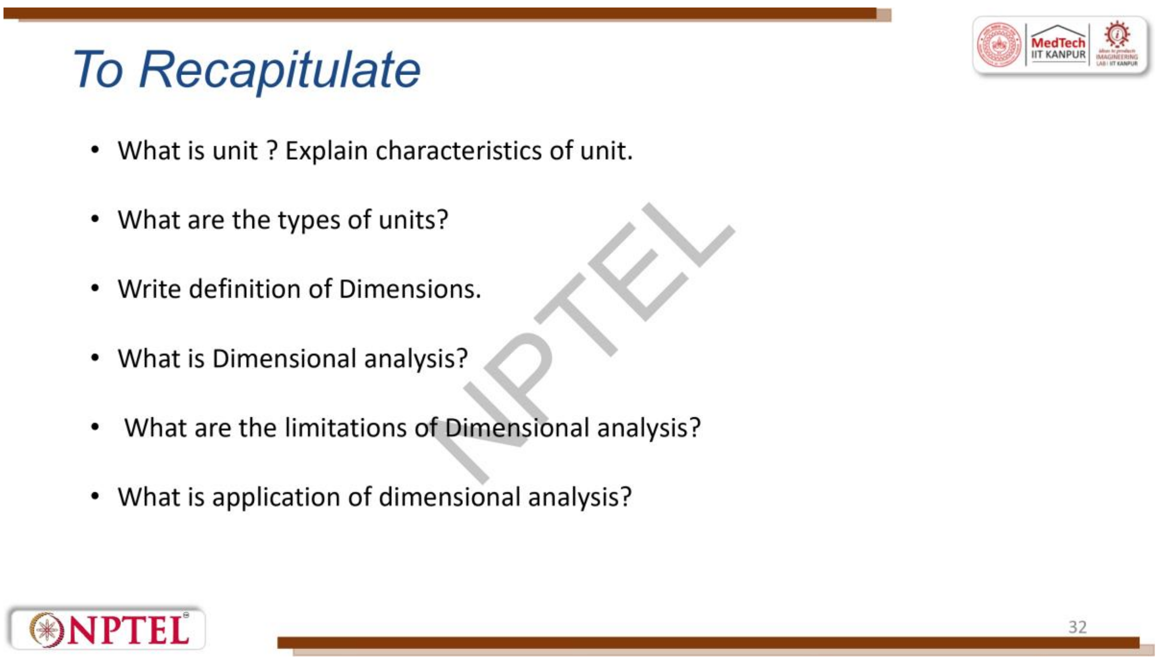
g (acceleration due to gravity) = $[L^1T^{-2}]$

Hence, the dimensions of terms $2\pi\sqrt{l/g}$ on R.H.S

$= (L^1/ L^1T^{-2}])^{1/2} = [T^1] = [M^0L^0T^1]$

Thus, the dimensions of the terms on both sides of the relation are the same i.e., $[M^0L^0T^1]$.

Therefore, the relation is correct.



So in this lecture, we tried to look into what is unit. We tried to see the characteristics of unit. Then we saw different types of units.

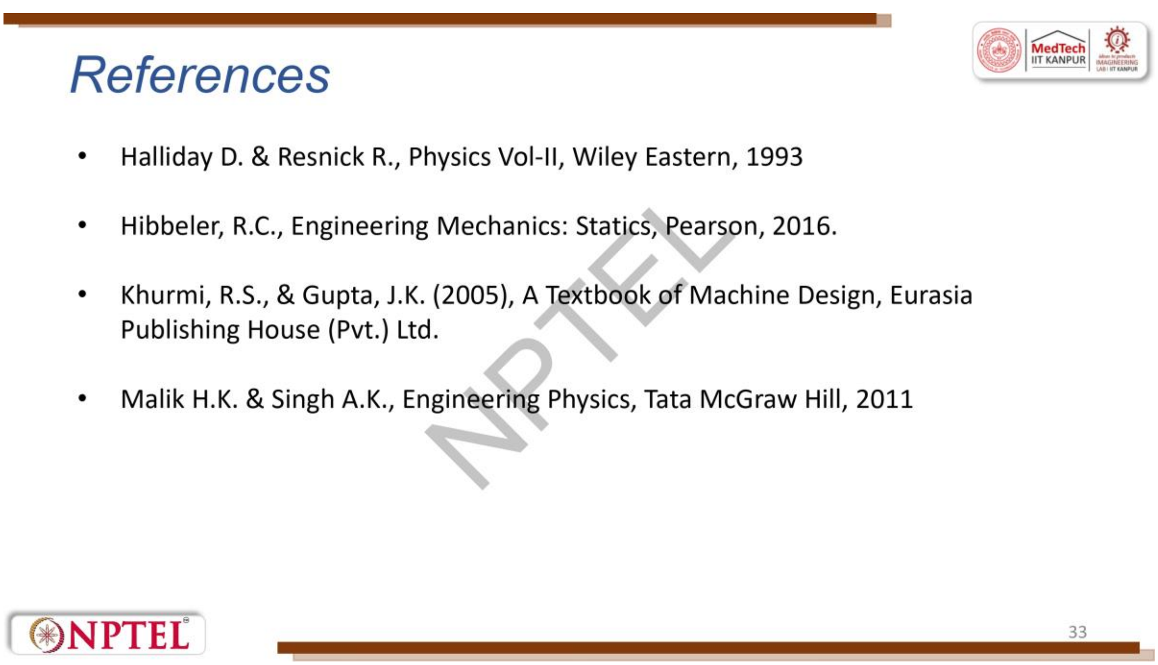
Then we defined dimension. Then we saw what is dimensional analysis. Then we found out the limitation of dimensional analysis. And finally we saw what is the application of dimensional analysis. So before I conclude I would like to give you 3 or 4 problems which are real time application oriented.

Try to measure the dimension of a chair which we use, right. So try to measure it and you should measure it in any scale, right and try to measure in three different types of units may be in centimeter, millimeter and in inches, you see, then you try to measure the pressure of air what you fill in tyre and see the units.

So it will be x units, so what are the units and try to convert this unit into three different types in different CGS system, MKS system, FPS system whatever it is, try to convert it and see the next one is going to be try to do dimensional analysis for three different derived units. Okay, these are the exercise which you will try to do for yourself.

And if you have difficulty, you can discuss during the tutorial hours. Otherwise, this is for your self understanding. You will see how does the units change? How does the magnitude change? See inches and millimeter.

There is a difference in terms of magnitude. So you should watch out for that and then see all the number game.



So these are some of the references we have used for this lecture. Thank you very much.