Basics of Mechanical Engineering-1

Prof. J. Ramkumar

Dr. Amandeep Singh

Department of Mechanical Engineering

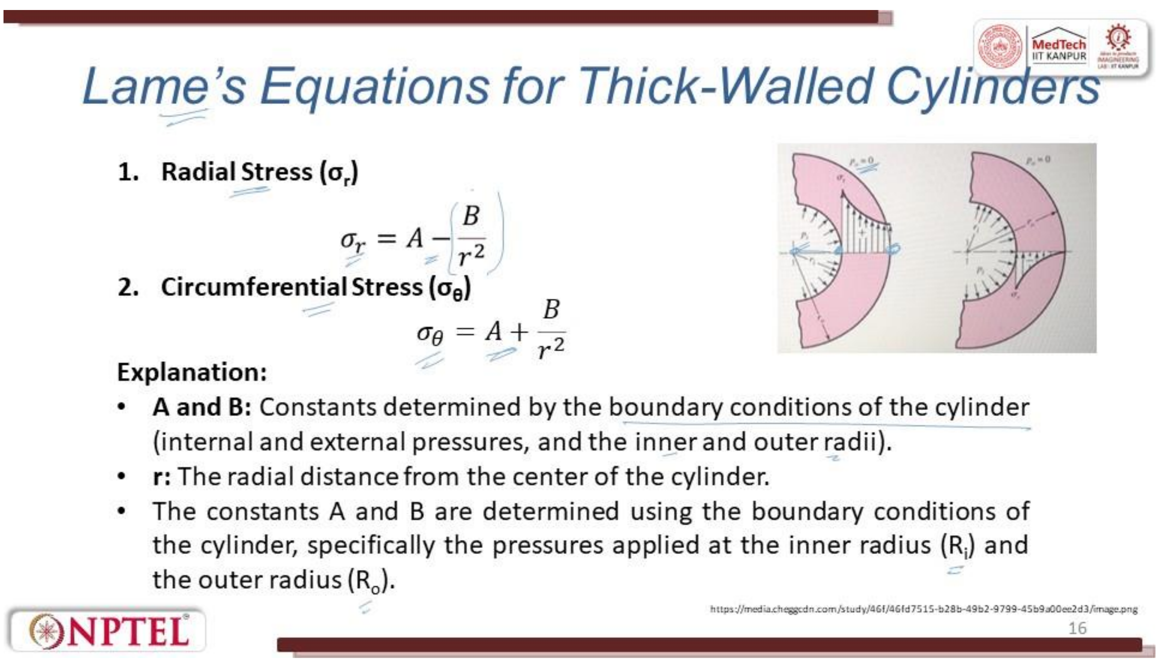
Indian Institute of Technology, Kanpur

Week 05

Lecture 20

Stress In Cylinders and Spheres (Part 2 of 2)

Welcome to the next lecture on Stress in Cylinders and Spheres.



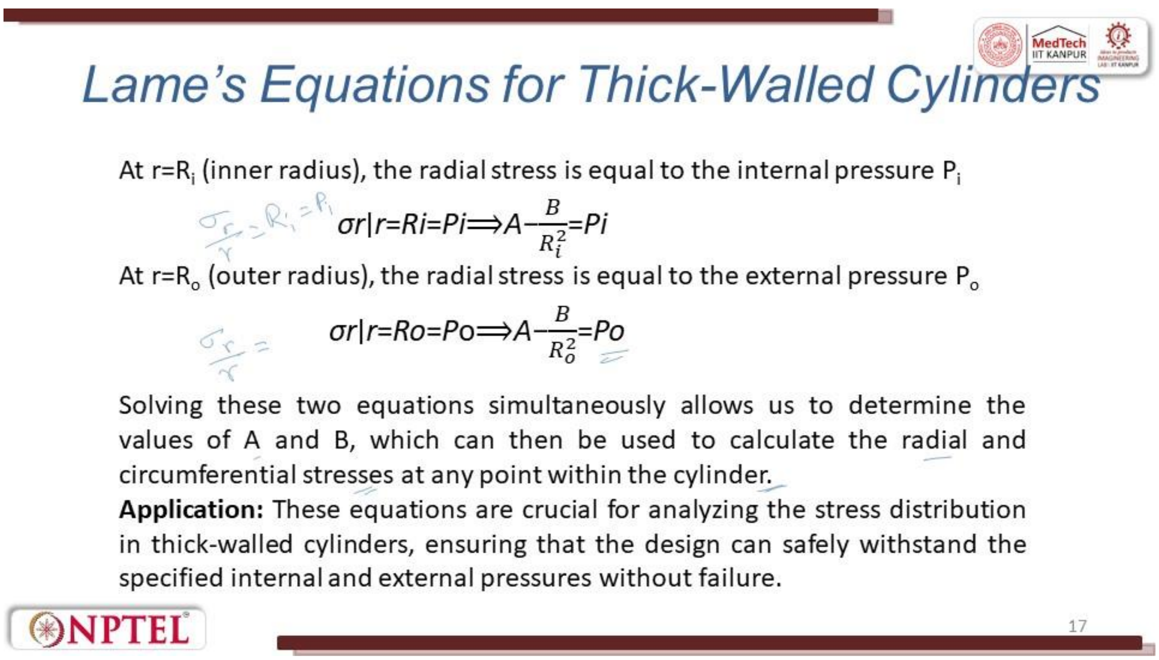
What is Lame's equation for Thick-Walled Cylinders? The Radial Stress $\sigma_r = A - \frac{B}{r^2}$.

So, what is A and B? A and B are constants determined by the boundary conditions of the cylinder, internal and external pressures and the internal and the outer radius.

So here if you see, we have clearly written it, right. So, these are the pressures which are acting, these are the radius, these are the pressure. So P1, P0 and this is how the stress gets distributed within the material. R is the radius from the centre of the cylinder. The constants A and B are determined by using boundary conditions of the cylinder, specifically the pressures applied at the inner radius Ri and the outer radius R. So inner radius is this, outer radius is this.

So, the Circumferential, this is Radial. The Circumferential Stresses are $\sigma_\theta = A + \frac{B}{r^2}$.

So please note down the difference and the radial stresses. So, through this, you can try to find out what are the Radial Stresses and Circumferential Stresses for a thick cylinder.

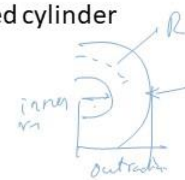


When at r=R_i (inner radius), the radial stress is equal to the internal pressure P_i. So, you should read it like $\frac{\sigma_r}{r} = R_i = P_i$, ok.

So, when the outer radius $\sigma r/r=Ro=Po$ and this is how the equation is. Solving these two equations simultaneously allows us to determine the values of A and B, which can then be used to calculate the radial and circumferential stresses at any point within the cylinder. So, this is for a thick cylinder case. So, we see for Thin-Cylinder case. So, this is for thin cylinder type and here what we are trying to solve is for thick cylinder type. This is Lame's equation which is also very important used in thick wall cylinders.

Numerical Problem

Calculate the radial and circumferential stresses for a thick-walled cylinder with the following parameters:



- Internal Pressure (Pi): 5 MPa
- External Pressure (Po): 0 MPa (atmospheric pressure)
- Inner Radius (Ri): 0.1 m
- Outer Radius (Ro): 0.2 m
- Radial Distance (r): 0.15 m (point where stress is to be calculated)

Determine constants A & B using boundary conditions.

① at $r = R_i$ (inner radius) $= \sigma_r = P_i$

$A - B/R_i^2 = P_i \Rightarrow A - B/(0.1)^2 = 5\,MPa.$

$A - 100B = 5$ .... ①

Numerical Problem

② at $r = R_o$ (outer radius) $\sigma_r = P_o$. $\rightarrow A - \frac{B}{(0.2)^2} = 0\,MPa.$

$A - B/R_o^2 = P_o$

$A - 25B = 0$ or $A = 25B$ ——— ②

From ① + ②

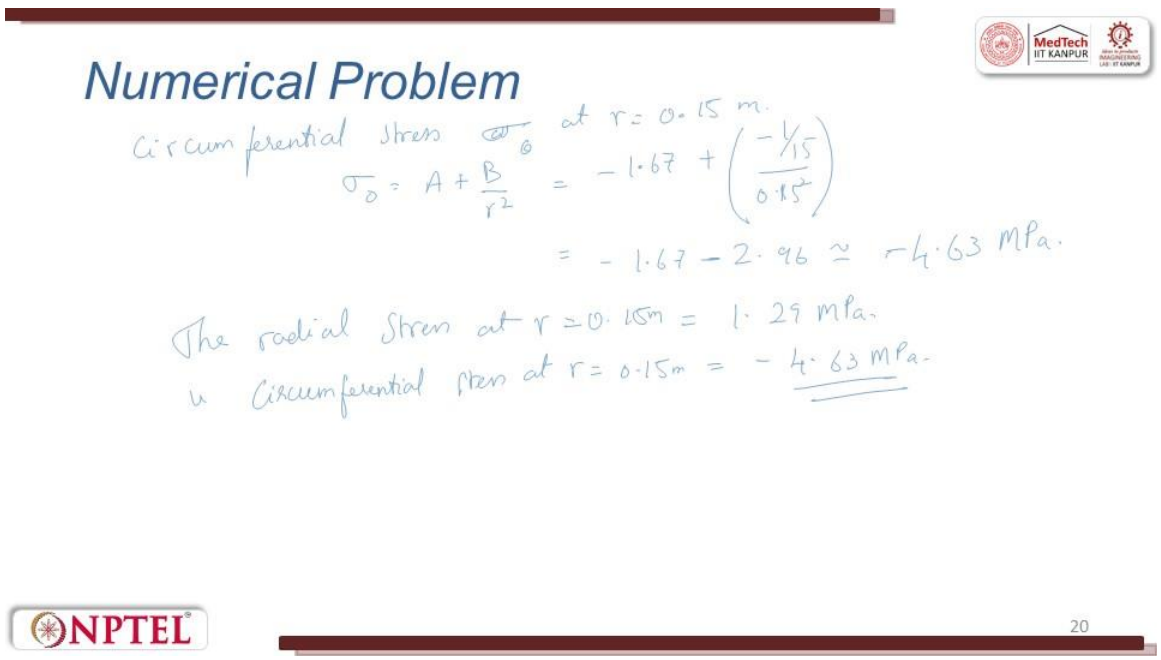
$25B - 100B = 5$

$-75B = 5 \quad B = -1/15$

$A = 25 \times -1/15 = -5/3 = -1.67\,MPa$

Calculating Radial stress (σ_r) at $r = 0.15\,m$

$\sigma_r = A - B/r^2$; $-1.67 - \left(\frac{-1/15}{0.15^2}\right) = -1.67 + 2.96 = 1.29\,MPa$



Now let us try to solve a numerical using the Thick-Wall Cylinder equation. So, calculate the radial and the circumferential stress for a thick-walled cylinder whose internal pressure Pi is given, external pressure P0 is given. Then we have inner radius 0.1, outer radius 0.2, the radial distance is 0.15. So, what we are trying to say is this is inner, this is outer. This is again outer radius and the radial distance is r, right. So, that is what is 0.15 point where stresses to be calculated. So, what are we supposed to do?

Determine constants A and B using boundary conditions:

O at r = R_i (inner radius) $\sigma_r = P_i$

$A - B / R_i^2 = P_i$ → $A - B / (0.1)^2 = 5$ Mpa

A – 100 B = 5 …..(1)

$\sigma_r = -1.67 + \frac{1}{15 \times 0.0225}$

$\sigma_r = -1.67 + \frac{1}{0.3375} = -1.675 + 2.96 = 1.29$ Mpa (Ans.)

3. Calculate circumference stress (σ_o) at r = 0.15 m

Formula = $\sigma_o = A - \frac{B}{r^2}$

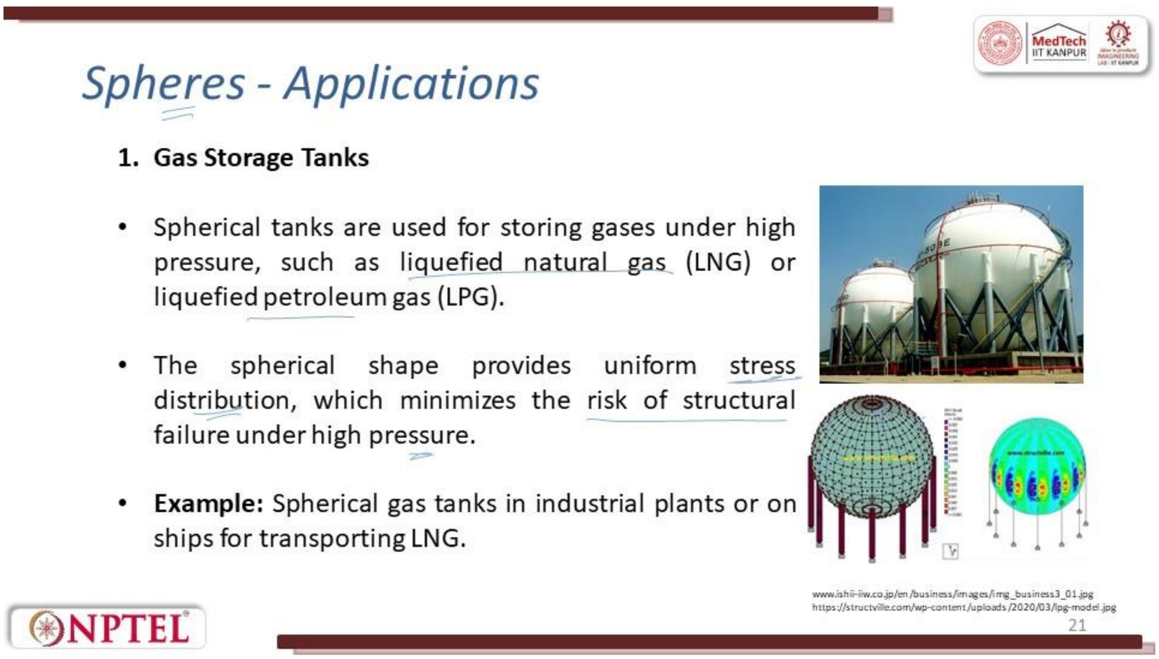
$\sigma_o = -1.67 + \frac{-1/15}{(0.15)^2}$

$\sigma_o = -1.67 - \frac{1}{0.3375} = -1.675 - 2.96 = -4.63$ Mpa (Ans.)

Radial Stress (σr) at r = 0.15 m; = 1.29 Mpa

Circumference stress (σ_o) at r = 0.15 m; -4.63 Mpa

We are trying to find out the constants and we are trying to solve the problem.

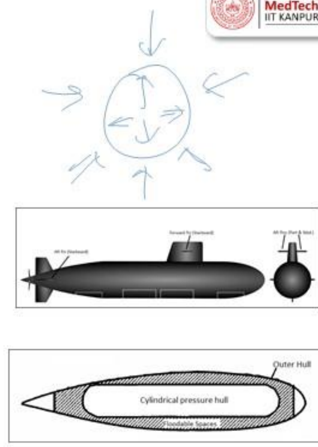


Now, let us move into Spheres application. Spheres application is predominantly found out in gas storage tanks. The spherical tanks are used for storing gas under high pressure such as Liquefied Nitrogen Gas (LNG) or Liquefied Petroleum Gas (LPG). So, you can see there all these gas storing tanks will be spherical. So, it will be rested on cylinders and then you form a sphere. The spherical shape provides Uniform Stress Distribution. Uniform stress distribution along all the directions which minimize the risk of structural failure under high pressure. The examples we have already seen used in plants.

Spheres - Applications

2. Submarine Hulls

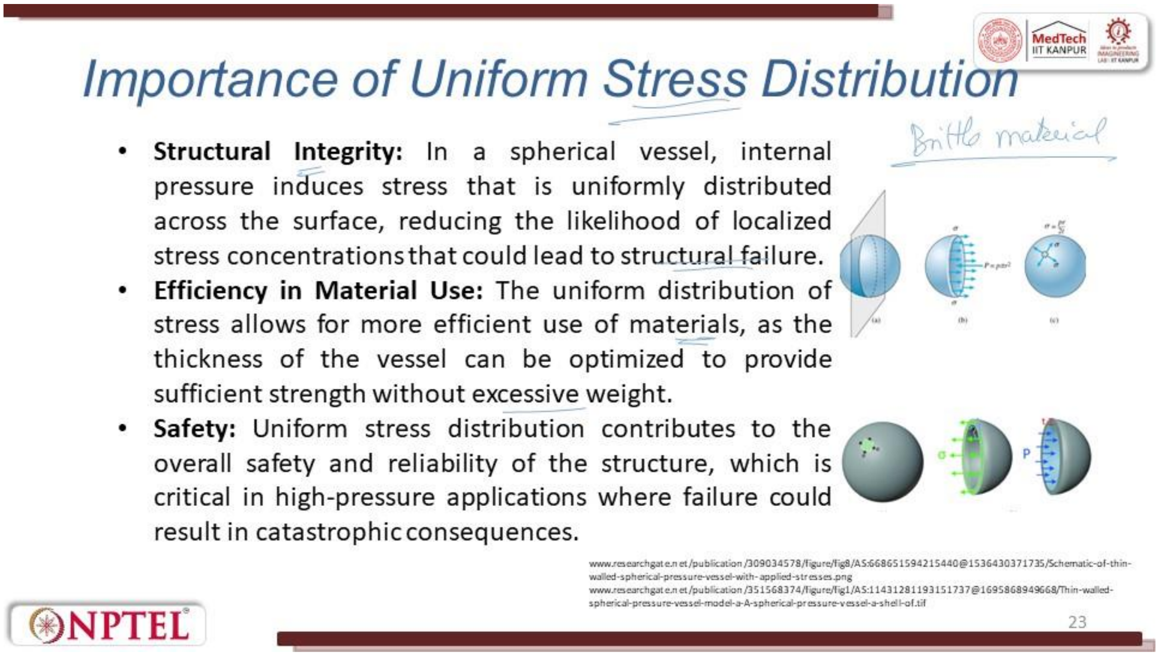
- The hulls of submarines are often designed with spherical sections to withstand the immense pressure exerted by deep-sea environments.
- Uniform stress distribution in spherical sections of the hull enhances the submarine's ability to endure deep-sea pressures without deformation or failure.
- **Example:** Pressure-resistant compartments in deep-diving submarines.



www.marineinsight.com/wp-content/uploads/2016/10/sub8.png
www.marineinsight.com/wp-content/uploads/2016/10/sub2.png

When we talk about other applications submarine hull submarine hull is also a spherical one the hull of a submarine are often designed with a spherical section to withstand the immense pressure exerted under deep sea. The Uniform Stress Distribution in spherical sections of the hull enhances the submarine's ability to endure deep sea pressure because in sea, hydrostatic pressure there will be uniform pressure around. Now, you have to maintain this pressure. You are going inside water. Pressures are going very high.

With respect to height or depth, if you go, the pressures are very high. Then you will have to counter this fellow. Otherwise, what will happen? It will get deformed. So, spheres are very important to be considered when you are trying to talk or think about underwater. So, the pressure resistance compartment in the deep diving submarines are always used.

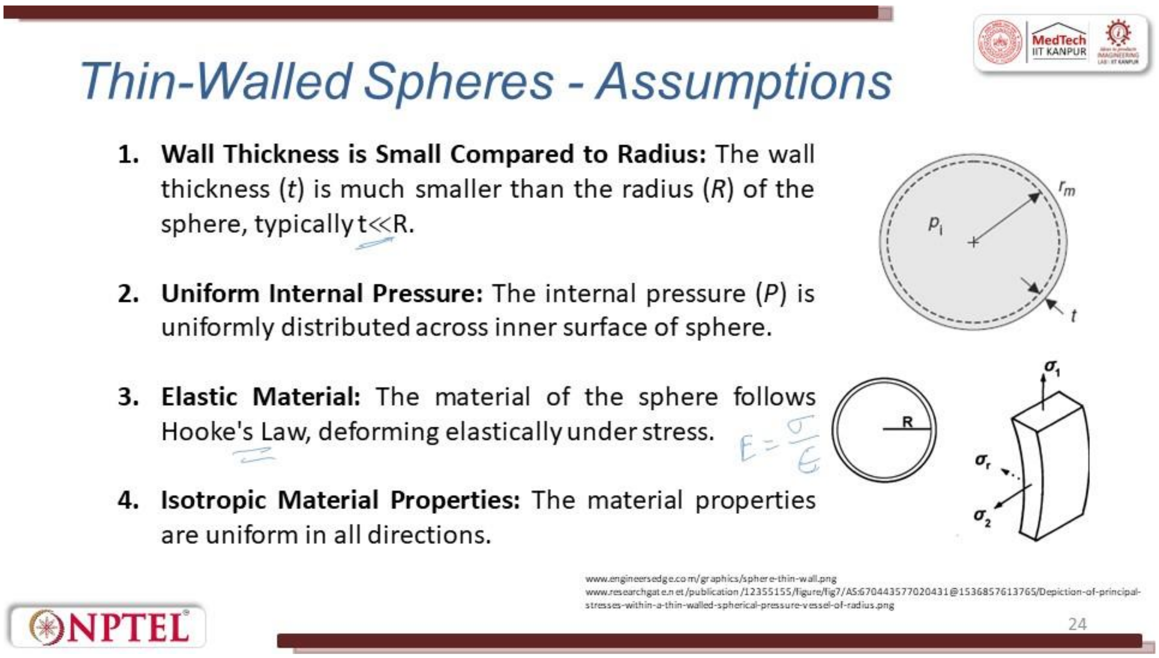


The importance of Uniform Stress Distribution, this always helps to maintain the structural integrity. In a spherical vessel, internal pressure induces stress that is uniformly distributed across the surface reducing the likelihood of localized stress concentration that could lead to structural failures. Many a times when we start doing machining of a brittle material.

Machining of a brittle material like glass, so what we do is we always try to take it inside a chamber where in which there is Uniform Stress Distribution around the piece. So, there will be pressure at this point of the work piece. When I try to machine what will happen is it will try to convert the brittle fracture into ductile fracture. So, when you are trying to work on silicon machining. By chance you are trying to work on a single point cutting machine, then it always depends on that stress distribution.

We are establishing a new facility at IIT Kanpur on 'Hydrostatic machining operations'. So, wherein which the cylinder will be made, inside a cylinder pressure will be exerted, the workpiece will be kept inside the cylinder such that we get Uniform Stress Distribution. The efficiency in material use, the uniform distribution of stress allows for more efficient use of material such as the thickness of the vessel can be optimized to provide sufficient strength without excess weight. So, you do not have to put a thick cylinder rather than that we put a thin cylinder and we apply pressures uniform. Then the next one is Safety; the Uniform Stress Distribution contributes to the overall safety and reliability of the structure.

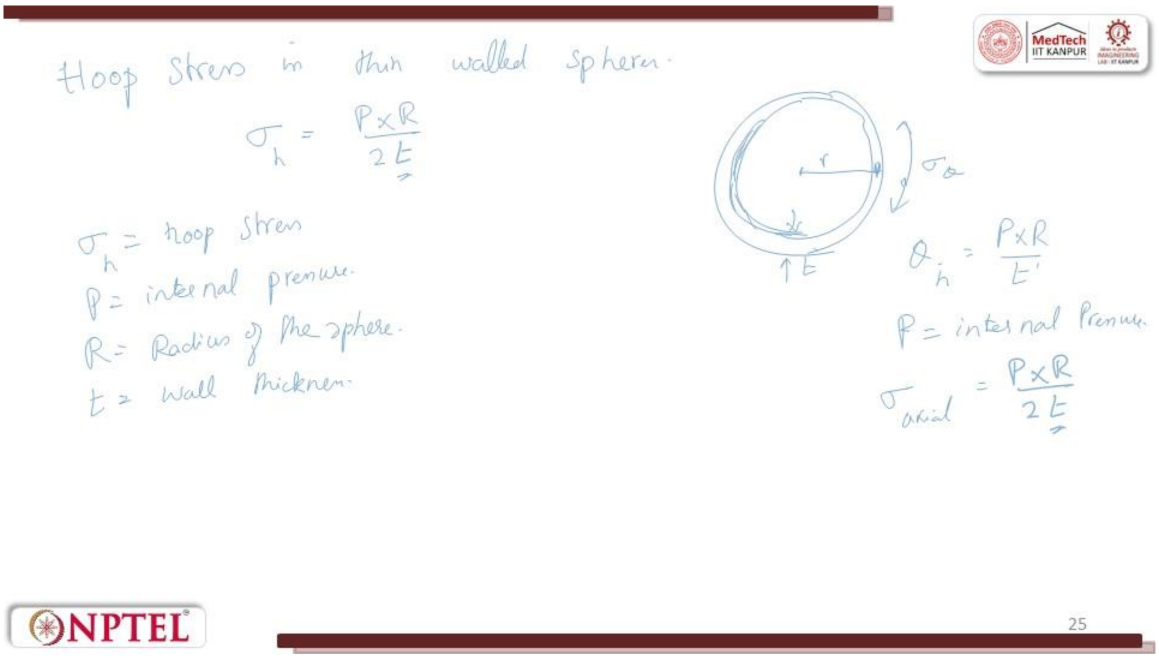
So, for structural integrity, for efficiently using material and for safety, we always try to do this thin wall and thick wall calculations and we try to maintain Uniform Stress Distribution. Again, in sphere, you have thin wall and thick wall.



So thin-walled spheres, the wall thickness is small compared to the radius. The wall thickness t is smaller than the radius R of the sphere. It has to be extremely small as compared to that of the radius.

Uniform Internal Pressure. The internal pressure P is uniformly distributed across the inner surface sphere. So, when we try to take a section, you will see here, this is σ1, σ2, σr. The elastic material, the assumption, elastic material of the sphere follows the Hooke's law. So, Hooke's law, what is Hooke's law? Hooke's law is, we can try to stress by strain.

This is strain. The isotropic material property, the material property are uniform in all directions. When we try to use a homogeneous material, we always try to simplify the calculations by using isotropic material property.



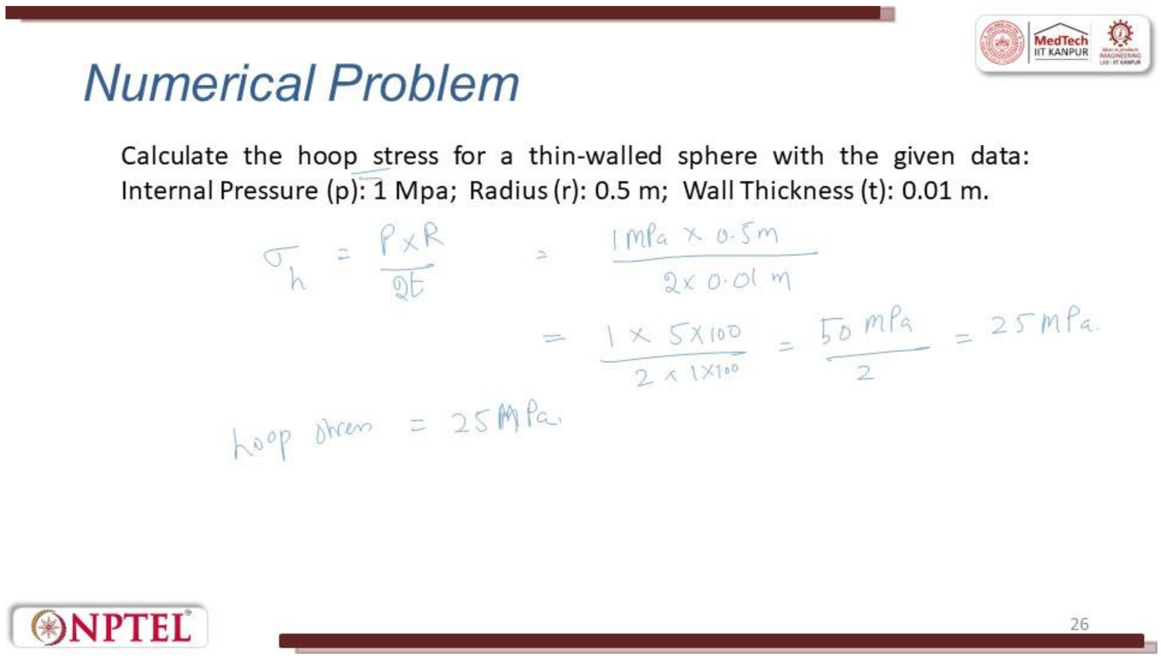
So, let us try to derive or write down the equation for Hoop stress or the Circumferential stress in thin-walled spheres. So, it is expressed as Hoop stress is

$$\sigma_h = \frac{P \cdot R}{2t}$$

- σ_h: Hoop stress, or circumferential stress, which acts along the surface of the sphere.
- P: Internal pressure within the sphere.
- R: Radius of the sphere.
- t: Wall thickness of the sphere.

The formula for hoop stress in a thin-walled sphere is derived under the assumption that the wall thickness is small compared to the radius, leading to a simplified stress distribution where the hoop stress is considered uniform.

Now, the formula of the Hoop stress in the thin wall sphere is derived under the assumption that the wall thickness is extremely small as compared to that of the thickness.



So now let us try to calculate the hoop stress for a thin-walled sphere with the given data: Internal Pressure (p): 1 Mpa; Radius (r): 0.5 m; Wall Thickness (t): 0.01 m.

Hoop stress

$$\sigma_h = \frac{P \times R}{2t}$$

$$\sigma_h = \frac{1\ MPa \times 0.5\ m}{2 \times 0.01 m}$$

$$= \frac{1 \times 5 \times 100\ m}{2 \times 1 \times 100} = \frac{50}{2} = 25\text{MPa}$$

Hoop stress = 25 MPa

Thick-Walled Spheres - Assumptions

1. **Non-Negligible Wall Thickness:** The wall thickness is significant compared to the radius of the sphere, making it important to account for in stress calculations.
2. **Elastic Material:** The material of the sphere behaves elastically, following Hooke's Law.
3. **Uniform Material Properties:** The material is homogeneous and isotropic, meaning its properties are consistent throughout.
4. **Internal and External Pressures:** The sphere is subjected to both internal pressure (P_i) and possibly external pressure (P_o).
5. **Spherical Symmetry:** The stress distribution is assumed to be spherically symmetric due to the geometric symmetry of the sphere.

So now moving forward, what is the assumptions for a thick-walled sphere? The assumptions are non-negligible wall thickness. The wall thickness is significantly compared to the radius of the sphere, making it important to account for in-stress calculation. Next is it follows Hooke's law. It has uniform material property. The internal pressure and the external pressure, the sphere is subjected to internal pressure as well as external pressure. The symmetry is also followed.

Thick-Walled Spheres

Radial Stress (σr) $= \frac{P_i R_i^3 - P_o R_o^3}{R^3{}_i - R_i^3} \cdot \frac{R_o^3}{r^3} - \frac{P_o R_o^3 - PiR_i^3}{R_0^3 - R_i^3} \cdot \frac{Ri^3}{r^3}$

Where:

- R_i and R_o are the inner and outer radii of the sphere.
- r is the radial distance from the center.

2. Circumferential Stress (Hoop Stress, σ_θ)

$$= \frac{P_i R_i^3 - P_o R_o^3}{R^3{}_i - R_i^3} \cdot \frac{R_i^3}{r^3} + \frac{P_o R_o^3 - PiR_i^3}{R_0^3 - R_i^3} \cdot \frac{Ro^3}{r^3}$$

Where:

- P_i is the internal pressure.
- P_o is the external pressure.

So, when we try to calculate the thick-walled sphere radial stress, it is the same calculation like whatever we did like last time

$$\boldsymbol{\sigma_r} = \frac{P_i R_i^3 - P_o R_o^3}{R3i - R_i^3} \cdot \frac{R_o^3}{r^3} + \frac{P_o R_o^3 - PiR_i^3}{R_0^3 - R_i^3} \cdot \frac{Ri^3}{r^3}$$

Where,

- R_i and R_o are the inner and outer radii of the sphere.
- r is the radial distance from the center.

When we calculate the Circumferential strength,

$$\boldsymbol{\sigma_\theta} = \frac{P_i R_i^3 - P_o R_o^3}{R3i - R_i^3} \cdot \frac{R_i^3}{r^3} + \frac{P_o R_o^3 - PiR_i^3}{R_0^3 - R_i^3} \cdot \frac{Ro^3}{r^3}$$

Where,

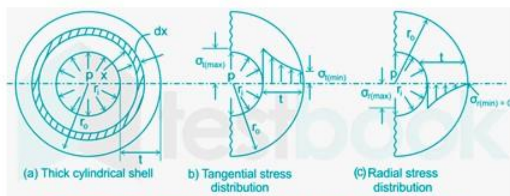
- P_i is the internal pressure.
- P_o is the external pressure.

Stress Distribution in Thick-Walled Spheres

Radial Stress (σr) = $\dfrac{PiR_{\mathrm{i}}^3 - P_o\, R_o^3}{R_0^3 - R_i^3} \cdot (\dfrac{Ro^3}{r^3} - 1)$

Where:

- P_i: Internal pressure.
- P_o: External pressure.
- R_i: Inner radius.
- R_o: Outer radius.
- r: Radial distance from the center.



Stress distribution in thick cylindrical shells subjected to internal pressure

https://storage.googleapis.com/tb-img/production/20/04/F1_M.J_Madhu_04.04.20_D4.png

So, the stress distribution in a thick wall cylinder can be calculated by the Radial stress is nothing but

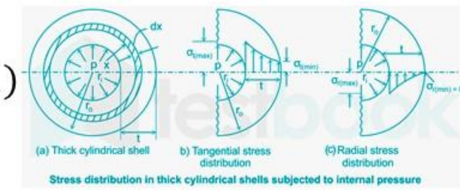
$$\sigma_r = \frac{PiR_{\mathrm{i}}^3 - P_o R_o^3}{R_0^3 - R_i^3} \cdot (\frac{Ro^3}{r^3} - 1)$$

The difference you should see, it is -1, in hoop, it becomes + 1, rest all it is the same.

Stress Distribution in Thick-Walled Spheres

Circumferential Stress (Hoop Stress, σ_θ)

$$= \frac{P_i R_i^3 - P_O R_O^3}{R_0^3 - R_i^3} \cdot \left(\frac{R_o^3}{r^3} + 1 \right)$$



Stress distribution in thick cylindrical shells subjected to internal pressure

Explanation:

- The **radial stress (σ_r)** decreases from the internal surface to the external surface of the sphere. It is highest at the inner radius and reduces to the external pressure at the outer radius.
- The **circumferential stress (σ_θ)** is typically higher than the radial stress and reaches its maximum at the inner surface of the sphere. This stress is a result of the internal pressure trying to expand the sphere.

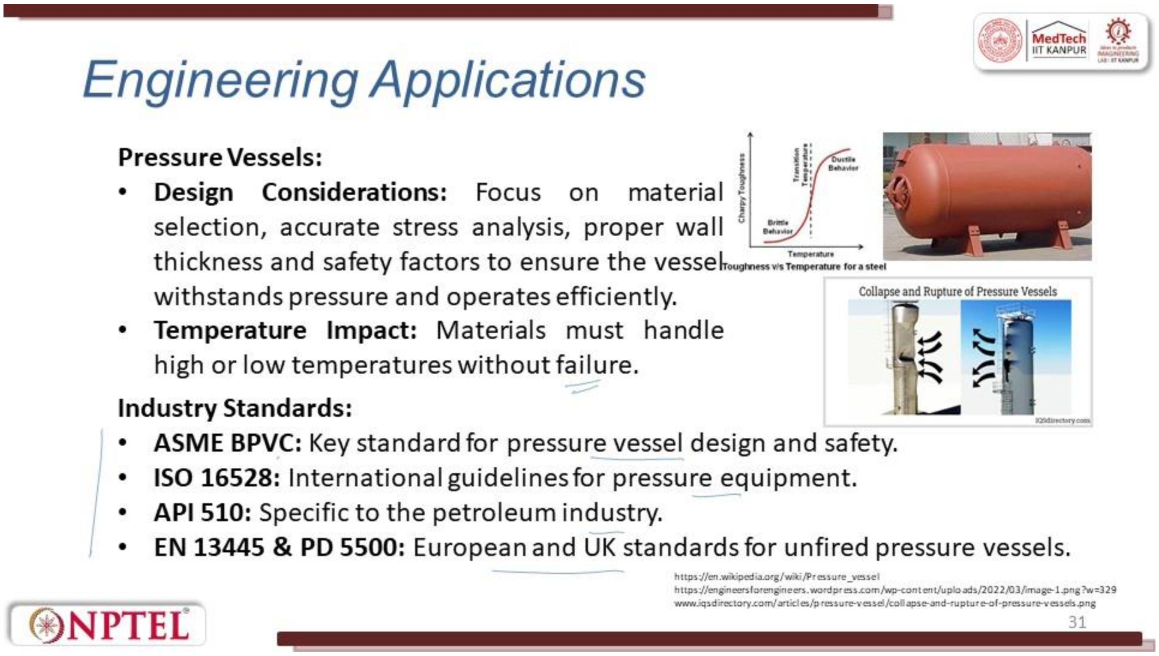
https://storage.googleapis.com/tb-img/production/20/04/F1_M.J_Madhu_04.04.20_D4.png

When we talk about hoop, it is same. The radial stress decreases with the internal surface to the external surface of the sphere. It is highest at the internal sphere.

So, that is why we do a calculation. It always be representative. So, the stress at this point is higher as compared to this point. So, this is inner and this is outer. Here also you can represent it in an exponential function.

Inner surface to the external surface of the sphere. It is highest at the inner radius and reduces to the external pressure at the outer radius. The circumferential. So, it is interesting in a sphere radius is 1, circumference is 1. So let us be little careful when we do the circumferential stress is typically higher than the radial stress and reaches its maximum value at the inner surface of the sphere.

This stress is a result of internal pressure trying to expand the sphere. So, you have circumferential stress, radial stress. This is how you calculate the radial stress. This is how you calculate the circumferential stress.



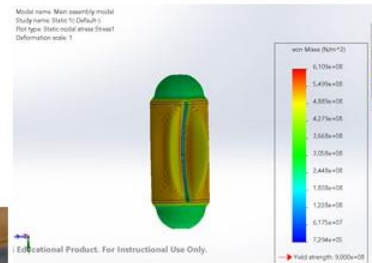
So where is it exhaustively used? We use it in pressure vessels for in the design consideration. It is focused on material selection, accurate stress analysis, proper wall thickness and factor of safety to ensure the vessel withstands the pressure and operating efficiency. The temperature impact is also high. The material must handle high or low temperature without failure. There are several industrial standards which are there.

ASME BPVC is a standard which is used for pressure vessel design. ISO has a pressure vessel design equipments. It is separate. Then API is specific for petroleum industry. EN 13445 and PD 5500 is the European and UK standards for unfired pressure vessels.

Case Study Example

Failed Pressure Vessel Analysis:

- A pressure vessel used in a chemical plant ruptured due to unexpected internal pressure buildup. Post-failure analysis revealed that the wall thickness was insufficient to handle the peak pressure during operation.

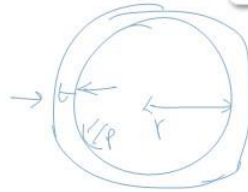


https://preview.redd.it/very-new-to-fea-im-trying-to-do-a-simple-static-structural-v0-vu8ll611e2ya1.png?width=1057&format=png&auto=webp&s=663f00a4d92e00125f2ffd5e498bf34a7687a15d
www.redriver.team/wp-content/uploads/elementor/thumbs/image004-1-1-qf3d49dh9iy16tcl8qpembclhb7dnf7w0abgimxrao.webp

So let us take a case study. The case study for our discussion will be a pressure vessel used in a chemical plant ruptured due to unexpected internal pressure builds up. The post-failure analysis reveals that the wall thickness was insufficient to handle the peak pressure during operation. So same way when you take a balloon, you keep on blowing a balloon. After a certain pressure, what happens is the internal pressure is quite high.

The wall thickness is very small. It pushes the wall thickness. At one point of time, the balloon bursts. So, in the same way, this is a common failure which we always see in pressure vessels. So, the internal pressures are very high, the wall thickness is very low.

Case Study Example



Lessons Learnt:

Importance of Accurate Stress Calculations:

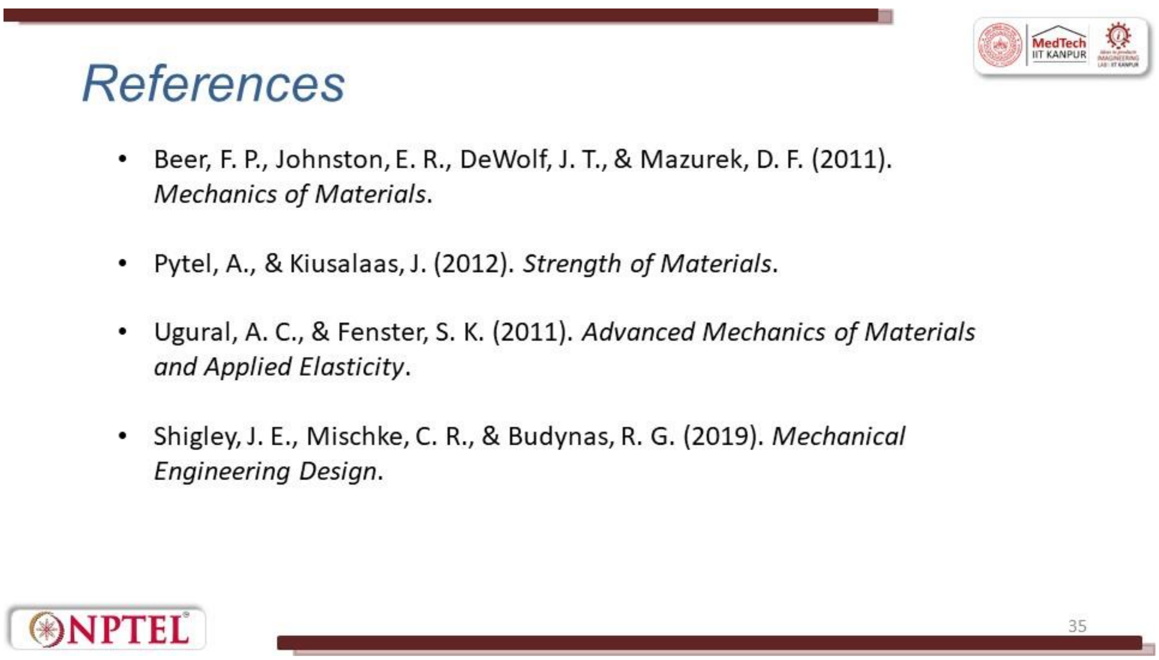
- The failure highlighted the critical need for precise stress analysis and appropriate safety factors in the design phase. Inaccurate calculations or assumptions can lead to catastrophic failure, emphasizing the importance of adhering to industry standards and rigorous testing protocols.

So, the lesson learnt from this case study is the failure highlighted the critical need for precise stress analysis and appropriate safety factor in the design phase. Inaccurate calculations or assumptions can lead to catastrophic failure. Emphasizing the importance of adhering to industrial standards and rigorous testing protocol. So predominantly what happens when we talk about a sphere, we try to talk about the radius and the wall thickness and the wall thickness. This two is very important t and here the internal stresses whatever it is getting built.

To Recapitulate

- What is the primary purpose of studying stress in cylinders & spheres in engg.?
- How do you differentiate between thin-walled and thick-walled cylinders?
- What is the formula for hoop stress in a thin-walled cylinder, and what are the parameters involved?
- How does longitudinal stress in a thin-walled cylinder differ from hoop stress?
- What are Lame's equations for thick-walled cylinders and how are they derived?
- Provide a numerical example of calculating hoop stress in a thin-walled cylinder.
- Describe the stress distribution in thick-walled spheres.
- Provide a numerical example of calculating radial and circumferential stress in a thick-walled sphere.
- Explain a real-world case study involving the failure of a pressure vessel.

To recap whatever, we have seen in this lecture, what is the primary purpose of studying stress in cylinders and spheres? How do you differentiate between thin wall and thick wall cylinders? What are the formulas used to calculate the hoop stress in thin wall and thick wall? What are the important parameters? How does longitudinal stress in a thin wall cylinder differ from a hoop stress? What are Lame's equation for thick wall cylinders? We underwent a numerical study. Then we saw the stress distribution for a thick-walled sphere. Then a numerical problem was solved. Finally, we saw a real case study wherein which failure has happened in a pressure vessel. The importance of internal pressure getting developed, wall thickness are very important when we try to work on either cylinders or spheres.



These are the references which we have used while preparing for this lecture.

Thank you very much.